ANALYSIS OF DISTRIBUTED RESOURCES POTENTIAL IMPACTS ON ELECTRIC SYSTEM EFFICACY

By

Paul Robinson

A Thesis

Submitted to the Faculty

of the

WORCESTER POLYTECHNIC INSTITUTE

In partial fulfillment of the requirements for the

Degree of Master of Science

in

Electrical and Computer Engineering

By

____

December 2009

Approved by:

____

Professor Alexander E. Emanuel, Major Advisor

____

Slobodan Pajic, Review Committee

Abstract

The intent of this Thesis is to study the potential of distributed resources to increase the efficacy of the electric system without decreasing the efficiency of the system. Distributed resources (DR) are technologies that provide an increase in power or a decrease in load on the distribution system. An example of DR is a storage device that uses electricity during low use periods to store energy and then converts the stored energy to power during high use periods.

The energy storage being studied is for the purpose of peak shaving or the ability to shift small amounts of load to a more optimum time. In particular the concept of load curve leveling is explored. DR options are studied to determine how size, location, and storage losses impact the overall system efficacy and efficiency. This includes impacts on system losses, capacity utilization, and energy costs.

Acknowledgements

I would like to thank my wife Mara and my children Kyle and Marissa for their support and understanding during this endeavor. Also, I would like to thank Prof. A. Emanuel and other WPI faculty for their guidance and encouragement throughout this process. Lastly, I would like to thank National Grid for the educational reimbursement program that made funding this research possible.

Contents

1 Introduction

The intent of this Thesis is to study the potential of distributed resources to increase the efficacy of the electric system, while not decreasing the efficiency of the system. The system should have enough capacity to reliably supply the varying demand for electricity 100% of the time, with the least amount of equipment and energy input. Having higher efficacy may not be desirable if the system efficiency is lowered as a result. The ability to decrease system losses during peak periods to offset increased losses from energy storage used to increase the efficacy of the system will be explored.

For the purposes of this analysis, the efficacy of the system can be determined by the capacity utilization factor. The capacity utilization factor is the capacity factor times the load factor. The load factor is the ratio of the average load over a designated time period to the peak load occurring during that time period. The capacity factor is the ratio of the total energy served over a designated time period to the energy that would have been served if the system had operated continuously at its maximum rating.

To provide some context on what distributed resources are, the following is a simplified overview of the major topology categories of the electric system, as shown in Figure 1.1. First, there is generation which uses some form of energy to produce electricity. Next, there is transmission which delivers electricity at high voltage from the large generators to the rest of the system. Then, there is distribution which is connected to transmission and supplies the electricity to the consumer. The amount of electricity required by the consumer is also referred to as the electric load or demand. To satisfy electric demand the electricity must be produced by the generators and delivered to the load when it is needed.

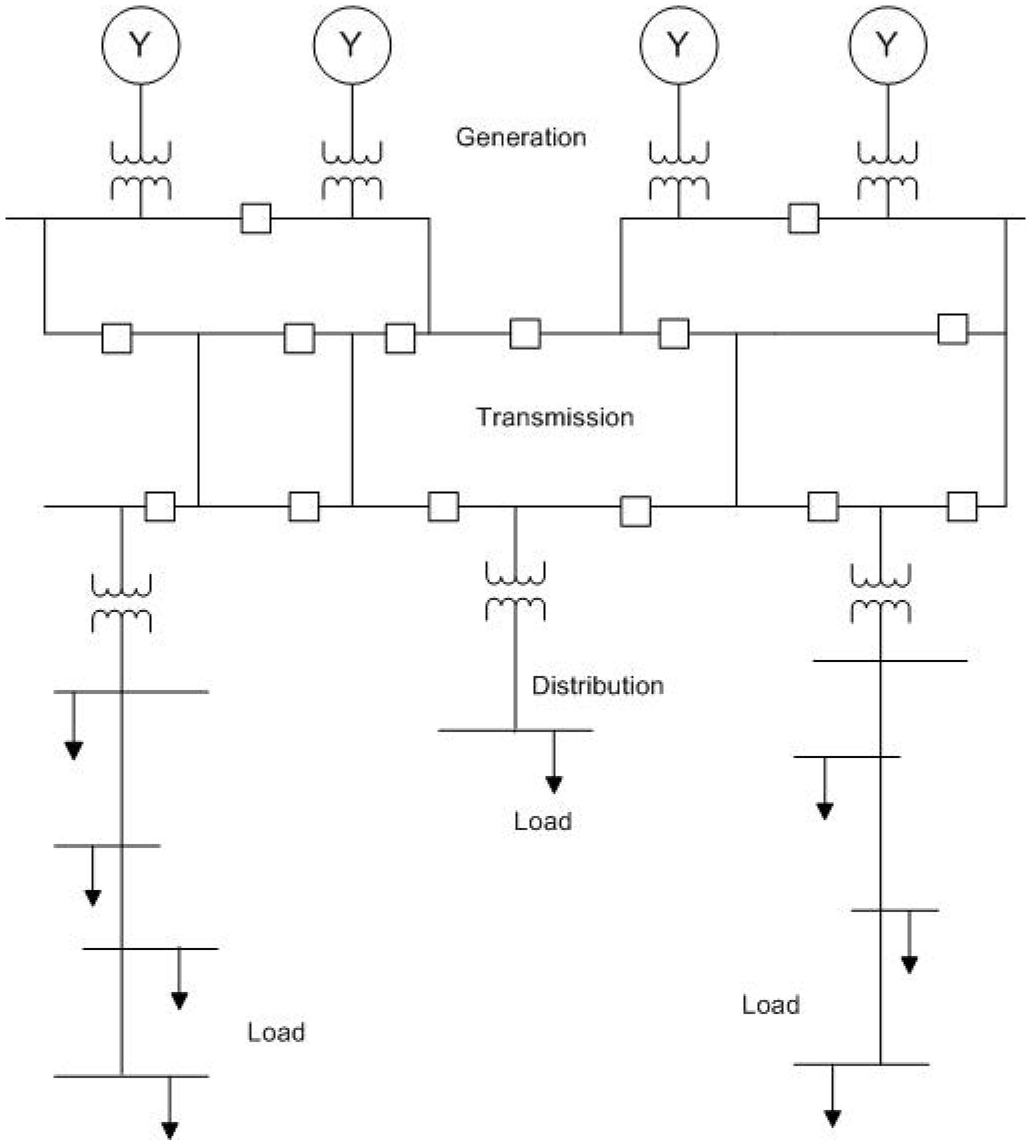

Figure 1.1 – Diagram of electric system topology

Most of the energy generated in the system is provided by large units, (hundreds of MWs), connected to the transmission system. The concept of also having multiple small generation sources distributed along the distribution system has been referred to as distributed generation (DG). If energy storage devices are included with DG it has been known as dispersed storage and generation (DSG). If other technologies to provide an increase in power or a decrease in load are also included, then it is being collectively referred to as distributed resources (DR).

Inherent in the system is a varying level of power being produced and delivered as the amount of load changes. In other words, the electricity needed by the system increases as electric consuming devices are turned on, and conversely, the amount of electricity decreases as electric consuming devices are turned off. For the system to be stable there must be a match between the amount of electricity being consumed, including losses, and the amount of electricity being produced. This leads to a system that must have the capacity to meet the highest level of demand even if that level is only reached for a short time. Also, the amount of electric loss in the system is greater at the higher load levels then it is at lower load levels.

For most of its history, the electric system was designed to reliably supply the load requirements of the electricity consumers. This has led to a system that is built to have the capacity necessary to satisfy the highest, or peak, amount of demand even if that demand is only reached for a few hours. Parts of the system can be strained to their capacity limits for a short time, but they may sit idle or under utilized for the majority of time. The amount of load usually varies by time of day, day of week, and weather conditions. A plot of the amount of electric load on the system over time is called a load curve. An example load curve for one day normalized to the peak demand is shown in Figure 1.2.

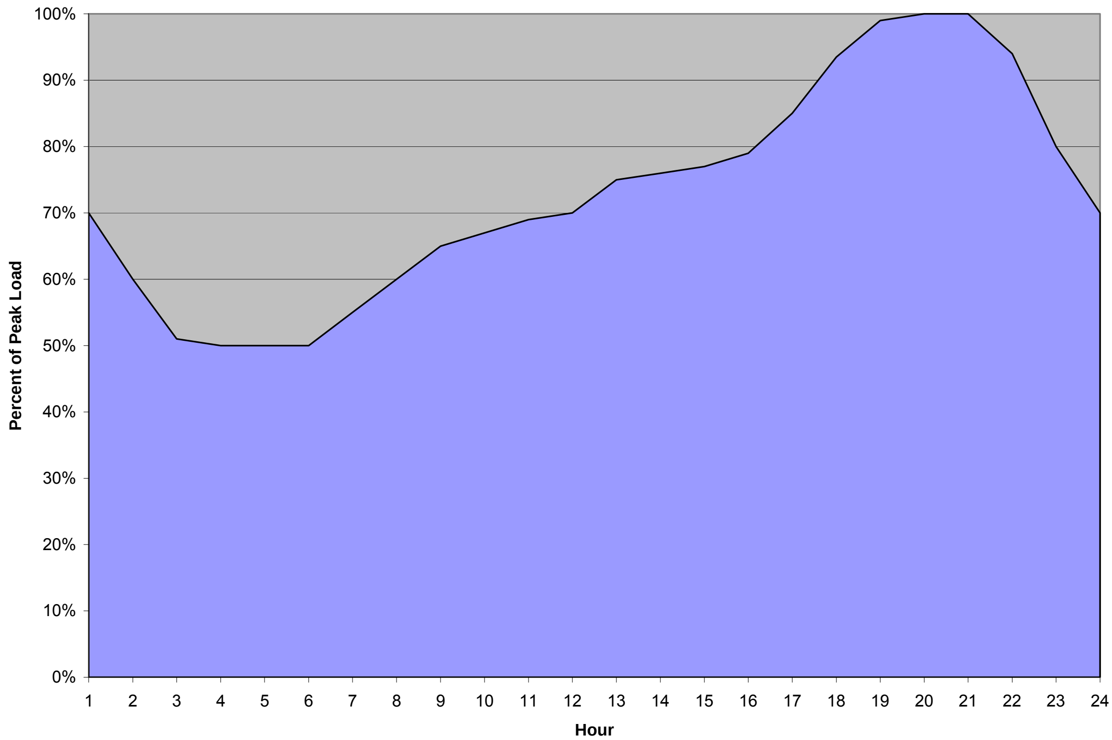

Figure 1.2 – Example of a 24 hour Load Curve

From an asset utilization view, it would be desirable to have a more constant level of load. Reducing the peak amount of load and shifting it to times of lower load would flatten the load curve. The total amount of energy delivered would be the same but the capacity utilization factor would decrease. The result is the ability to deliver the same amount of energy with less or smaller capacity infrastructure. It could also allow for more energy to be delivered without increasing the capacity infrastructure. An example of load shifting is shown in Figure 1.3.

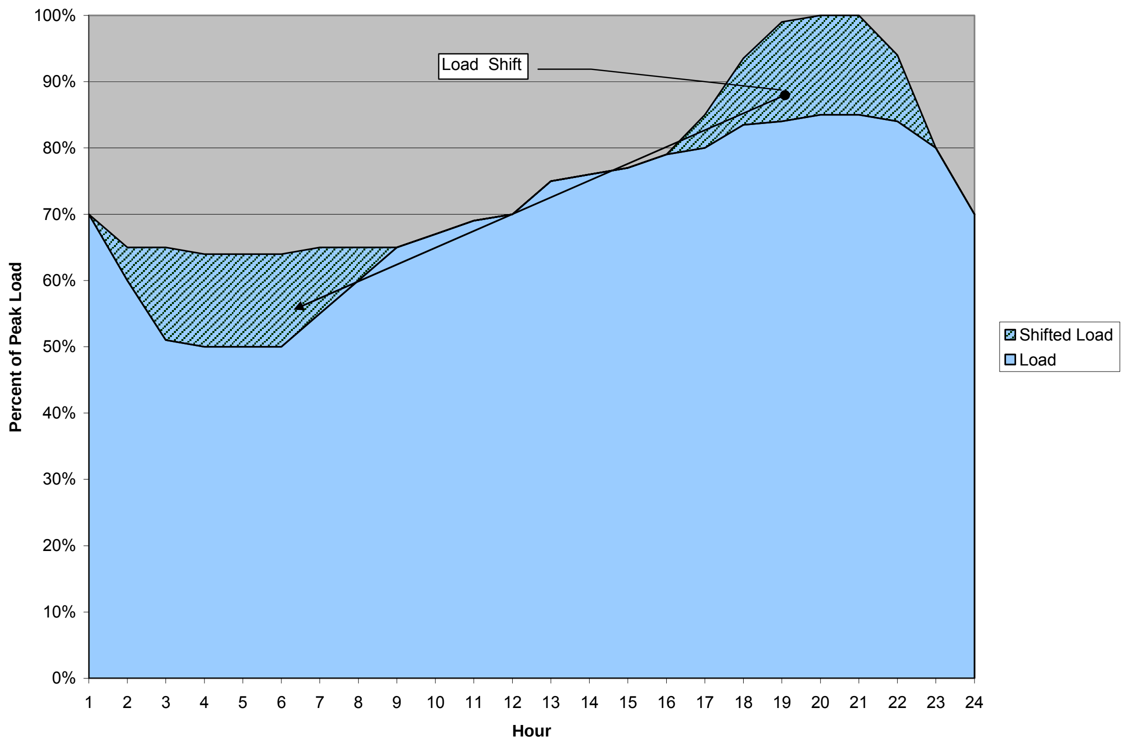

Figure 1.3 – Example of Load Shifting

There are a few factors in the electric system that makes load shifting or peak shaving advantageous. One factor is that usually not all generation has the same efficiency or costs. If load can be shifted to a time when the generation is more efficient then the overall efficiency is improved. Another factor is that the delivery system needs to be sized so the highest level or peak power usage can be delivered. Infrastructure must be built to handle the highest load levels, so if this level is only reached occasionally then its capacity is being under utilized.

A more subtle factor is that system losses are proportional to the current squared. An electric system with twice as much current has four times as much loss. If the load cycle for a 24 hour period is examined it is typical that there is a curve that has valleys and peaks. If the overall energy load was spread out with same amount being used each hour then the curve would be a straight line. The amount of energy used is the same for the load cycle with the valleys and peaks as for the straight line cycle. However, the system loss is higher for the cycle with valleys and peaks. This is due to the nonlinear relation between the power loss and current.

From and electric system standpoint there are several benefits to having DR: 1) it can provide voltage support and reduce system losses by having the supply source closer to the load, 2) the ability to reduce system peaks which reduces capacity needs, and 3) the ability to maximize the use of the most efficient supplies through storage. These features can help lead to a more efficacious system. To maximize the benefit obtained from these resources requires the integration of DR into the system. The size and location of the DR has an impact on the overall efficacy of the system.

An example of DR is a storage device that uses electricity during low use periods to store energy and then converts the stored energy to power during high use periods. This can be accomplished by such techniques as charging batteries, pumping water, compressing air or creating hydrogen from electrolysis. For the purpose of this analysis, DR consists of resources of less than 10 MW and usually around 1 MW on the distribution system.

The energy storage being studied is for the purpose of peak shaving or the ability to shift small amounts of load to a more optimum time. Usually the shift is from a period of high electric use or system constrained time to a lower use period. Energy storage options are studied to determine how storage losses compare to system loss reduction. In particular the concept of load curve leveling is explored.

The DR generation being studied could be from co-generation or from renewable resources such as wind and solar. In the current environment the ability to utilize renewable energy sources has taken on greater importance than in the past. The integration of small generation sources onto the electric distribution system poses a number of technical implications. These can range from system stability and control to protection schemes and metering. This analysis will focus on the impact that a distributed resource can have on capacity utilization and system efficacy.

Most of the focus of energy efficiency efforts has been on having devices use less energy. A device is more efficient if it uses less energy to produce work than a device that uses more energy to produce the same amount of work. The amount of energy is usually measured at the terminals of the device and does not include the system that provides the energy. The reference point can be broadened beyond the terminal of the device to also consider the system that provides the energy. If we think of the consuming devices as part of the system then it can change how we view efficiency and the supply and demand relationship.

One area of interest is the idea of distributed end user device storage to flatten the load curve. Local storage could allow for charging during low use periods and drain during higher use periods. If devices that utilize AC/DC power supplies could also store small amounts of energy then additional charging losses would be minimized. The stored energy of hundreds of thousands of devices could be utilized randomly or controlled over the projected peak period to reduce it. The consumer would get the benefit of the work produced by the device during the peak period and the system would benefit from the reduced peak load.

2 Background

The AC electric system in the United States can be traced back to 1885 in Great Barrington Massachusetts. George Westinghouse had bought American patents covering the AC transmission system developed by L. Gaulard and J. D. Gibbs of Paris. An associate of Westinghouse, William Stanley tested single phase transformers in his laboratory and supplied 150 lamps in the town. In 1888, Nikola Tesla presented a paper on two-phase induction and synchronous motors. In 1893 Tesla demonstrated a two-phase AC distribution system at the Columbian Exposition in Chicago. The advantages of polyphase AC and especially three-phase AC over DC became apparent. By January 1894 there were five polyphase generating plants in the United States. [1]

The ability to step-up or step-down the voltages with the transformer was the biggest reason that AC was adopted over DC early on in the industry. The ability to transmit electricity at high voltages allows for transmission lines to deliver more power than at lower voltages. This allowed for large amounts of power to be transmitted at high voltages and then stepped-down to lower voltages for use.

The modern transmission system is operated at very high voltage levels in the range of 115 kV – 765kV. The system is configured as a network with multiple connection points, so the loss of one element in the system will not interrupt the supply. Large generating facilities in the range of 100 MW – 1200 MW are connected to the transmission system.

Distribution systems are usually operated at voltage levels in the range of 13kV – 34 kV. Some distribution systems in cities are connected in a network but most distribution systems are radial with one normal supply point and backup connections that are normally open. The distribution system is supplied from substations that have the bulk power transformers that are supplied at a transmission voltage level and then supply the distribution system at a lower distribution voltage level. The distribution system is used to supply point of use transformers, or distribution transformers, to convert the voltage to levels used by consumers. The voltages used by consumers are usually 120/240 V or 120/208 V for residential and small commercial users and 277/480V three phase for larger commercial users.

The demand for electricity has continued to grow throughout the history of the electric system. The system has been built to have adequate capacity to meet the peak demand of the load. The total net summer generation capacity in the U.S. from 1971 to 2007 is shown in Figure 2.1.[2] The increase in capacity is not only due to population growth it is also due to increasing usage per person. In 1982 the average power usage per person in the U.S. was 1.1 kW [3], in 1996 it was 1.3 kW [4], and in 2007 it was 1.6 kW.

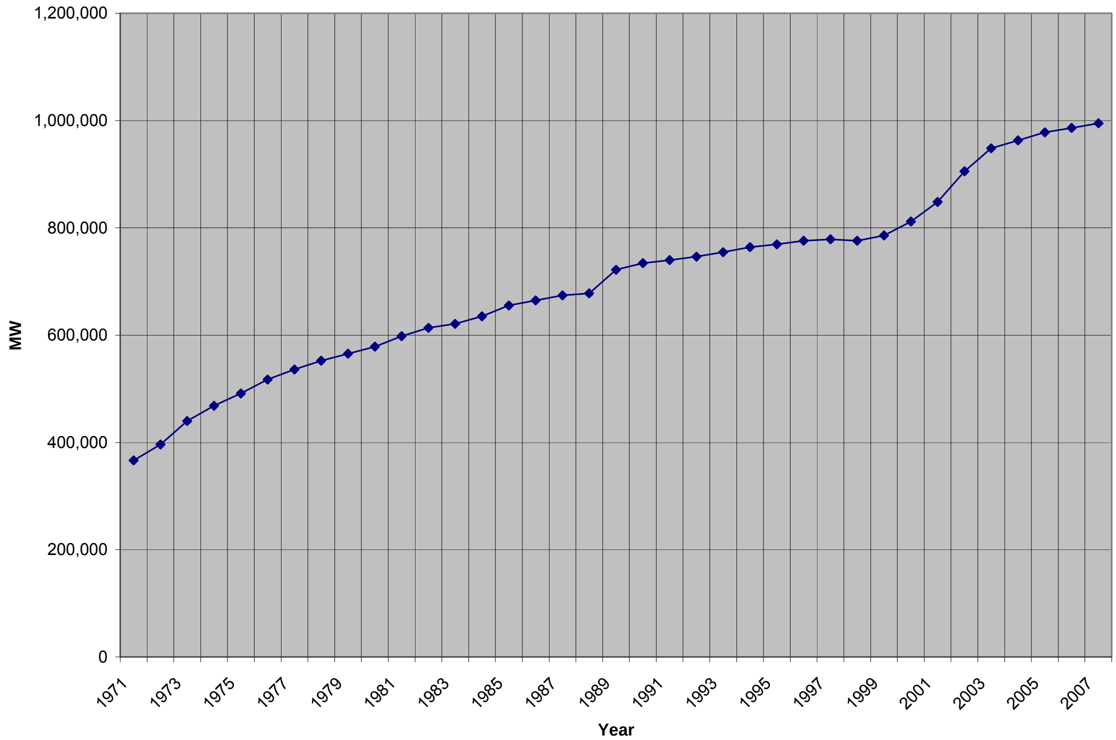

Figure 2.1 – Total U.S. summer generation capacity [2]

The plant or use factor of the system is the ratio of the total actual energy produced over a designated time period to the energy that would have been produced if the plant had operated continuously at its maximum rating [6]. A diagram of the plant factor of the U.S. system derived from DOE data, included in appendix H, is shown in Figure 2.2

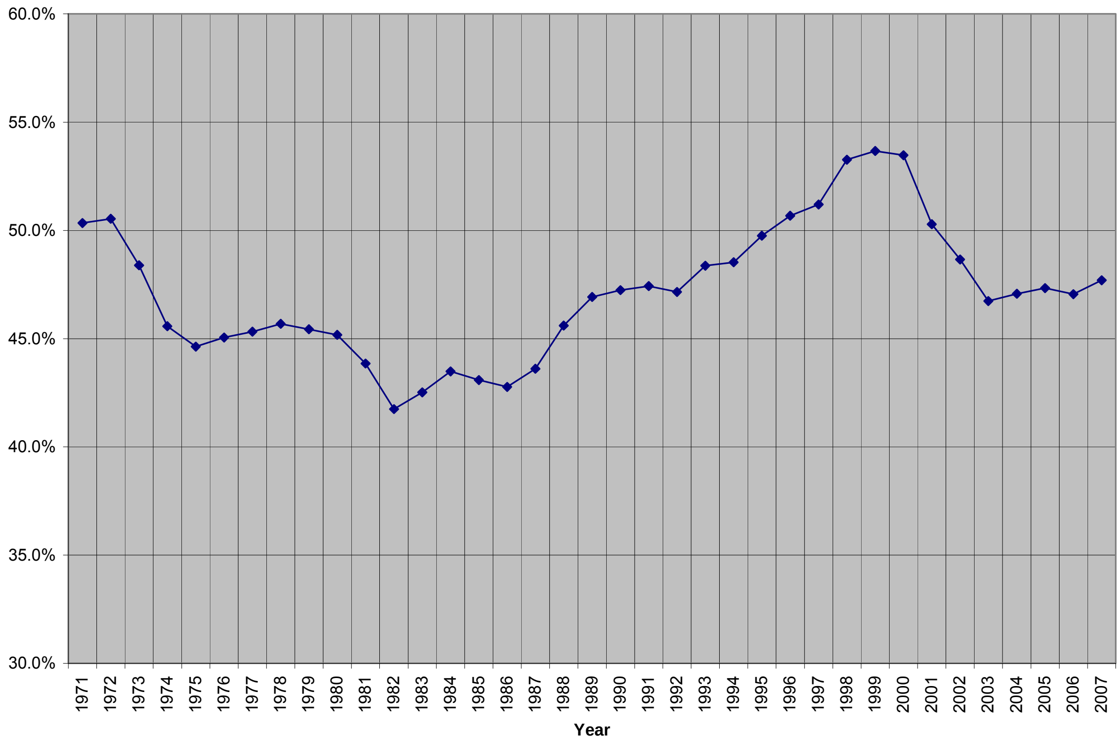

Figure 2.2 – U.S. plant factor of the electric system [2]

One can see from the data that the plant utilization factor has historically been between roughly 40-55%. This has been driven by the goal of always having capacity to meet demand. Since the demand has a peaking load curve, the capacity needed for the peak also sits underutilized about half the time.

The electric system has been designed with large generation facilities connected to the bulk transmission network which supplies the sub-transmission system. For the most part the sub-transmission system feeds radial distribution substations that provide electricity to the distribution system and consumers. With the growth of small renewable power generation capabilities such as wind and solar, there is the potential of having many smaller sources of power on the distribution system. This poses both opportunities and challenges for the electric industry.

A basic premise of the power system is to have the generation available to meet the load of the system. As the load of the system changes, the power required to be generated changes along with it. For the most part the load changes as the devices using the power are utilized. With most devices the benefit derived from the electricity being used is obtained when the device is using the electricity from the system supplying it. So if you want to use the electricity from the system when it is most efficient, then the benefit of the device must be used when the supply is the most efficient. However, if the device could store the energy when it is supplied most efficiently and use it during less efficient supply times then the system overall is more efficient.

The vast majority of electric systems utilize AC to generate and supply electricity. One drawback to AC is that it can not be directly stored for later use; it must be generated and used at the same time. To store the electricity it must be converted from AC to another form of energy, however energy is lost during this conversion process. If more energy is lost when storing electricity than would be in direct use, then any efficiency benefit is reduced. A device that converts electricity into useful work is deemed more efficient if it requires less energy while producing the same amount of work. The electric system can be deemed more efficacious if it requires less energy and equipment to supply the electricity to the end users.

Over the past several decades the utilization of power electronics has grown significantly. The advances in semiconductor fabrication technology have enabled higher voltage and current handling and switching speeds of power semiconductor devices. This has enabled electronic controllers to improve the efficiency of devices that utilize power electronics. It has been estimated by electric utilities that since 2000, over half of the electrical load is supplied through power electronics. [5]

The result of the change in a large portion of the load utilizing power electronics is that the system has become increasingly an AC/DC hybrid system. The bulk of the power is generated and delivered as AC and then the majority of end use devices convert it to DC. The system now has a large amount of AC/DC conversion happening at the end use device location. One of the ideas presented in this paper is the untapped potential of this DC power to be harnessed as a distributed resource.

3 Design Elements

3.1 Simple radial electric system

A model of a simple radial electric system was created to study the potential impacts of DR on the efficacy of the system, shown in Figure 3.1. The model has one large generator, one transmission line and one substation transformer feeding a distribution feeder, or circuit. The feeder has 10 loads and 10 DR locations positioned at different points along the feeder. The DR points are modeled as small generators that can be either on or off, depending on the analysis scenario.

This model provides the ability to measure the impact that different levels of load and DR size and location has on the electric system. Parameters such as the amount of power flowing through an element of the system or total losses can measured and compared to reference measurements. It also provides the ability to determine if there is an optimum location and size of DR for a given system arrangement. Multiple power flow simulations of different combinations of load levels and DR locations and size were performed.

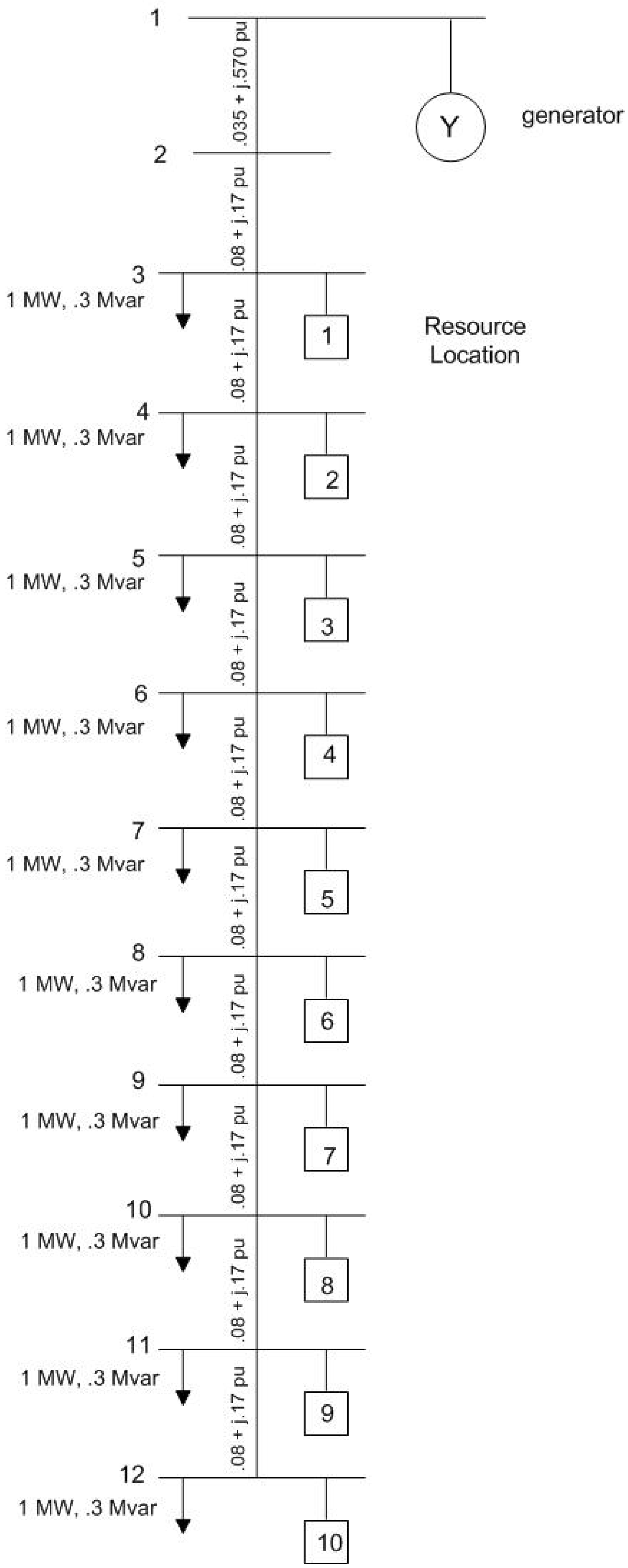

Figure 3.1 - Diagram of radial feeder model used for simulations

The electrically equivalent parameters of the proposed system were derived to be consistent with typical values found in the industry. The distribution line impedances were calculated assuming a typical open air overhead cross-arm construction as shown in Figure 3.2. The phase spacing was 44” and 336 AL wire was used.

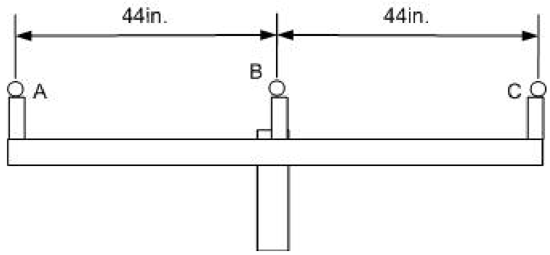

Figure 3.2 – Distribution feeder geometry used to calculate impedance

The inductance of the distribution feeder was calculated using geometric mean radius (GMR) [1]. The details of the calculation are shown in Appendix A. For the model feeder each segment is about 2750’ which gives a segment impedance of .08+j.17. The impedance of transformer was taken from typical values in the industry, .035+j.57 on a per unit basis.

3.2 Power Flow Methodology

To solve the power flow or “load flow” of the electric system a nodal analysis is performed. Each node, usually referred to as a “bus”, has four variables; voltage (V), voltage angle (θ), real power (P), and reactive power (Q). The buses are assigned a type depending on which variables are defined and which are to be calculated. The system has one reference bus or "slack bus" which has a specified V and θ. A load bus has known or specified P and Q values. A voltage controlled or generator bus has known or specified P and V values. The line data is represented in a matrix form with from and to bus, resistance and reactance per unit. The data is defined in an admittance matrix (Y).

For a total of N buses the calculated voltage at any bus k, where n ≠ k, and where P_k and Q_k are specified is: [1]

$$V_k = \frac{1}{Y_{kk}}\left(\frac{P_k - jQ_k}{V_k *} - \sum_{n=1}^{N} Y_{kn} V_n\right) \tag{3.1}$$

For a bus where voltage magnitude rather than reactive power is specified, the components of the voltage for n ≠ k are found from:

$$P_k - jQ_k = \left(Y_{kk} V_k + \sum_{n=1}^{N} Y_{kn} V_n\right) V_k * \tag{3.2}$$

For n=k

$$P_k - jQ_k = \left(V_k * \sum_{n=1}^{N} Y_{kn} V_n\right) \tag{3.3}$$

$$Q_k = -\mathrm{Im}\left(V_k * \sum_{n=1}^{N} Y_{kn} V_n\right) \tag{3.4}$$

The nonlinear equations for the nodal analysis can be solved with an iterative solution as shown in Figure 3.3.

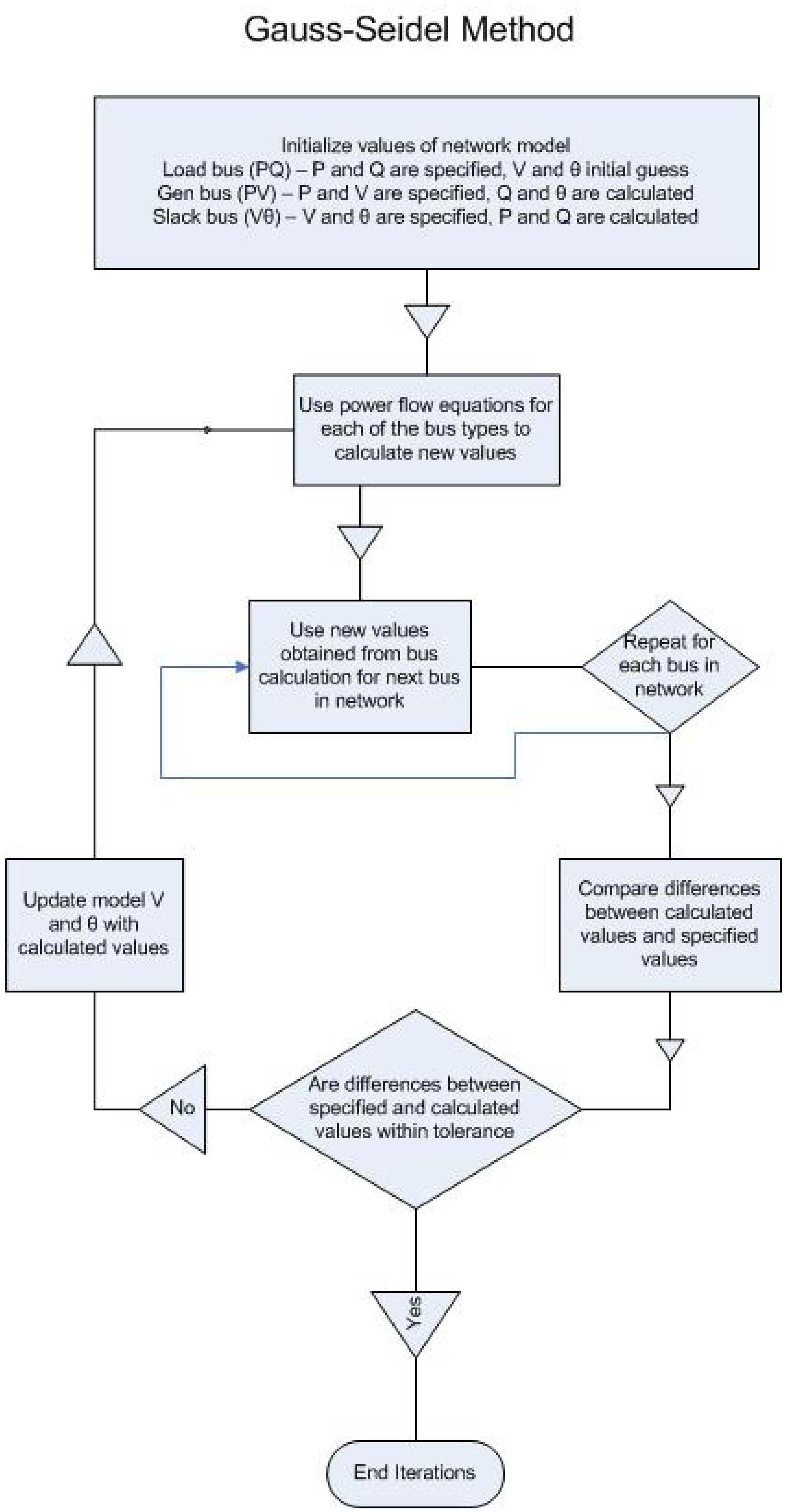

Figure 3.3 – Power flow methodology

The Gauss-Seidel method uses the admittance matrix representation of the line data to solve the I=YV equation, where I is the current, Y is the admittance, and V is the voltage. An iterative solution method starts with an initial guess of values to solve the unknowns. For the power flow equations the initial guess for voltages are 1 per unit and 0 for angles. The calculated values are compared to find the mismatch. If the mismatch is greater than the set tolerance, then the calculated values are used as the guesses to solve the equations again. This is repeated until the mismatch is within the tolerance or the max number of iterations is reached.

For a load bus guesses for the V and α are entered into the equation to calculate values for P and Q. These calculated values are compared to the known values of P and Q. If the mismatch between the calculated values and known values are within a set tolerance the iterations can stop. If the mismatch is greater than the tolerance the newly calculated values for V and θ are used and the process is repeated. For a generator or voltage controlled bus guesses for the Q and θ are entered into the equation to calculate values for P. The calculated value is compared to the known values of P.

A basic Gauss-Seidel load flow program was created in Matlab and is included in Appendix B. A more robust load flow program, Matpower V3.2, (http://www.pserc.cornell.edu/matpower/), was used for the majority of the analysis. The program was modified to provide the ability to scale the loads based on a scaling factor entered at program execution. The portion of modified code is included in Appendix C. This modification enabled a much more efficient program execution cycle for the multiple simulations required.

3.3 Multi-circuit system model

A second model of a small distribution system was created to study impacts on overall system efficacy beyond a simple feeder. The multi-circuit model used in this analysis is shown in figure 3.4. The model consists of a generator, transmission line, a substation bus supplied by a transformer, and three feeders. The bus has a peak shaving battery unit and two of the feeders have a peak shaving battery units at different locations. The feeders are segmented into three sections with 8 - 3MW 95% pf loads supplied by distribution transformers. Simulations were run comparing the losses with the different peak shaving unit locations and methods.

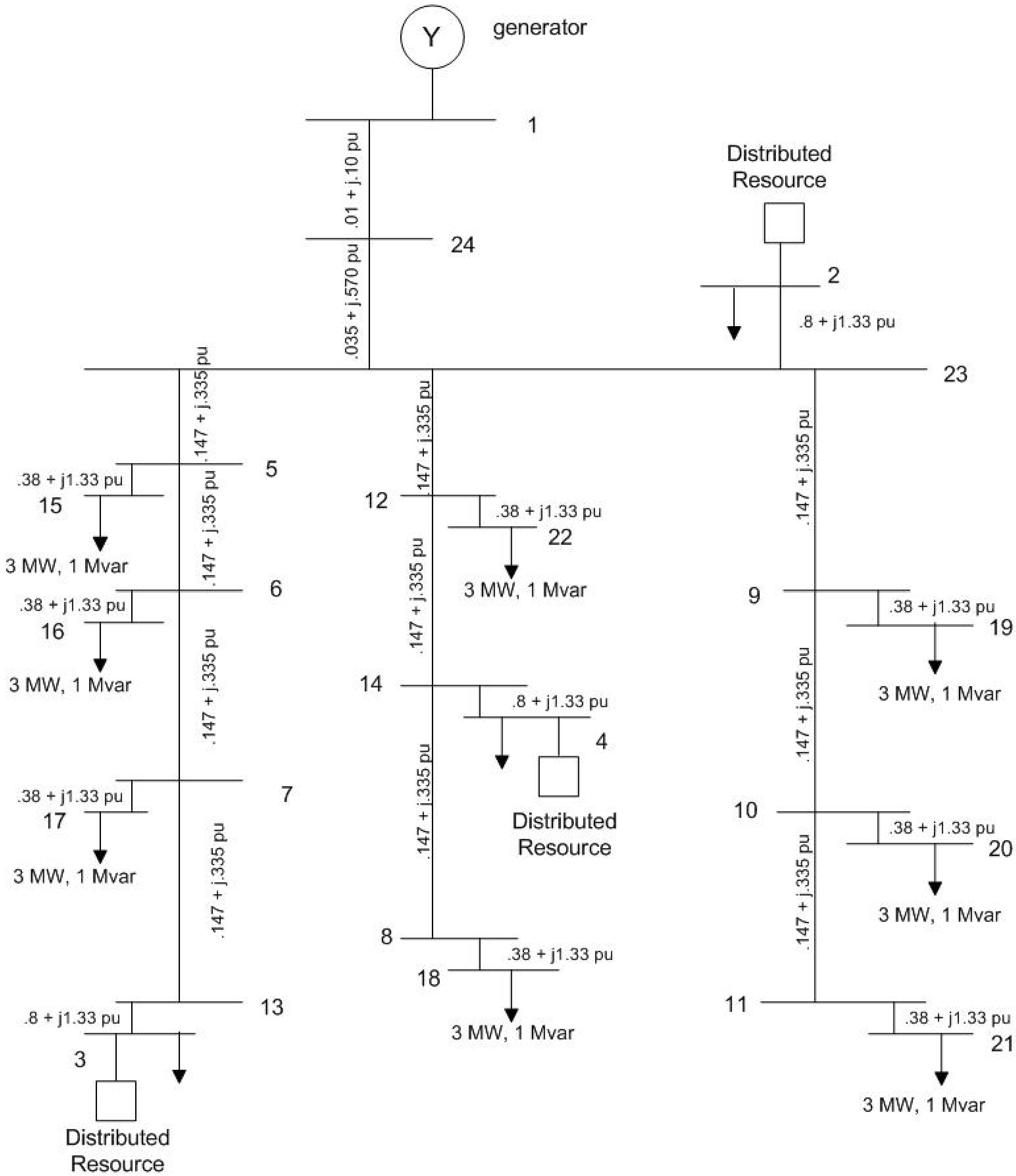

Figure 3.4 – Diagram of multi-circuit distribution system model

3.4 End use device based storage

For DR that utilizes charging from the system, the charging losses are a major factor in limiting efficacy. The biggest source of losses from battery storage is in the conversion of power from AC to DC and then from DC to AC. An approach introduced in this paper is to have the storage based at end use devices that already utilize AC/DC power supplies. This eliminates the need for extra conversion processes and their associated losses. This approach is also inherently scalable to the load since it is based in the load.

The concept is to have a small amount of energy stored in the end use device that would enable the device to use this energy to function for a short time. The device would charge during a low use period and would then use the stored energy during a high use period. If there are hundreds of thousands of these devices on a system then the amount of load that could be shifted is significant. The devices would operate randomly over the projected high and low use periods.

The impact that DR based on end use device storage has on system efficacy will be analyzed along side distribution based DR. Since the end use device DR is part of the load, it will be modeled as load. The result is a lower load during a peak period and a higher load during an off peak period. The concept of end use device storage is shown in Figure 3.5.

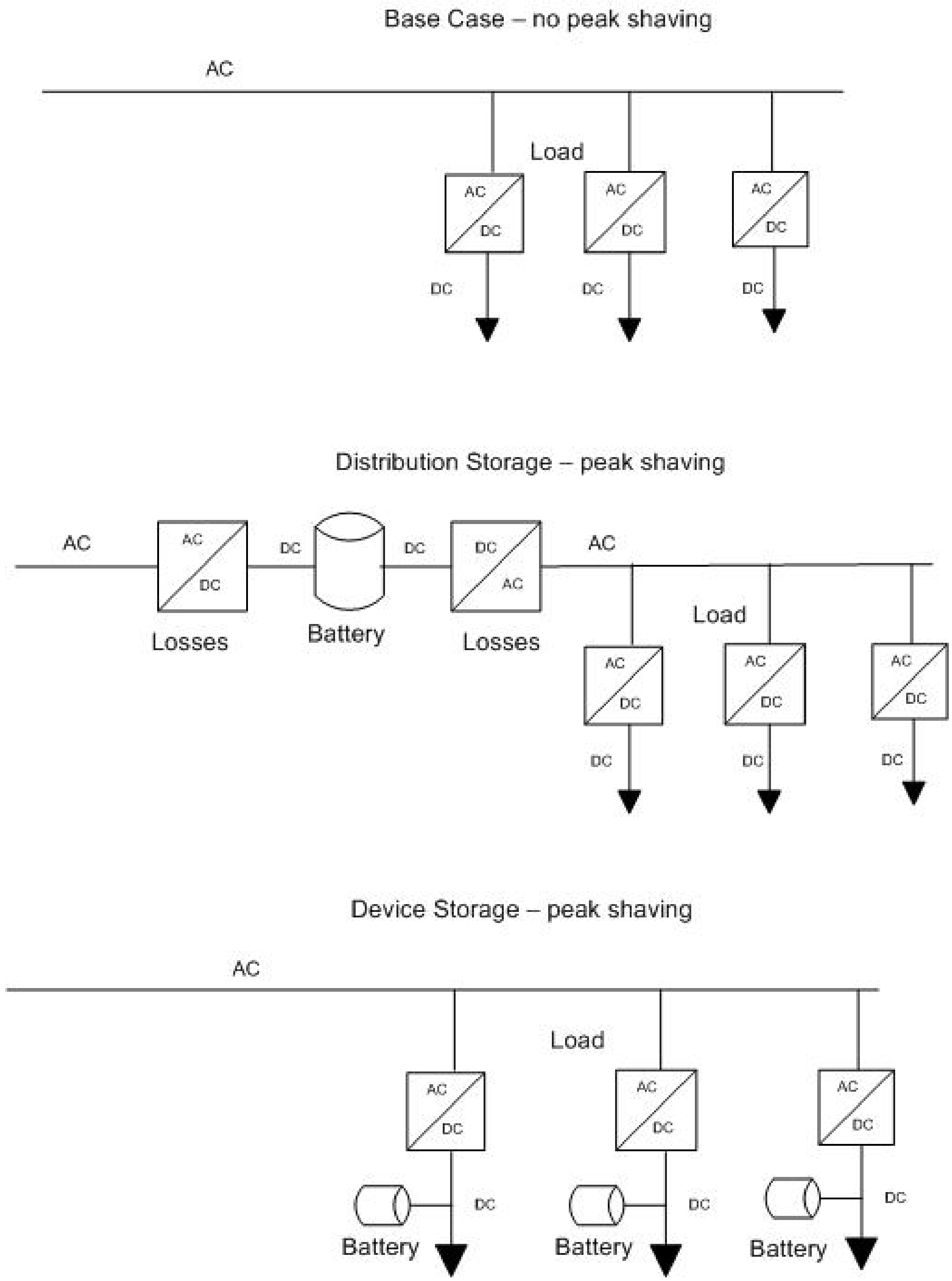

Figure 3.5 – Diagram of base case, distribution DR and end use device DR

4 Simulation Results

4.1 Simple radial electric system with DR supply

Load flow simulations were run on the distribution feeder model, shown in Figure 3.1. For the first set of simulations the feeder had a total load of 10 MW and 3 MVar with 1 MW and .3 MVar at each load point. The system losses were recorded with no DR and used for the normalized base line. Simulations were run with different DR locations and sizes to compare the effect they have on system efficacy and efficiency.

The load factor of the feeder is increased by having a lower peak load seen at the substation bus. For the case of having a 1 MW DR, the peak load on the feeder is 9 MW instead of 10 MW, but the total energy of the load has not changed. The result is the feeder now has extra capacity for load growth. The same goes for the other cases with their respective levels of DR. A graph showing the system losses versus DR location compared to the base case without DR is shown in Figure 4.1.

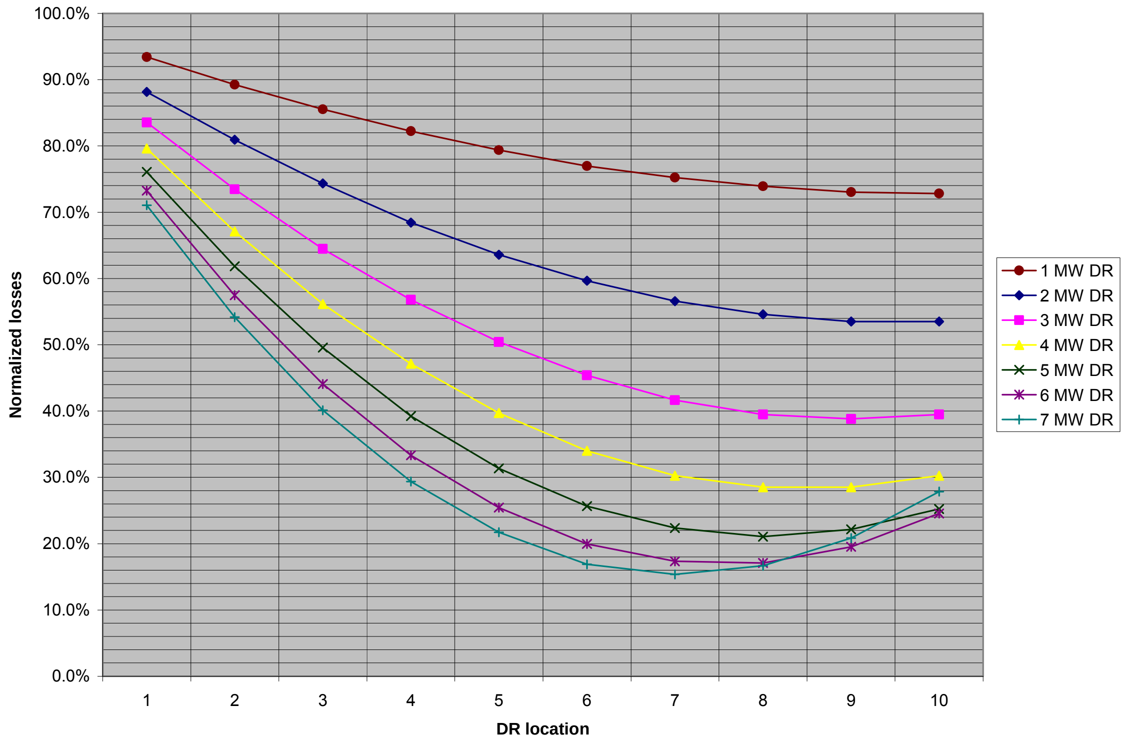

Figure 4.1 – Normalized system losses versus DR location for different DR sizes

It can be seen from Figure 4.1 that the size and location of the DR has an impact on the system losses. In all cases the overall system losses were reduced and the closer the DR was to the beginning of the feeder the less of a reduction there was. For the lower capacity DR the most system loss reduction is for locations at the end of the line. As the DR capacity increases the greater loss reduction moves from the end to the middle of the line.

If we assume no or minimal voltage drop, then the line losses can be expressed in generic terms with the following equations. [7]

The line loss without any DR present is defined as:

$$\text{Loss}_B = rL\frac{(P_L^2 + Q_L^2)}{3V_P^2} \qquad (4.1)\ [7]$$

The line loss from the source to the DR location can be expressed as:

$$\text{Loss}_{\text{ASD}} = rD\frac{(P_L^2 + Q_L^2 + P_D^2 + Q_D^2 - 2P_L P_D - 2Q_L Q_D)}{3V_P^2} \qquad \textit{(4.2) [7]}$$

The line loss form the DR location to the load can be expressed as:

$$\text{Loss}_{\text{ADL}} = r\frac{(L-D)(P_L^2 + Q_L^2)}{3V_P^2} \qquad \textit{(4.3) [7]}$$

The total line loss can be expressed as:

$$\text{Loss}_{\text{AT}} = rL\frac{(P_L^2 + Q_L^2 + (P_D^2 + Q_D^2 - 2P_L P_D - 2Q_L Q_D)(\frac{D}{L}))}{3V_P^2} \qquad \textit{(4.4) [7]}$$

Where

Loss_{B} = Line loss without any DR
Loss_{ASD} = Line loss from source to DR
Loss_{ADL} = Line loss from DR to load
Loss_{AT} = total line loss with DR
P_L = Real power of load
Q_L = Reactive power of load
P_D = Real power of DR
Q_D = Reactive power of DR
r = line resistance per phase, ohm/mile
L = Distance of distribution line, miles
D = Distance from source to DR location, miles
Vp = RMS phase voltage of the distribution line and load

Simulations were also run for equivalent amounts of demand reduction, for example 1 MW of DR supply was compared to 1 MW of DR demand reduction. A graph showing the system losses with DR supply normalized to the equivalent DR demand reduction is shown in Figure 4.2.

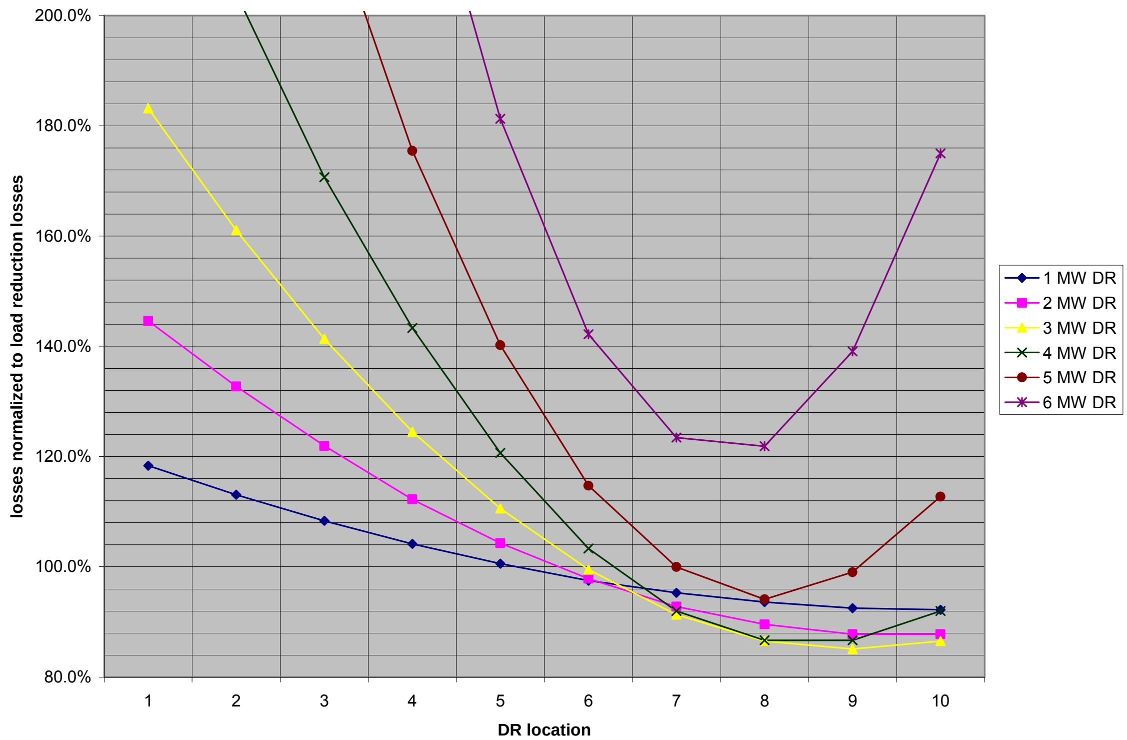

Figure 4.2 – System losses with DR supply normalized to losses from peak load reduction versus DR location

It can be seen from Figure 4.2 that the size and location of the DR supply has an impact on the system losses compared to equivalent amounts of load reduction. In all cases the overall system losses were higher for DR supply closer to the beginning of the feeder compared to equivalent amounts of load reduction. For the lower capacity DR supply, the system losses for locations at the end of the line are lower than equivalent load reduction. As the DR capacity increases the system losses are greater for DR supply compared to equivalent amounts of load reduction.

Since it was determined that the location and size of the DR source has an impact on system losses, it was of interest to see the effect of adding another variable to the analysis. It was decided to also look at the concentration of the DR source. For example, if the 1 MW source was now made up of two 0.5 MW sources located at different positions on the feeder. Having multiple DR supply sources also has significant implications when it comes to capacity planning. For planning purposes it is standard to consider the capacity of the system with an N-1 condition. If there are two DR sources then capacity credit could still be claimed for one of them.

Multiple simulations were run with two different DR locations providing a supply source. Figure 4.3 shows the percent of normalized losses for two 0.5 MW DR supply points for 3 combinations of locations. Positions 10, 8, and 6 plus each of the 10 points individually are shown. For example, a case with a 0.5 MW source at location 10 and 1 was solved. This looks like a 9 MW feeder load at the substation bus. The losses on the system were recorded and normalized to a percentage of the base line loss level. Next, another simulation was run with sources at position 10 and 2. The process was repeated for each of the other combinations listed. The results were summarized in a graph and a line showing the percent of normalized loss for a 9 MW feeder load is shown for comparison.

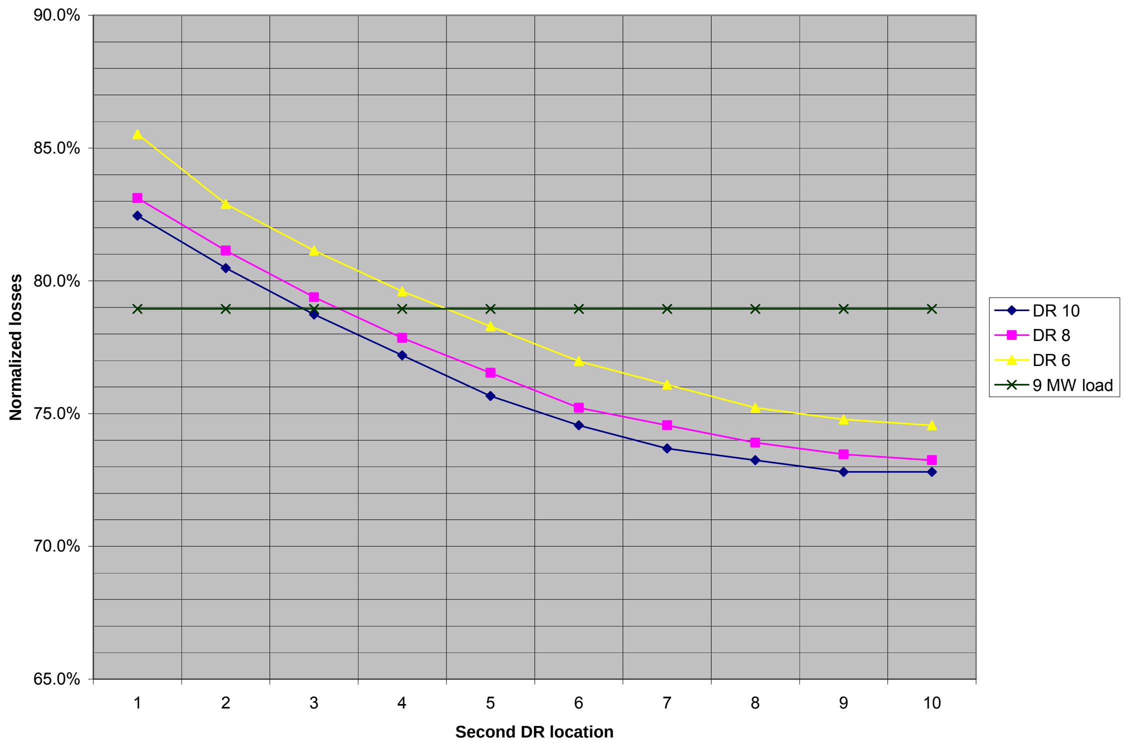

Figure 4.3 – Normalized system losses versus DR locations for two 0.5 MW DRs

Again it can be seen from the results that system losses are reduced with the presence of the DR and the reduction is dependent on the location of the DR. The system losses for the equivalent 9MW feeder load are lower than the DR source when the DR is close to the feeder's beginning. The system loss reduction is greater when the DR is at the end of the feeder and is lower than the system losses with the equivalent 9MW feeder load.

Figure 4.4 shows the percent of normalized losses for two 1.0 MW DR supply points for 3 combinations of locations. Positions 10, 8, and 6 with each of the 10 points are shown. A line showing the percent of normalized loss for an 8 MW feeder load is shown for comparison.

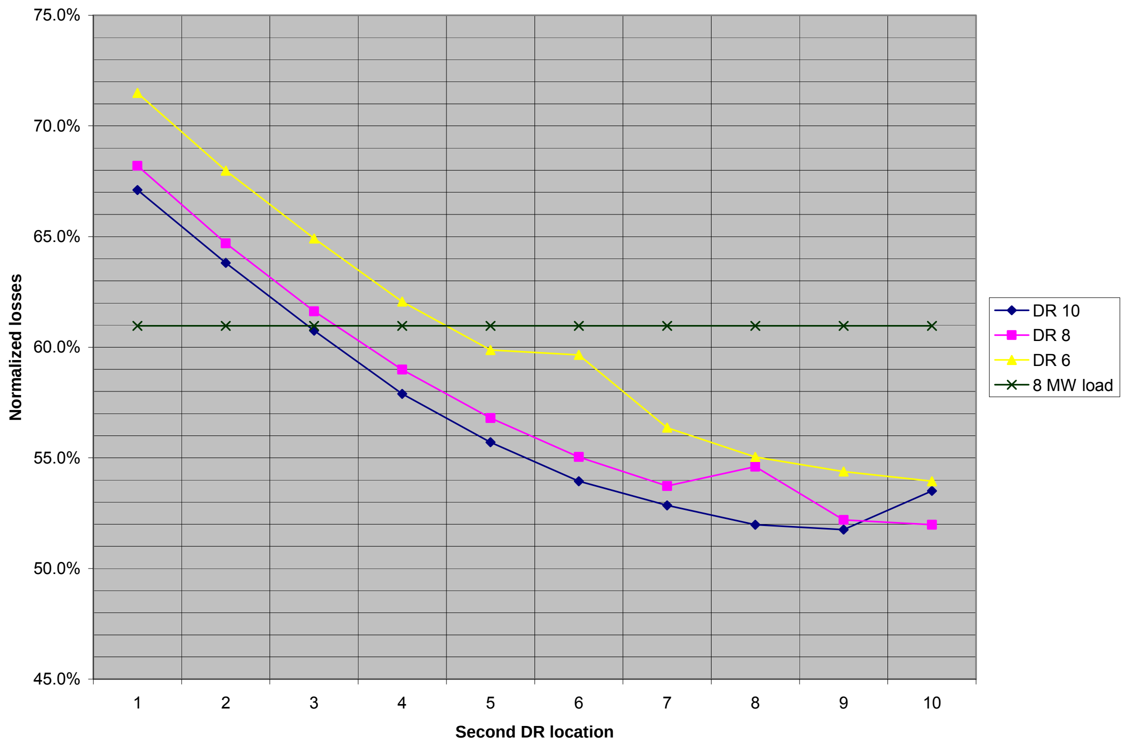

Figure 4.4 – Normalized system losses versus DR locations for two 1.0 MW DRs

It can be seen in Figure 4.4, that when the DR sources are located at the same location, there is less of a system loss reduction. This shows that from a system loss reduction perspective having multiple DR locations is more beneficial than having one DR location. It can also be seen that the system loss reduction is greater for locations near the end of the feeder.

Figure 4.5 shows the percent of normalized losses for two 2.0 MW DR supply points for 3 combinations of locations. Positions 10, 8, and 6 with each of the 10 points are shown. A line showing the percent of normalized loss for a 6 MW feeder load is shown for comparison.

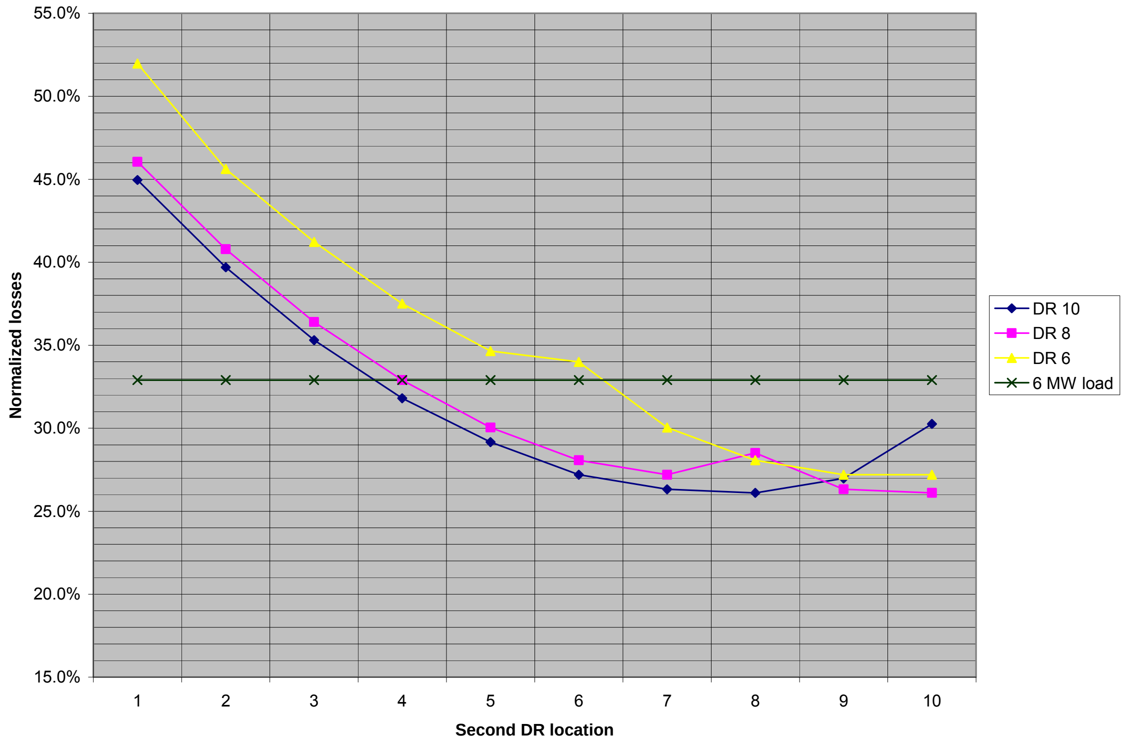

Figure 4.5 – Normalized system losses versus DR locations for two 2.0 MW DRs

Again it can be seen in Figure 4.5, that when the DR sources are located at the same location, there is less of a system loss reduction. It can also be seen that the system loss reduction is greater for locations separated near the end of the feeder.

Figure 4.6 shows the percent of normalized losses for two 3.0 MW DR supply points for 3 combinations of locations. Positions 10, 8, and 6 with each of the 10 points are shown. A line showing the percent of normalized loss for a 4 MW feeder load is shown for comparison.

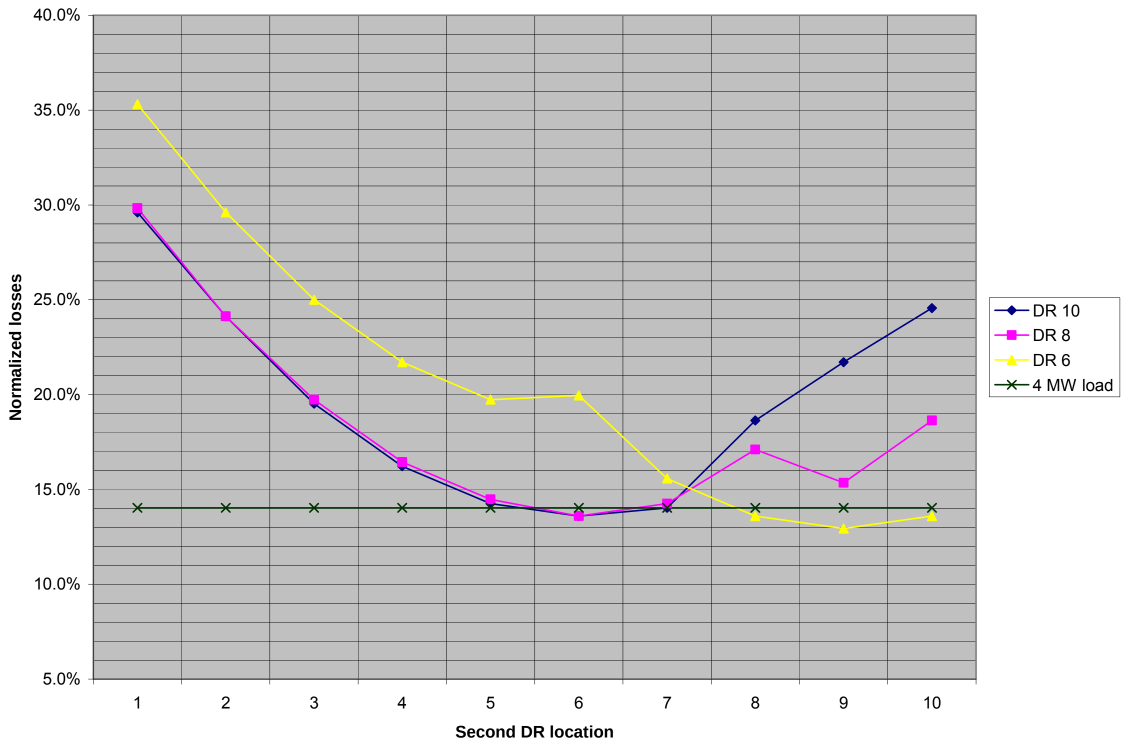

Figure 4.6 – Normalized system losses versus DR locations for two 3.0 MW DRs

The results show that as the capacity of the DR sources increase compared to the feeder load, the lowest system losses are obtained when the DR are located between the middle and end of the feeder. Also, it can be seen that the equivalent amount of load reduction has about the same amount of system losses as the lowest DR locations.

In all cases the peak load seen at the substation bus is decreased. From a capacity standpoint this is equivalent to increasing the unused capacity of the system by the amount of the DR plus the reduced losses. In the case of existing equipment, this can defer the need for system upgrades such as a larger substation transformer or feeder upgrade. By reducing the peak load and keeping the total amount of energy the same over a given period, the load factor is increased. This in turn increases the capacity utilization factor and frees up system capacity and allows for additional load to be served, which is more efficacious.

In the case of planning new equipment, the reduced peak can allow for a smaller amount of capacity installation. This enables the ability to obtain a higher capacity utilization factor than could be obtained with a lower load factor. The amount of DR can be added to capacity calculations or subtracted from peak load projections. The same amount of energy can be delivered over a given time with less installed capacity.

4.2 Simple radial electric system with DR storage

Simulations were run to also capture the system losses due to charging the DR in an off peak period. The feeder load was assumed to be 40% of peak and the charging was set to be 100% efficient. The combined losses of the peak supply and the off peak charge are shown in Fig 4.7.

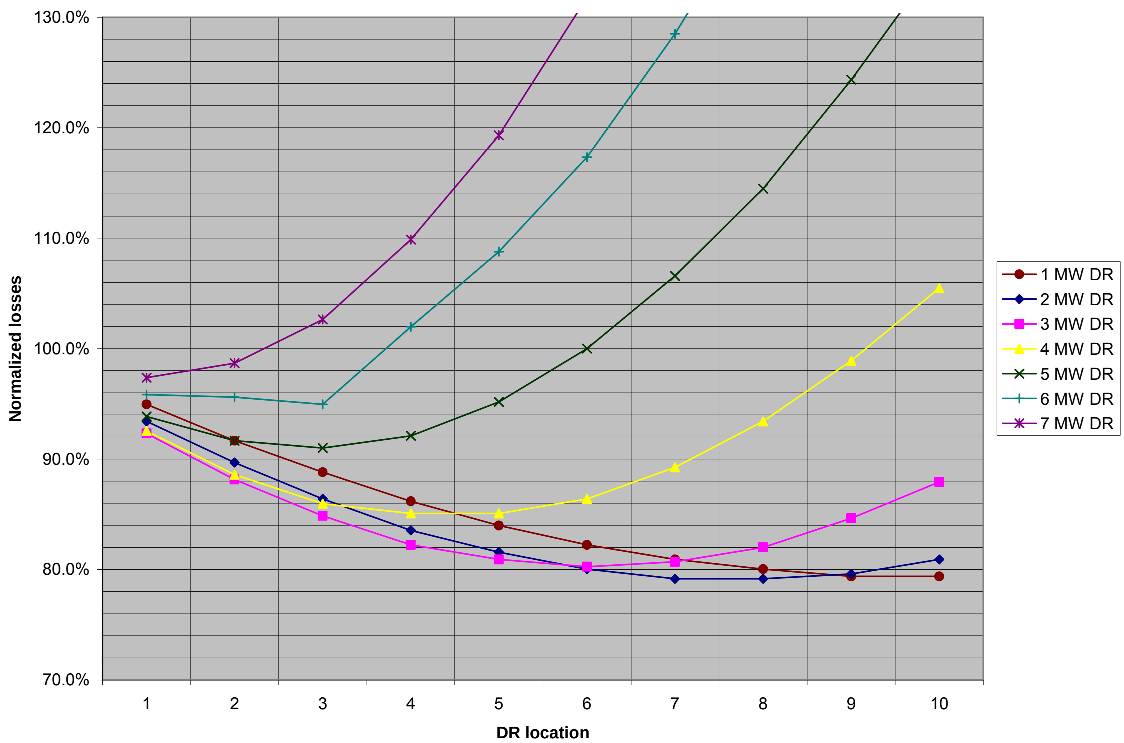

Figure 4.7 – Normalized system losses including system losses from charging (100% efficient) versus DR location for different DR sizes

It can be seen from Figure 4.7 that the size and location of the DR has an impact on the system losses. It can be seen that when system losses from charging are included the overall loss reduction is reduced and highly dependent on DR size and location. For the lower capacity DR the most system loss reduction is for locations at the end of the line. As the DR capacity increases the greater loss reduction moves from the end to the middle of the line.

Simulations were run to also capture the system losses due to charging the DR in an off peak period. The equivalent amount of load reduction was also charged in the off peak period. The feeder load was assumed to be 40% of peak and the charging was set to be 100% efficient. The combined losses of the peak supply and the off peak charge are shown in Fig 4.8.

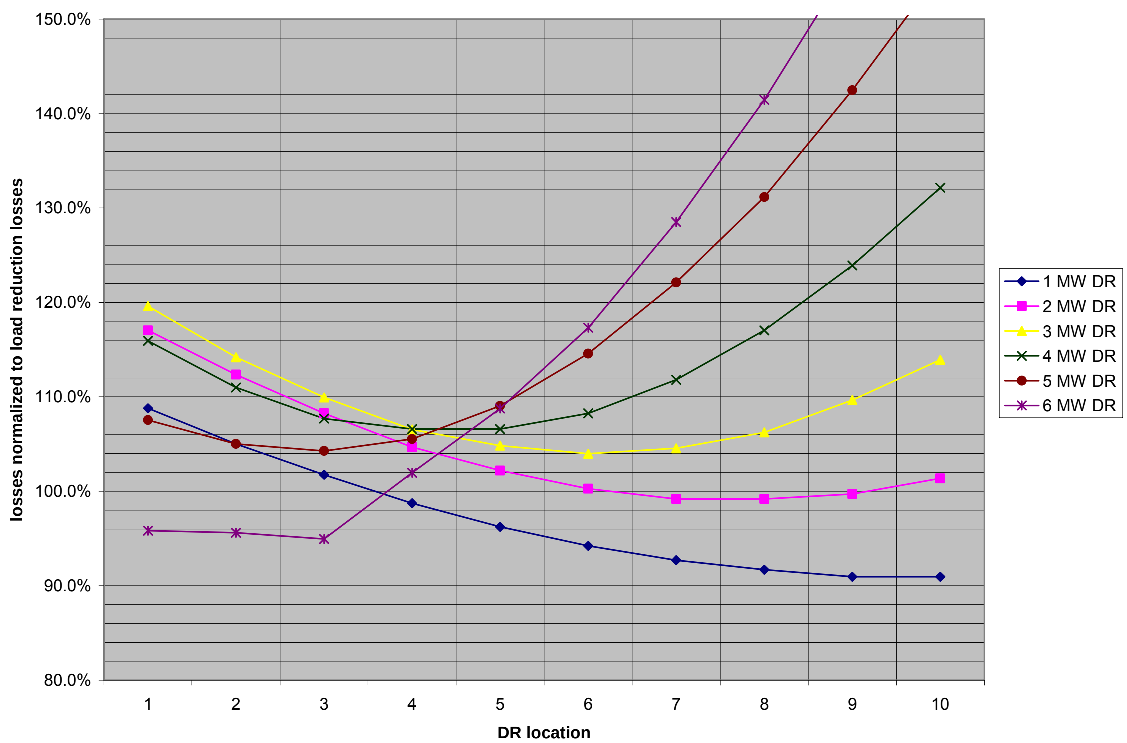

Figure 4.8 – System losses with DR supply including losses from charging (100% efficient) normalized to losses from load shifting versus DR location

It can be seen from Figure 4.8 that the size and location of the DR has an impact on the system losses. It can be seen that when system losses from charging are included the overall loss reduction is reduced and highly dependent on DR size and location. For the lower capacity DR the most system loss reduction is for locations at the end of the line. As the DR capacity increases the greater loss reduction moves from the end to the middle of the line.

Next simulations were run were the feeder load was assumed to be 40% of peak and the charging of the distribution DR was set to be 80% efficient. The load shifting DR was set to be 90% efficient. This was done since the load based DR has less conversion losses since it eliminates an AC to DC and DC to AC conversion. The combined losses of the peak supply and the off peak charge are shown in Fig 4.8.

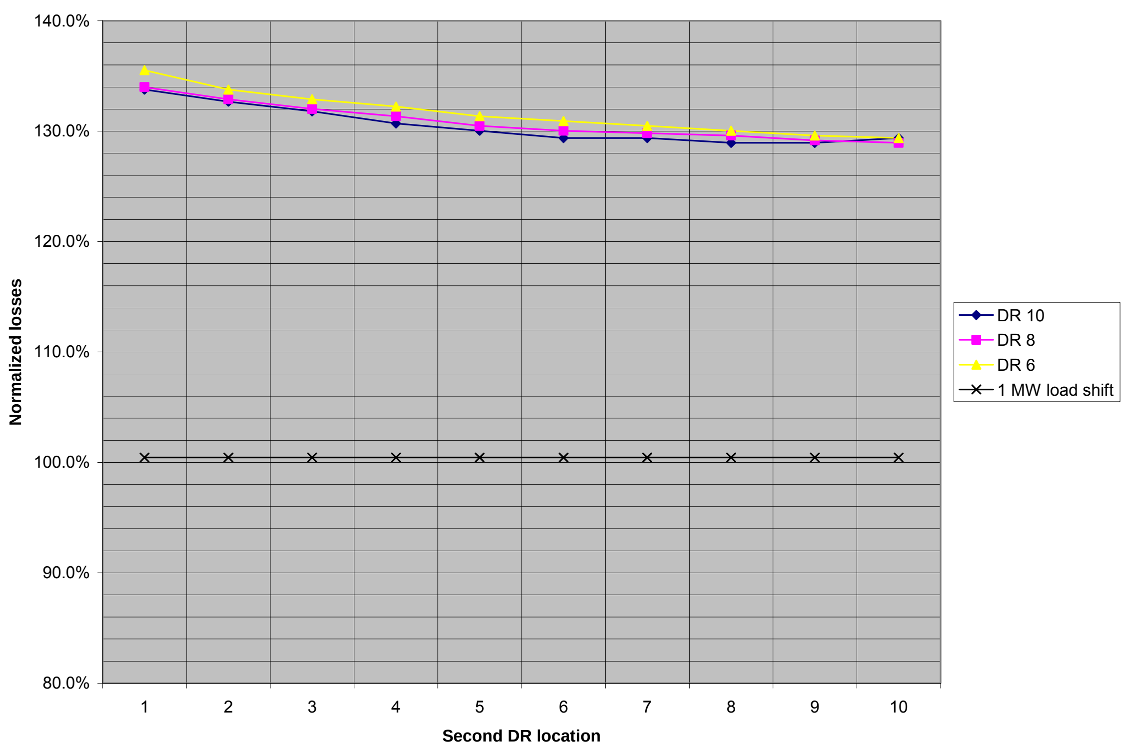

Figure 4.9 – Normalized system losses with charging versus DR locations for two 0.5 MW 80% efficient DRs and 90% efficient load shift.

It can be seen from Figure 4.9 that for smaller amounts of DR, the locations near the end of the feeder have lower losses than DR at the beginning. It can be seen that when system losses from charging are included the overall loss reduction is reduced and in this case the overall losses are increased. The end use device based DR has lower overall losses than the distribution based DR.

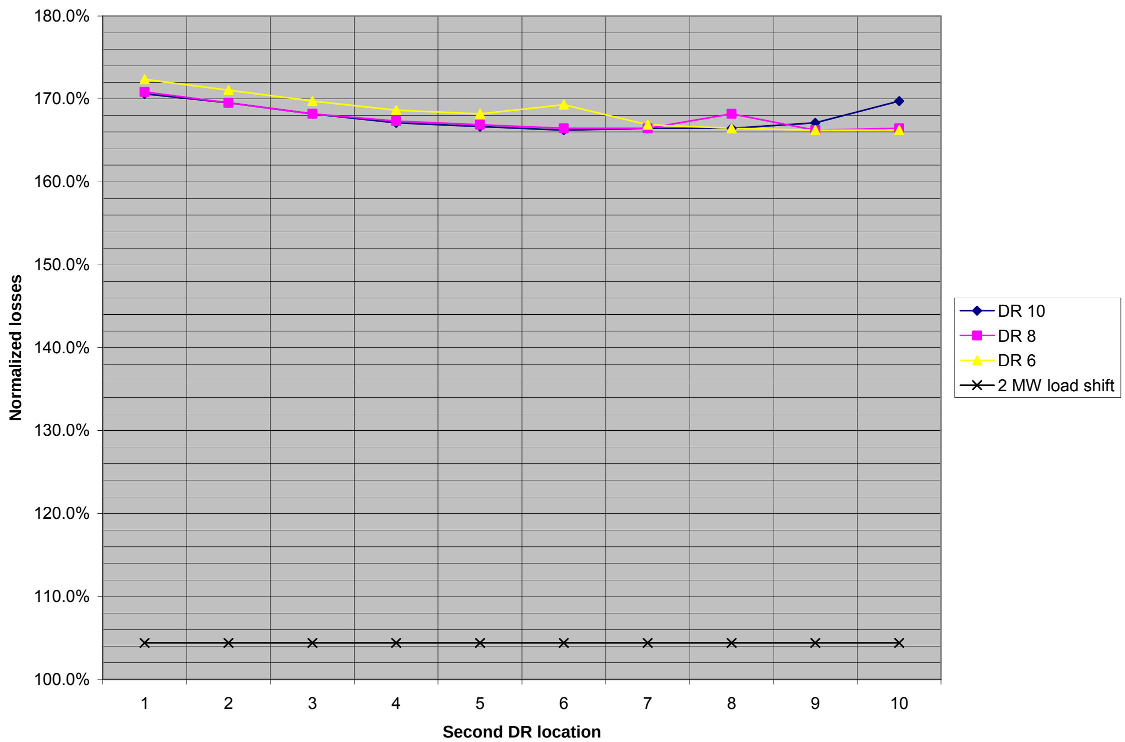

Figure 4.10 – Normalized system losses with charging versus DR locations for two 1.0 MW 80% efficient DRs and 90% efficient load shift.

It can be seen from Figure 4.10 that for moderate amounts of DR, the locations near the middle and end of the feeder have lower losses than DR at the beginning. It can be seen that when system losses from charging are included the overall loss reduction is reduced and in this case the overall losses are increased. The end use device based DR has lower overall losses than the distribution based DR.

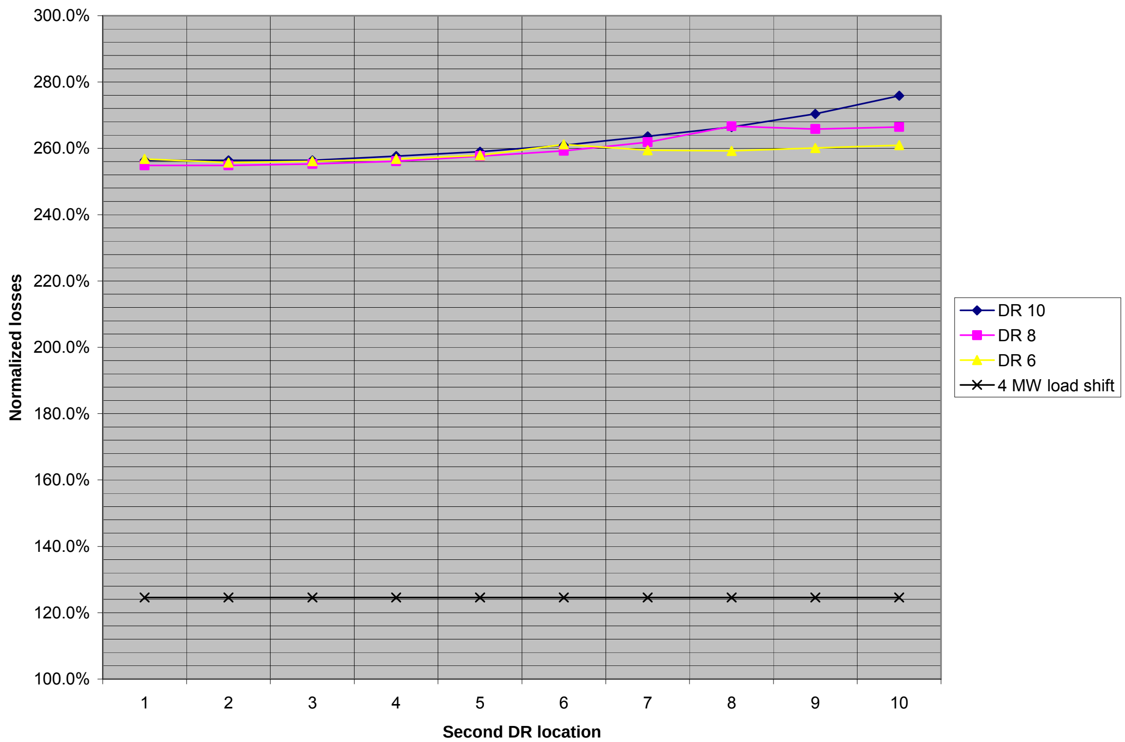

Figure 4.11 – Normalized system losses with charging versus DR locations for two 2.0 MW 80% efficient DRs and 90% efficient load shift.

It can be seen from Figure 4.11 that for larger amounts of DR, the locations near the beginning of the feeder have lower losses than DR at the end. It can be seen that when system losses from charging are included the overall loss reduction is reduced and in this case the overall losses are increased. The end use device based DR has lower overall losses than the distribution based DR.

4.3 Multi-circuit system model with DR storage

Simulations were run on the second model which is a representation of a simple distribution system. The model was shown in Figure 3.4; it has one large generator, one transmission line, one substation transformer, one substation bus, three distribution feeders, and three DR locations. The DR locations were picked to be at the substation bus, near the beginning of a feeder, and near the end of a feeder. The distribution load was modeled as large 3 MW loads located at several points along the feeders. The impedances of the distribution transformers were also included in the model, shown in figure 3.4.

The system was analyzed with a 24MW peak load and with the base load shape shown in Figure 4.7. All loads were scaled to the percent of peak load for the corresponding hour. The system load flow was solved for each hour and the system losses for each hour were recorded. This was used as the base for losses and was normalized to 100% for comparison to different load shapes and DR impacts.

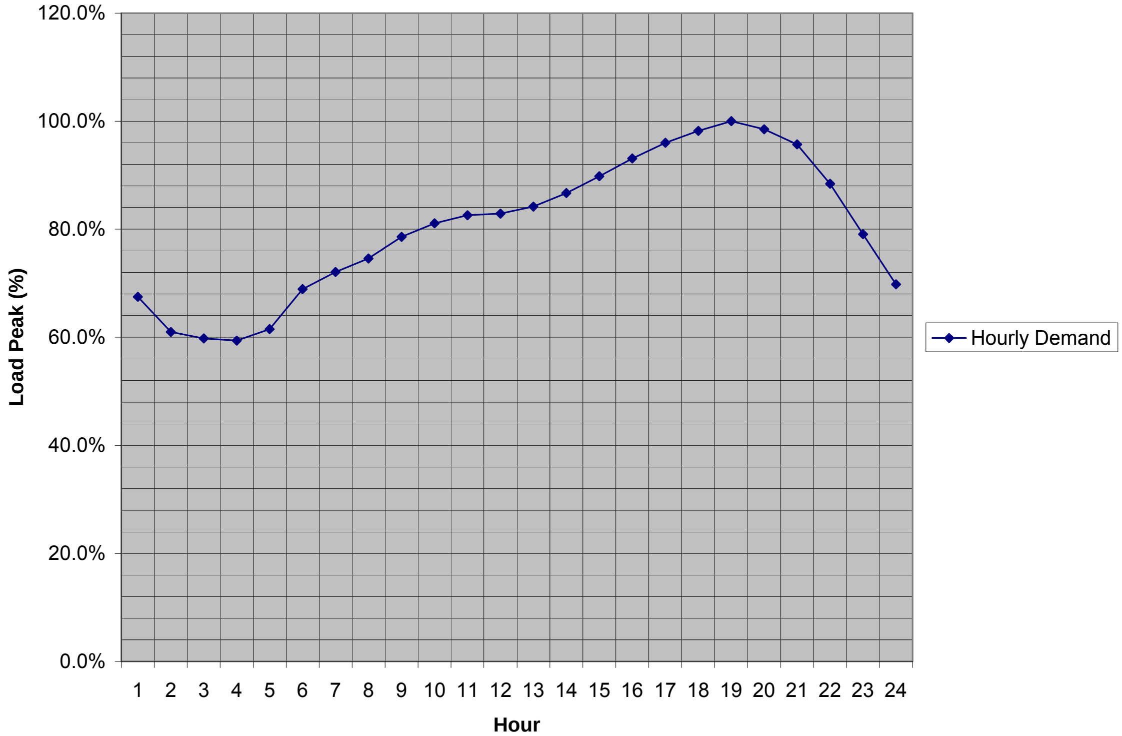

Figure 4.7 – Hourly demands used for base case 3 feeder system analyses

The impact that DR used for load shifting or peak shaving has on system efficacy was determined through load flow analysis. The DR locations were modeled as small generators connected to the distribution feeders. The size and on/off status was varied to simulate different combinations of DR on the system. To determine the losses associated with charging a battery, the DR location was replaced with a load. The results of varying DR locations and capacities on the system were recorded.

The total kWh delivered to the load in the 24 hour period was kept the same for all the simulations. However the total energy generated varied by a little due to the different amount of system losses. In other words, the 24 hour load curve as seen from the substation bus varied but the total energy delivered to the consumer remained the same. This was done to show that how energy is used over the time period has an impact on the system efficacy and efficiency.

For the first set of simulations 3 MWh was shifted from the highest load level to the lowest load level assuming 100% efficiency of load shifting. In other words it takes 3 MWh to charge a storage device that delivers 3 MWh. This was done system wide by reducing each of the highest 3 hours by 1.0 MW in aggregate and increasing each of the lowest 5 hours by 0.6 MW in aggregate.

The 3 MWh was also shifted using DR in different locations on the system. A DR of 1 MW was supplied to the system for the highest 3 hours. It was charged from the system by serving as a load of 0.6 MW for the lowest 5 hours. This was done individually for each DR location in the model and for all the locations simultaneously.

The data from the simulations are shown in Appendix G. The capacity utilization factor of each case can be calculated based on a system capacity of 25 MW and are as follows.

Case	Capacity utilization factor
	-
base case	62.05%
1 MW DR	64.75%
3 MW DR	70.92%
6 MW DR	82.73%

A chart summarizing the system energy losses for the cases with 100% storage efficiency is shown in Figure 4.8.

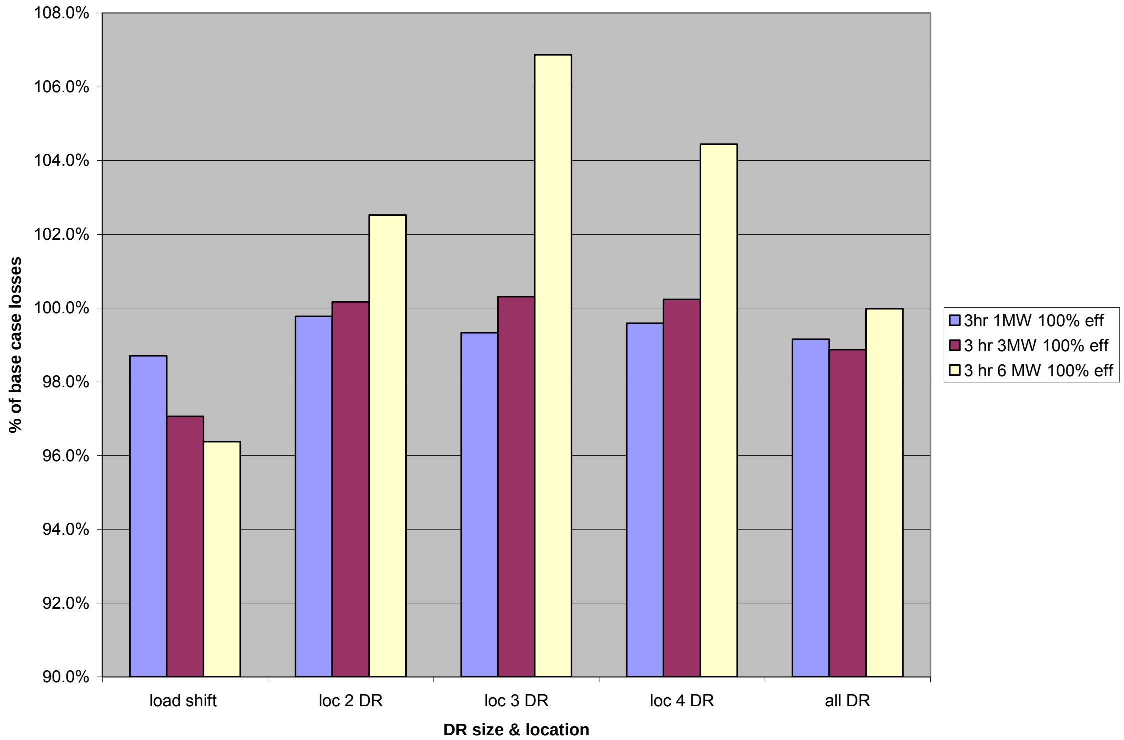

Figure 4.8 – Normalized system losses with different 100% efficient DR locations

It can be discerned that with a 100% efficient shift, the system wide load shift has the most decrease in overall system losses. The reduced system losses with the DR locations during peak usage is offset by increased system losses during off peak charging. As the size of the charging and distance from the substation increases, the greater the contribution of system losses is from charging. It can also be discerned that as the DR is spread out amongst multiple locations the overall system losses are lower. The more concentrated the DR is, the higher the system losses are due to the DR. There is also a trade off between the benefit of a storage DR at the end of a feeder for supply and the greater losses with having to charge the DR at the end of the feeder.

For the next set of simulations 3 MWh was shifted from the highest load level to the lowest load level assuming 90% efficiency of load shifting. This was done system wide by reducing each of the highest 3 hours by 1.0 MW in aggregate and increasing each of the lowest 5 hours by 0.667 MW in aggregate. The 3 MWh was also shifted using DR in different locations on the system. A DR of 1 MW was supplied to the system for the highest 3 hours. It was charged from the system by serving as a load of 0.667 MW for the lowest 5 hours. This was done individually for each DR location in the model and for all the locations simultaneously.

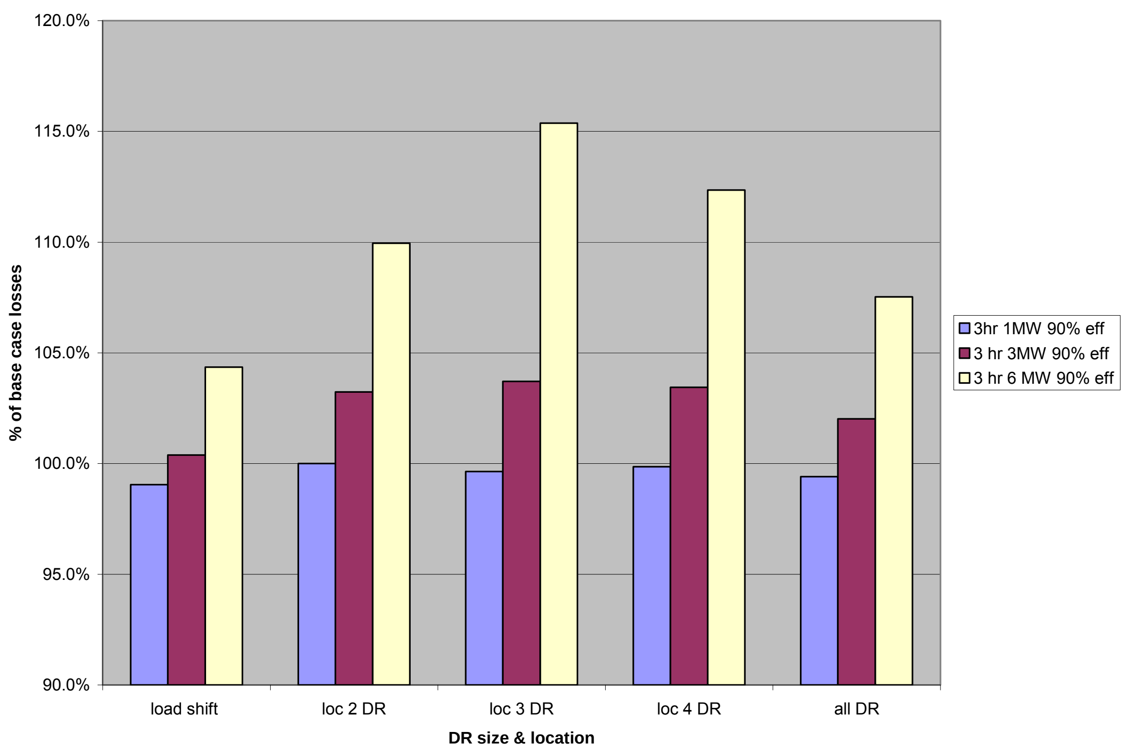

Figure 4.9 – Normalized system losses with different 90% efficient DR locations

It can be discerned that for DR storage with 90% storage efficiency, it is still possible to decrease overall system losses. The impact on overall system losses is highly dependent on the DR size and location. The overall losses associated with the DR are lower for smaller multi location applications than for larger single applications of DR. It is possible to lower overall system losses with DR storage even when the storage is not 100% efficient.

System losses are often expressed as a fraction of the system load in terms of percent of demand or percent of delivered energy. Defining the ratio of the total saved system losses to the peak load can be expressed by: [8]

$$\frac{L - L_s}{Peak\,Load} = \alpha k \frac{2(1 - G(d/\eta) - \alpha(1 + d/\eta)}{1 + G^2(d/\eta))} \qquad \textit{(4.5) [8]}$$

Where

L = system losses without any DR
L_s = system losses with DR
η = net AC energy efficiency of the DR storage system
Ro, Rp are the equivalent T&D resistances during peak and off-peak periods, respectively
Io, Ip are the load current during peak and off-peak periods, respectively
Is current provided locally by the DR storage device
d = Ro/Rp
G = Io/Ip
α = Is/Ip
k= system losses (L)/Peak Load

For each system configuration and load characteristics, there is a maximum storage size of DR when the losses due to charging equal the reduced system losses with the DR. This size is a function of the location of the DR and the ratio of peak to off-peak load. The results have been from analyzing actual load flow simulations, but general equations can also be used for an approximation of maximum storage size.

$$\frac{Max_{ss}}{Gap_{pop}} = \frac{1 - G(d/\eta)}{(1-G)(1+(d/\eta))} \tag{4.6}$$

[8]

Where

Max_{ss} = Maximum storage size
Gap_{pop} = Gap between peak and off-peak
η = net AC energy efficiency of the storage system
d = Ro/Rp
G = Io/Ip
Ro, Rp are the equivalent T&D resistances during peak and off-peak periods, respectively
Io, Ip are the load current during peak and off-peak periods, respectively

The saved losses can be expressed as a fraction of the storage size as follows:

$$\frac{L - L_s}{StorageSize} = k\frac{2(1 - G(d/\eta) - \alpha(1 + d/\eta)}{1 + G^2(d/\eta))} \tag{4.7}$$

[8]

Where

L = system losses without any DR
L_s = system losses with DR
η = net AC energy efficiency of the DR storage system
Ro, Rp are the equivalent T&D resistances during peak and off-peak periods, respectively
Io, Ip are the load current during peak and off-peak periods, respectively
Is current provided locally by the DR storage device
d = Ro/Rp
G = Io/Ip
α = Is/Ip
k= system losses (L)/Peak Load

So far the analysis has focused on system losses, to determine if DR can be implemented to increase the system capacity utilization factor without decreasing the system efficiency. There are other benefits to obtaining a flatter load curve and increasing the capacity utilization factor. The ability to reduce system capacity requirements has enormous economic implications. The capacity of the system is sized to meet the peak demand of the system, which includes generation, transmission, and distribution. A lower system peak can also lower the cost of power by lowering the incremental costs. More detailed cost implications will be examined in the next section.

5 Impact on System Costs

The impact that DR can have on distribution costs goes beyond just reduced losses. It also includes investment cost of the feeders and system capacity. By lowering the peak load of the feeder, extra capacity is freed up to allow for additional load which can delay the need for upgrades. If there is wide spread use of DR on the system, then the overall system peak load can also be lowered. This has a ripple effect to all parts of the system, distribution feeders, substations, transmission lines and generation. When DR is used to shift load from peak periods to off peak periods it can help flatten the load curve.

A flatter load curve can lower the overall cost of power. This is mainly due to the fact that not all generating units have the same efficiency or cost structure. Usually very large plants require a huge capital investment to build. These large costs can be offset by lower costs of fuel to produce power, such as coal and nuclear plants. These plants can not start quickly and are most economical when they are run continuously and are referred to as base load units. Other plants may require less capital to build but have a higher fuel cost such as oil and natural gas. These plants can usually start quicker and can be dispatched to follow load.

In a power market where generation suppliers bid for supplying power, the bids are accepted from lowest to highest. However, all of the suppliers receive the price set by the highest accepted bidder. Figure 4.10 and 4.11 show example graphs of costs associated with a portfolio of generating plants. The units that can operate profitably for \$10/MWh, run continuously but never receive less than \$15/MWh and can receive as much as $70/MHh. The units that can only operate profitably at $70/MWh, run only during the peak period and receive $70/MWh. This is pricing model is used to reward the most cost efficient units and encourage new units to be more cost efficient to maximize profits.

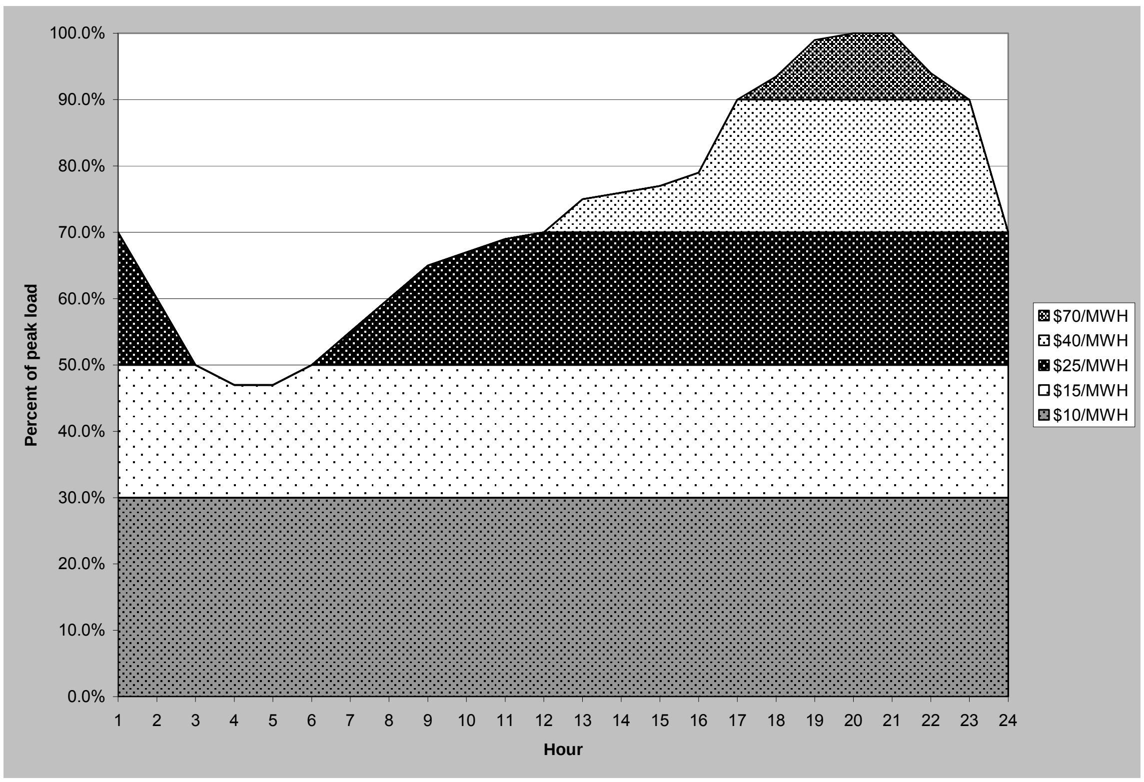

Figure 5.1 – Example graph of generation portfolio cost without DR

For example, a system needs 100 MW for its peak hour to meet its load demand. There are 30 MW at $10/MWH, 20 MW at $15/MWH, 20 MW at $25/MWH, 20 MW at $40/MWH and 10 MW at $70/MWH. Since the price for all the power used is set by the highest bid accepted, the price of all 100 MWh is $70/MWH, totaling $7000 for that hour. The same analysis is done for each hour and the total cost for the day is $71,095.

If DR is used to shift 10 MW of load from the peak hour to an off peak time, then the system only needs 90 MWh for the peak. Now there is no need for the $70/MWH generation and the highest price is $40/MWH, totaling $3600 for that hour. The off peak load has increased as a result but the price of this load is $25/MWH. The total energy cost for the day is now $57,738, an almost 19% reduction. This simplified example illustrates the ability a small amount of peak load has to dramatically impact price.

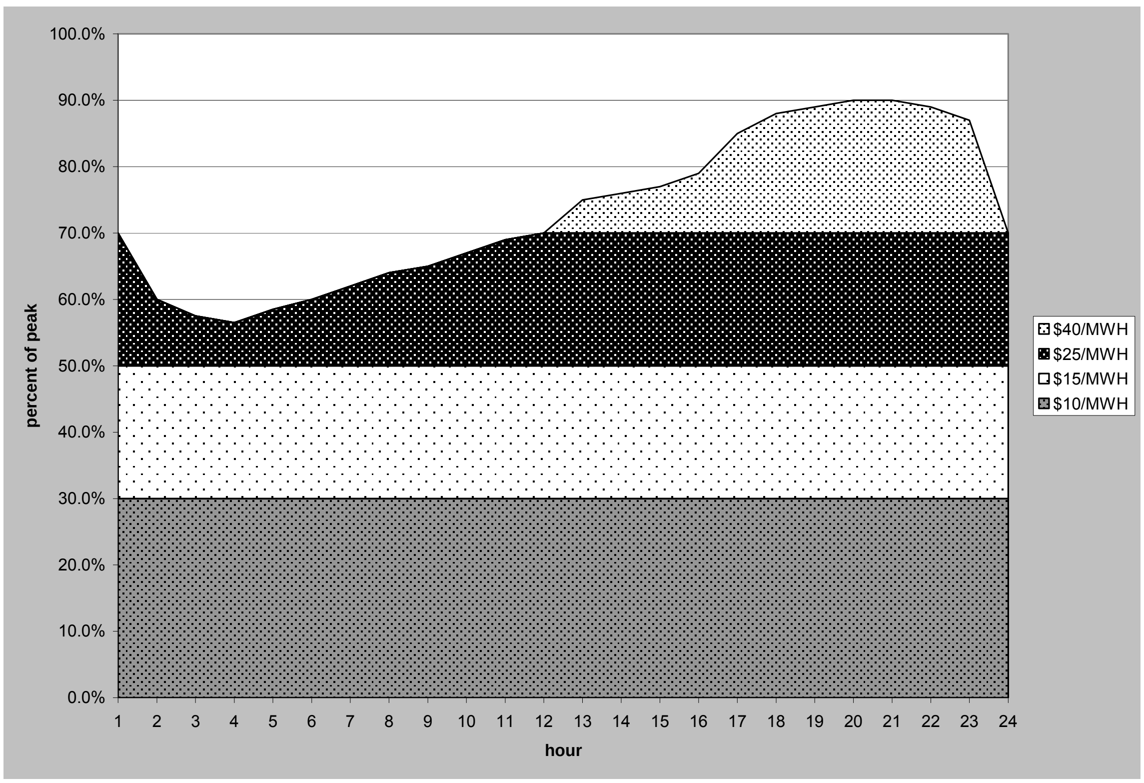

Figure 5.2 - Example graph of generation portfolio cost with DR peak shave/shift

As mentioned previously, the cost of the generation to produce the electricity is only one piece of the total cost to the consumer. The cost of the infrastructure capacity needed to deliver the electricity must also be considered. There are different methods for analyzing the cost of electric system elements. One method to analyze distribution costs is the total annual equivalent cost method. [6]

$$\mathbf{TAC = AIC + AEC + ADC\ \$/mi}$$

TAC is the total annual equivalent cost of the feeder
AIC is the annual equivalent cost of the investment cost of the feeder
AEC is the annual equivalent cost of the I^2R losses
ADC is the annual equivalent cost of the system capacity to supply I^2R losses of the feeder

$$\mathbf{AIC = IC_F \times i_F\ \$/mi}$$

$\mathbf{IC_F}$ is the annual equivalent investment cost of a given size feeder
$\mathbf{i_F}$ is the annual fixed charge rate or carrying rate of the feeder

$$\mathbf{AEC = 3\ I^2R \times EC \times F_{LL} \times F_{LS} \times F_{LSA} \times 8760\ \$/mi}$$

EC is the cost of energy \$/kWh
$\mathbf{F_{LL}}$ is the load location factor
$\mathbf{F_{LS}}$ is the loss factor defined as the ratio of the average annual power loss to the peak annual power loss
$\mathbf{F_{LSA}}$ is the loss allowance factor, which is an allocation factor that allows for additional losses due to transmission from generating plants to distribution substations

$$\mathbf{F_{LL} = S/L}$$

S is the point on the feeder for an assumed lumped feeder load
L is the distance in miles

$\mathbf{F_{LS} = 0.3\ F_{LD} + 0.7\ F^2_{LD}}$, for urban areas

$\mathbf{F_{LS} = 0.16\ F_{LD} + 0.84\ F^2_{LD}}$, for rural areas

$\mathbf{F_{LD}}$ is the load factor, which is the ratio of the average load over a designated period of time to the peak load occurring in that period

$$\mathbf{ADC = 3\ I^2R \times F_{LL} \times F_{PR} \times F_R \times F_{LSA}\ [(C_G \times i_G) + (C_T \times i_T) + (C_S \times i_S)]\ \$/mi}$$

$\mathbf{F_{LL}}$ is the load location factor

F_{PR} is the peak responsibility factor, which is a per unit value of the peak feeder losses that are coincident with the system peak demand
F_R is the reserve factor, which is the ratio of total generation capability to the total load and losses to be supplied
F_{LSA} is the loss allowance factor
C_G is the cost of peaking generation ($/KVA)
i_G is the annual fixed charge rate applicable to the generation system
C_T is the cost of transmission ($/KVA)
i_T is the annual fixed charge rate applicable to the transmission system
C_S is the cost of the distribution substation ($/KVA)
i_S is the annual fixed charge rate applicable to the distribution substation

The analysis presented in this paper focuses on how DR can impact the efficacy of the system which directly affects the cost of the system. The TAC equation for calculating distribution feeder costs can be used as a basis for analyzing how changes in the equation variables affect the costs. The equation can be modified and augmented to accommodate given or calculated data values such as the system energy costs (CE). Central to this approach is the notion that the cost of energy changes with its time of use (TOU). There has been an increasing emphasis in the industry to implement TOU rates to send better pricing signals to consumers. TOU cost structures can be used to modify TAC analysis to include the total cost of energy.

For this modified TAC analysis 24 hourly load flow simulations were run and the energy plus losses for each hour were calculated and recorded. The cost of energy for each hour was assigned and summed for the daily cost of energy. If the annual cost of DR is included then the TAC equation can be expressed as the following.

$$\mathbf{TAC = [IC_F \times i_F] + [CE \times 365] + [F_{PR} \times F_R\, [(C_G \times i_G) + (C_T \times i_T) + (C_S \times i_S)]] + [IC_{DR} \times i_{DR}]\ \$/kVA}$$

The equation was solved for the base case with no DR and the cost was used as the basis for comparison with the DR options. The cost of energy per MW for a day was calculated based on each hour of the load shape and assigned values as shown in the Appendix. The peak responsibility factor was calculated from the losses at the peak hour normalized to the equivalent system load at that hour. Figure 5.3 and 5.4 show the TAC costs for a 1 MW load shift with 100% and 90 % efficiency respectively.

	base	1 MW shift	1 MW #2 DR	1 MW #3 DR	1 MW #4 DR	1 MW all DR
Icf	50	50	50	50	50	50
if	0.12	0.12	0.12	0.12	0.12	0.12
CE	984.79	980.17	981.08	980.58	980.88	980.58
Fpr	1.070	1.021	1.027	1.023	1.025	1.023
Fr	1.15	1.15	1.15	1.15	1.15	1.15
Cg	200	200	200	200	200	200
ig	0.14	0.14	0.14	0.14	0.14	0.14
Ct	150	150	150	150	150	150
it	0.1	0.1	0.1	0.1	0.1	0.1
Cs	130	130	130	130	130	130
is	0.12	0.12	0.12	0.12	0.12	0.12
Icdr	0	0	400	400	400	400
idr	0.14	0.14	0.14	0.14	0.14	0.14
TAC	$359,527	$357,836	$358,227	$358,044	$358,150	$358,044
Savings		0.47%	0.36%	0.41%	0.38%	0.41%

Figure 5.3 – Table of TAC for 1 MW load shift with 100% efficiency

	base	1 MW shift	1 MW #2 DR	1 MW #3 DR	1 MW #4 DR	1 MW all DR
Icf	50	50	50	50	50	50
if	0.12	0.12	0.12	0.12	0.12	0.12
CE	984.79	980.75	981.63	981.17	981.42	981.13
Fpr	1.070	1.021	1.027	1.023	1.025	1.023
Fr	1.15	1.15	1.15	1.15	1.15	1.15
Cg	200	200	200	200	200	200

ig	0.14	0.14	0.14	0.14	0.14	0.14
Ct	150	150	150	150	150	150
it	0.1	0.1	0.1	0.1	0.1	0.1
Cs	130	130	130	130	130	130
is	0.12	0.12	0.12	0.12	0.12	0.12
Icdr	0	0	400	400	400	400
idr	0.14	0.14	0.14	0.14	0.14	0.14
TAC	$359,527	$358,049	$358,424	$358,257	$358,348	$358,242
Savings		0.41%	0.31%	0.35%	0.33%	0.36%

Figure 5.4 – Table of TAC for 1 MW load shift with 90% efficiency

Figure 5.5 and 5.6 show the TAC costs for a 3 MW load shift with 100% and 90 % efficiency respectively.

	base	3 MW shift	3 MW #2 DR	3 MW #3 DR	3 MW #4 DR	3 MW all DR
Icf	50	50	50	50	50	50
if	0.12	0.12	0.12	0.12	0.12	0.12
CE	984.79	971.50	974.13	973.54	973.88	973.04
Fpr	1.070	0.924	0.942	0.935	0.939	0.935
Fr	1.15	1.15	1.15	1.15	1.15	1.15
Cg	200	200	200	200	200	200
ig	0.14	0.14	0.14	0.14	0.14	0.14
Ct	150	150	150	150	150	150
it	0.1	0.1	0.1	0.1	0.1	0.1
Cs	130	130	130	130	130	130
is	0.12	0.12	0.12	0.12	0.12	0.12
Icdr	0	0	1200	1200	1200	1200
idr	0.14	0.14	0.14	0.14	0.14	0.14
TAC	$359,527	$354,666	$355,793	$355,580	$355,702	$355,397
Savings		1.35%	1.04%	1.10%	1.06%	1.15%

Figure 5.5 – Table of TAC for 3 MW load shift with 100% efficiency

	base	3 MW shift	3 MW #2 DR	3 MW #3 DR	3 MW #4 DR	3 MW all DR
Icf	50	50	50	50	50	50
if	0.12	0.12	0.12	0.12	0.12	0.12

CE	984.79	973.17	975.75	975.29	975.50	974.67
Fpr	1.070	0.924	0.942	0.935	0.939	0.935
Fr	1.15	1.15	1.15	1.15	1.15	1.15
Cg	200	200	200	200	200	200
ig	0.14	0.14	0.14	0.14	0.14	0.14
Ct	150	150	150	150	150	150
it	0.1	0.1	0.1	0.1	0.1	0.1
Cs	130	130	130	130	130	130
is	0.12	0.12	0.12	0.12	0.12	0.12
Icdr	0	0	1200	1200	1200	1200
idr	0.14	0.14	0.14	0.14	0.14	0.14
TAC	$359,527	$355,274	$356,386	$356,218	$356,295	$355,990
Savings		1.18%	0.87%	0.92%	0.90%	0.98%

Figure 5.6 – Table of TAC for 3 MW load shift with 90% efficiency

The results show that when the total cost of energy is included in the analysis it is possible to have a lower TAC with DR implemented on the feeder, even when the DR is not 100% efficient. The major driver for the decreased costs is the replacement of high cost electricity with lower cost electricity. The replacement of peak system losses with off peak system losses also contribute a little to the savings. The ratio of peak load to off peak load and the ratio of peak electricity cost to off peak electricity cost will be the limiting factors in the ability of DR to lower the TAC.

6 Conclusion

The ability of DR, used for peak shaving, to improve system efficacy was explored. Models representing the basic elements of electric systems were created. Load flow analysis was used on the models to obtain simulation results of the system performance. Base case simulations were run without DRs in the models and used as the basis to compare to system configurations with DR. The amount of system losses was analyzed with different sizes and locations of DR to determine impact on efficiency. The ability of DR to impact the load factor and capacity utilization factor were explored. The ability of DR to impact the cost of energy and infrastructure was examined.

The capacity utilization factor was used as a gage to measure changes in system efficacy. DR can improve the capacity utilization factor of an existing system while the total energy delivered remains the same. This enables the system to have an increase in unused capacity to serve future increases in load. DR can be used in analysis of future system expansion to allow for higher capacity utilization projections. Therefore, capacity requirements are smaller for projected energy delivery needs.

It was shown that the type, size, and location of the DR have an impact on system losses and hence efficiency. DR applications that are supplying power, such as distributed generation, have the greatest ability to reduce system losses. For sizes up to about 50% of a uniformly distributed feeder load, locations at the end of the feeder reduce the losses the most. For sizes over 50%, the greatest loss reduction is seen as the location is moved from the end to the middle of the feeder.

It was shown that having more than one DR location reduces the losses more than having one location of an equivalent size. This is due to the ability of having more of the current produced closer to the varied load locations. The closer the source is to the load the less the line losses are compared to the base case. Also, having more than one DR on a feeder allows for N-1 firm capacity to include the remaining DR in calculations.

For DR that stores energy for peak shaving, the charging or storage losses becomes the driving factor in its ability to reduce overall system losses. It was shown that peak shaving can shift the load and losses to an off peak time. For DR with close to 100% efficiency the savings in peak system losses makes the devices a lossless storage device to the system. For DR with lower storage efficiency the reduction in peak load also has benefits beyond a loss analysis. If the differential between the cost of energy during the peak and off peak is enough, then overall cost can still be lowered.

The advantages of load based DR verse distribution based DR were presented. Load based DR provides the ability to partially decouple the devices electric use from the system use. This gives the consumer the ability to obtain the benefit of the device when desired instead of going without at peak times. Storage losses are greatly reduced compared to other techniques where the storage requires energy conversion to store and energy conversion to supply. There are no system protection implications since the storage is load based with no additional supply points. The amount of DR is inherently scalable with the load since it is part of the load.

The analysis in this paper has shown that there are several benefits to having DR: 1) it can provide voltage support and reduce system losses by having the supply source closer to the load, 2) the ability to reduce system peaks which reduces capacity needs and peak losses, 3) the ability to maximize the use of the most efficient supplies through storage, and 4) small amounts of off peak storage losses can be offset by reduced peak losses.

These features can help lead to a more efficacious system. To maximize the benefit obtained from these resources requires the integration of DR into the system. The size and location of the DR has an impact on the overall efficacy and efficiency of the system. The ability to obtain a flatter load curve without increasing system losses is an enabling factor in obtaining a more efficacious electric system.

References

1. Stevenson, William D. Jr., “Elements of Power Systems Analysis”, Fourth Edition, McGraw-Hill Book Co., New York 1982, pg. 2, 54-60
2. U.S. Department of Energy, “Annual Energy Review, 2008”, June 2009.
3. Bergen, Arthur R., “Power Systems Analysis”, Prentice-Hall, Inc., New Jersey 1986, Pg. 3-5
4. Bergen, Arthur R., Vittal, Vijay, “Power Systems Analysis”, Second Edition, Prentice-Hall, Inc., New Jersey 1986, Pg. 3-5
5. Mohan, Ned, Undeland, Tore M., Robbins, William P., “Power Electronics, Converters, Applications, and Design”, Third Edition, John Wiley & Sons, Inc., United States 2003, pg. 3
6. Gönen, Turan, “Electric Power Distribution System Engineering”, Second Edition, CRC Press, Florida 2008, Pg. 357
7. Chiradeja, P., “Benefit of Distributed Generation: A Line Loss Reduction Analysis”, IEEE/PES Transmission and Distribution Conference & Exhibition: Asia and Pacific Dalina, China, 2005
8. Nourai, Ali, et al, “Load Leveling Reduces T&D Line Losses”, IEEE transactions on power delivery, Vol 23, No. 4, October 2008

Appendices

Appendix A: Impedance calculations

The distribution line impedances were calculated assuming a typical overhead cross-arm construction as shown in Figure 2. The phase spacing was 44" and 336 AL wire was used.

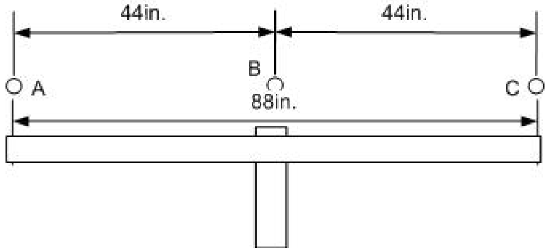

Figure 2 – Distribution feeder geometry

The inductance of the feeder was calculated using geometric mean radius (GMR), designated as D_s (Stevenson, p54). For 336 kcmil conductor the geometric mean radius Ds = .0243', from Stevenson Table A.1. Since the feeder has three phases the average inductance (La) is used. The average inductance is calculated by the following equation.

$$La = 2x10^{-7} \ln \frac{Deq}{Ds} \text{ H/m}$$ *[Stevenson p60]*

Deq is derived from the following equation.

$$Deq = \sqrt[3]{DabxDbcxDac}$$ *[Stevenson p60]*

For the model feeder

$$Deq = \sqrt[3]{44x44x88} = 55.44\,in = 4.62\ ft$$

$$La = 2x10^{-7} \ln \frac{4.62}{.0243} = 10.5x10^{-7} \text{ H/m}$$

This can be converted to inductive reactance (X) and English units with the following.

$$X = 2\pi\, fL = 2\pi 60x1609x10.5x10^{-7} = .636\,\Omega / mi = .121\,\Omega / 1000\ ft$$

The resistance (R) for 336 AL conductor is

$R = .2737\,\Omega / mi = .053\,\Omega / 1000\ ft$

This can be represented in impedance form (Z).

$Z = R + jX = .053 + j.121\,\Omega / 1000\ ft$

In order to use these values in the power flow model, they must be converted to per unit values. For the model a base of 100 MVA and 13.8kV was chosen. This gives a base impedance.

$$Zbase = \frac{Vbase}{VAbase} = \frac{13.8kV}{100MVA} = 1.9044$$

$$Zpu = \frac{.053 + j.121}{1.9044} = .0278 + j.3353\Omega / 1000\ ft\ pu$$

For the model feeder each segment is about 2750 ft which gives a segment impedance of .08+j.17. The impedances of transformer were taken from typical values in the industry, .035+j.57 on a per unit basis.

Appendix B: Matlab program to solve Gauss-Seidel

Matlab program to solve power flow of feeder model

```
clear
%Paul Robinson
%Gauss-Seidel power flow program
%This program solves a power flow with Gauss-Seidel solution
%
%bus data: bus num, type, voltage, angle , Pl, Ql, Pg, Qg
%bus type: 0=load P&Q, 1=Gen Q hold V limit,2=hold V Var limit,
%3=hold V&ang swing or slack
% bus data in pu at 100 MVA
busd =...
   [1  3 1 0 0     0     0 0;
    2  0 1 0 0     0     0 0;
    3  0 1 0 0.010 0.003 0 0;
    4  0 1 0 0.010 0.003 0 0;
    5  0 1 0 0.010 0.003 0 0;
    6  0 1 0 0.010 0.003 0 0;
    7  0 1 0 0.010 0.003 0 0;
    8  0 1 0 0.010 0.003 0 0;
    9  0 1 0 0.010 0.003 0 0;
    10 0 1 0 0.010 0.003 0 0;
    11 0 1 0 0.010 0.003 0 0;
    12 0 1 0 0.010 0.003 0 0];
%line format: from bus, to bus, Rpu Xpu
lined =...
[1   2 .035+0.57j;
 2   3 .08+0.17j;
 3   4 .08+0.17j;
 4   5 .08+0.17j;
 5   6 .08+0.17j;
 6   7 .08+0.17j;
 7   8 .08+0.17j;
 8   9 .08+0.17j;
 9  10 .08+0.17j;
 10 11 .08+0.17j;
 11 12 .08+0.17j];
% bus count and lines
nbuses = 12;
nlines = size(lined,1);
fbus = lined(:,1);
tbus = lined(:,2);
Rpu = real(lined(:,3));
Xpu = imag(lined(:,3));
ybr = 1./lined(:,3);

% calc admitance matrix
Ymat = zeros(nbuses);
for line = 1:nlines
   yline = ybr(line);
   f = fbus(line);
   t = tbus(line);
```

```
   Ymat(f,f) = Ymat(f,f) + yline;
   if( t ~= 0)
      Ymat(t,t) = Ymat(t,t) + yline;
      Ymat(f,t) = Ymat(f,t) - yline;
      Ymat(t,f) = Ymat(t,f) - yline;
   end
end
tol = 0.00001;
iter = 0;

% Calc the real and reactive power supplied to buses

for i = 1:nbuses
   busp(i) = busd(i,7) - busd(i,5);
   busq(i) = busd(i,8) - busd(i,6);

end
% voltage at bus
for i= 1:nbuses
    busv(i) = busd(i,3);
end
%perform iterations to solve all matrix values
while (iter < 5000)
  iter = iter + 1;
  busvo = busv;
%calc the updated bus voltage
  for i = 1:nbuses
     if busd(i,2)~= 3
% If gen bus, calc updated reactive
        if busd(i,2)== 2
           reac = 0;
           for k = 1:nbuses
            reac = reac + Ymat(i,k) * busd(k,3);
           end
            reac = conj(busd(i,3)) * reac;
            busq(i) = -imag(reac);
        end

% Calculate updated voltage at bus from all buses
        svi = 0;
        for k = 1:nbuses
            if i ~= k
              svi = svi - Ymat(i,k) * busd(k,3);
            end
        end
        svi = (busp(i) - j*busq(i)) / conj(busd(i,3)) + svi;
        busd(i,3) = 1/Ymat(i,i) * svi;

% If gen bus, update the volt mag
         if busd(i,2)== 2
            busd(i,3) = busd(i,3) * abs(busvo(i)) / abs(busd(i,3));
         end
      end
  end

 for i= 1:nbuses
```

```
     busv(i)= busd(i,3);
 end
 % check if within tolerence
   mism = busv - busvo;
   if max(abs(mism)) < tol
      break;
   end
end
%end of while statement

for i=1:nbuses
% if slack bus update the power
  if busd(i,2)==3
      slk = 0;
      for k = 1:nbuses
          slk = slk + Ymat(i,k)*busd(k,3);
      end
      slk = conj(busd(i,3))*slk;
      busp(i) = real(slk);
      busq(i) = -imag(slk);
  end
end

% reformat data for output
for i = 1:nbuses
% pu voltage
  mag = abs(busd(i,3));
  phase = angle(busd(i,3))*180/pi;
  busd(i,3)=mag;
  busd(i,4)=phase;
% Power load
  busd(i,5)=busd(i,5)*100;
  busd(i,6)=busd(i,6)*100;
% Power gen
%  P = (busp(i) +bus(i,7))*100;
%  Q = (busq(i) +bus(i,8))*100;
  P = (busp(i))*100;
  Q = (busq(i))*100;
  if busd(i,2)==0
      P=0;
      Q=0;
  end
  busd(i,7)=P;
  busd(i,8)=Q;
end
% print out bus matrix and iterations
fprintf('--------------------------------------------------------------
----------------\n');
fprintf('   Bus#       Type      Volt      angle       Pld       Qld
Pgen     Qgen\n');
fprintf('==============================================================
================\n');
busd
fprintf('--------------------------------------------------------------
----------------\n');
iter
```

```
% Calc and print losses header
fprintf('\n');
fprintf('                      Losses                          \n');
fprintf('-------------------------------------------------------\n');
fprintf('   Line          From           To       Loss      Loss    \n');
fprintf('    #             Bus          Bus       (MW)     (MVAR)   \n');
fprintf('=======================================================\n');

% Initialize total loss
Pltot = 0;
Qltot = 0;

for i = 1:nlines;
   fb = fbus(i);
   tb = tbus(i);
   il = (busv(fb) - busv(tb)) * Ymat(fb,tb);
   Pl = abs(il)^2 * Rpu(i)*100;
   Ql = abs(il)^2 * Xpu(i)*100;

   Pltot = Pltot + Pl;
   Qltot = Qltot + Ql;

% print losses
   fprintf('   %4d',i);
   fprintf('   %10d',fbus(i));
   fprintf('   %10d',tbus(i));
   fprintf('   %7.3f', Pl );
   fprintf('   %7.3f\n', Ql );
end

% print total losses
fprintf('-------------------------------------------------------\n');
fprintf('                          Totals:');
fprintf('   %7.3f', Pltot );
fprintf('   %7.3f\n', Qltot );
```

Appendix C: Modified Matpower program

Modified Matpower Matlab program to scale loads from program input

```
function [MVAbase, bus, gen, branch, success, et] = ...
                runspf(casename, mpopt, fname, scale, solvedcase)
%RUNSPF  Runs a power flow.
%
%   [baseMVA, bus, gen, branch, success, et] = ...
%           runspf(casename, mpopt, fname, scale, solvedcase)
%
%   Modified to read in scale factor to scale loads PER 03/29/09
%
%   Runs a power flow (full AC Newton's method by default) and optionally
%   returns the solved values in the data matrices, a flag which is true if
%   the algorithm was successful in finding a solution, and the elapsed time
%   in seconds. All input arguments are optional. If casename is provided it
%   specifies the name of the input data file or struct (see also 'help
%   caseformat' and 'help loadcase') containing the power flow data. The
%   default value is 'case9'. If the mpopt is provided it overrides the
%   default MATPOWER options vector and can be used to specify the solution
%   algorithm and output options among other things (see 'help mpoption' for
%   details). If the 3rd argument is given the pretty printed output will be
%   appended to the file whose name is given in fname. If solvedcase is
%   specified the solved case will be written to a case file in MATPOWER
%   format with the specified name. If solvedcase ends with '.mat' it saves
%   the case as a MAT-file otherwise it saves it as an M-file.
%
%   If the ENFORCE_Q_LIMS options is set to true (default is false) then if
%   any generator reactive power limit is violated after running the AC power
%   flow, the corresponding bus is converted to a PQ bus, with Qg at the
%   limit, and the case is re-run. The voltage magnitude at the bus will
%   deviate from the specified value in order to satisfy the reactive power
%   limit. If the reference bus is converted to PQ, the first remaining PV
%   bus will be used as the slack bus for the next iteration. This may
%   result in the real power output at this generator being slightly off
%   from the specified values.

%   MATPOWER
%   $Id: runpf.m,v 1.14 2006/09/29 19:23:07 ray Exp $
%   by Ray Zimmerman, PSERC Cornell
%   Enforcing of generator Q limits inspired by contributions
%   from Mu Lin, Lincoln University, New Zealand (1/14/05).
%   Copyright (c) 1996-2005 by Power System Engineering Research Center
(PSERC)
%   See http://www.pserc.cornell.edu/matpower/ for more info.

%%-----  initialize  -----
%% define named indices into bus, gen, branch matrices
[PQ, PV, REF, NONE, BUS_I, BUS_TYPE, PD, QD, GS, BS, BUS_AREA, VM, ...
    VA, BASE_KV, ZONE, VMAX, VMIN, LAM_P, LAM_Q, MU_VMAX, MU_VMIN] = idx_bus;
[F_BUS, T_BUS, BR_R, BR_X, BR_B, RATE_A, RATE_B, RATE_C, ...
    TAP, SHIFT, BR_STATUS, PF, QF, PT, QT, MU_SF, MU_ST, ...
    ANGMIN, ANGMAX, MU_ANGMIN, MU_ANGMAX] = idx_brch;
```

```
[GEN_BUS, PG, QG, QMAX, QMIN, VG, MBASE, GEN_STATUS, PMAX, PMIN, ...
    MU_PMAX, MU_PMIN, MU_QMAX, MU_QMIN, PC1, PC2, QC1MIN, QC1MAX, ...
    QC2MIN, QC2MAX, RAMP_AGC, RAMP_10, RAMP_30, RAMP_Q, APF] = idx_gen;

%% default arguments
if nargin < 5
    solvedcase = '';                %% don't save solved case
   if nargin < 4
       scale = 1.0;                 %% don't scale loads
    if nargin < 3
        fname = '';                 %% don't print results to a file
        if nargin < 2
            mpopt = mpoption;       %% use default options
            if nargin < 1
                casename = 'case9'; %% default data file is 'case9.m'
            end
        end
    end
   end
end

%% options
verbose = mpopt(31);
qlim = mpopt(6);                    %% enforce Q limits on gens?
dc = mpopt(10);                     %% use DC formulation?

%% read data & convert to internal bus numbering
[baseMVA, bus, gen, branch] = loadcase(casename);
[i2e, bus, gen, branch] = ext2int(bus, gen, branch);
%% scale PD and QD
ldbs = find(bus(:, BUS_TYPE)==1);
bus(ldbs, PD) = bus(ldbs, PD)*scale;
bus(ldbs, QD) = bus(ldbs, QD)*scale;
%bus(:, PD) = bus(:, PD)*scale;
%bus(:, QD) = bus(:, QD)*scale;
%% get bus index lists of each type of bus
[ref, pv, pq] = bustypes(bus, gen);

%% generator info
on = find(gen(:, GEN_STATUS) > 0);      %% which generators are on?
gbus = gen(on, GEN_BUS);                %% what buses are they at?

%%-----  run the power flow  -----
t0 = clock;
if dc                               %% DC formulation
    %% initial state
    Va0 = bus(:, VA) * (pi/180);

    %% build B matrices and phase shift injections
    [B, Bf, Pbusinj, Pfinj] = makeBdc(baseMVA, bus, branch);

    %% compute complex bus power injections (generation - load)
    %% adjusted for phase shifters and real shunts
    Pbus = real(makeSbus(baseMVA, bus, gen)) - Pbusinj - bus(:, GS) /
baseMVA;
```

```
    %% "run" the power flow
    Va = dcpf(B, Pbus, Va0, ref, pv, pq);

    %% update data matrices with solution
    branch(:, [QF, QT]) = zeros(size(branch, 1), 2);
    branch(:, PF) = (Bf * Va + Pfinj) * baseMVA;
    branch(:, PT) = -branch(:, PF);
    bus(:, VM) = ones(size(bus, 1), 1);
    bus(:, VA) = Va * (180/pi);
    %% update Pg for swing generator (note: other gens at ref bus are
accounted for in Pbus)
    %%      Pg = Pinj + Pload + Gs
    %%      newPg = oldPg + newPinj - oldPinj
    refgen = find(gbus == ref);             %% which is(are) the reference
gen(s)?
    gen(on(refgen(1)), PG) = gen(on(refgen(1)), PG) + (B(ref, :) * Va -
Pbus(ref)) * baseMVA;

    success = 1;
else                                %% AC formulation
    %% initial state
    % V0    = ones(size(bus, 1), 1);            %% flat start
    V0  = bus(:, VM) .* exp(sqrt(-1) * pi/180 * bus(:, VA));
    V0(gbus) = gen(on, VG) ./ abs(V0(gbus)).* V0(gbus);

    if qlim
        ref0 = ref;                         %% save index and angle of
        Varef0 = bus(ref0, VA);             %%   original reference bus
        limited = [];                       %% list of indices of gens @ Q
lims
        fixedQg = zeros(size(gen, 1), 1);   %% Qg of gens at Q limits
    end
    repeat = 1;
    while (repeat)
        %% build admittance matrices
        [Ybus, Yf, Yt] = makeYbus(baseMVA, bus, branch);

        %% compute complex bus power injections (generation - load)
        Sbus = makeSbus(baseMVA, bus, gen);

        %% run the power flow
        alg = mpopt(1);
        if alg == 1
            [V, success, iterations] = newtonpf(Ybus, Sbus, V0, ref, pv, pq,
mpopt);
        elseif alg == 2 | alg == 3
            [Bp, Bpp] = makeB(baseMVA, bus, branch, alg);
            [V, success, iterations] = fdpf(Ybus, Sbus, V0, Bp, Bpp, ref, pv,
pq, mpopt);
        elseif alg == 4
            [V, success, iterations] = gausspf(Ybus, Sbus, V0, ref, pv, pq,
mpopt);
        else
            error('Only Newton''s method, fast-decoupled, and Gauss-Seidel
power flow algorithms currently implemented.');
```

```
        end

        %% update data matrices with solution
        [bus, gen, branch] = pfsoln(baseMVA, bus, gen, branch, Ybus, Yf, Yt,
V, ref, pv, pq);

        if qlim             %% enforce generator Q limits
            %% find gens with violated Q constraints
            mx = find( gen(:, GEN_STATUS) > 0 & gen(:, QG) > gen(:, QMAX) );
            mn = find( gen(:, GEN_STATUS) > 0 & gen(:, QG) < gen(:, QMIN) );

            if ~isempty(mx) | ~isempty(mn)  %% we have some Q limit
violations
                if isempty(pv)
                    if verbose
                        if ~isempty(mx)
                            fprintf('Gen %d (only one left) exceeds upper Q
limit : INFEASIBLE PROBLEM\n', mx);
                        else
                            fprintf('Gen %d (only one left) exceeds lower Q
limit : INFEASIBLE PROBLEM\n', mn);
                        end
                    end
                    success = 0;
                    break;
                end
                if verbose & ~isempty(mx)
                    fprintf('Gen %d at upper Q limit, converting to PQ
bus\n', mx);
                end
                if verbose & ~isempty(mn)
                    fprintf('Gen %d at lower Q limit, converting to PQ
bus\n', mn);
                end

                %% save corresponding limit values
                fixedQg(mx) = gen(mx, QMAX);
                fixedQg(mn) = gen(mn, QMIN);
                mx = [mx;mn];

                %% convert to PQ bus
                gen(mx, QG) = fixedQg(mx);      %% set Qg to binding limit
                gen(mx, GEN_STATUS) = 0;        %% temporarily turn off gen,
                for i = 1:length(mx)            %% (one at a time, since
                    bi = gen(mx(i), GEN_BUS);   %%  they may be at same bus)
                    bus(bi, [PD,QD]) = ...      %% adjust load accordingly,
                        bus(bi, [PD,QD]) - gen(mx(i), [PG,QG]);
                end
                bus(gen(mx, GEN_BUS), BUS_TYPE) = PQ;   %% & set bus type to
PQ

                %% update bus index lists of each type of bus
                ref_temp = ref;
                [ref, pv, pq] = bustypes(bus, gen);
                if verbose & ref ~= ref_temp
                    fprintf('Bus %d is new slack bus\n', ref);
```

```
                end
                limited = [limited; mx];
            else
                repeat = 0; %% no more generator Q limits violated
            end
        else
            repeat = 0;     %% don't enforce generator Q limits, once is
enough
        end
    end
    if qlim & ~isempty(limited)
        %% restore injections from limited gens (those at Q limits)
        gen(limited, QG) = fixedQg(limited);    %% restore Qg value,
        for i = 1:length(limited)               %% (one at a time, since
            bi = gen(limited(i), GEN_BUS);      %%  they may be at same bus)
            bus(bi, [PD,QD]) = ...              %% re-adjust load,
                bus(bi, [PD,QD]) + gen(limited(i), [PG,QG]);
        end
        gen(limited, GEN_STATUS) = 1;               %% and turn gen back on
        if ref ~= ref0
            %% adjust voltage angles to make original ref bus correct
            bus(:, VA) = bus(:, VA) - bus(ref0, VA) + Varef0;
        end
    end
end
et = etime(clock, t0);

%%-----  output results  -----
%% convert back to original bus numbering & print results
[bus, gen, branch] = int2ext(i2e, bus, gen, branch);
if fname
    [fd, msg] = fopen(fname, 'at');
    if fd == -1
        error(msg);
    else
        printpf(baseMVA, bus, gen, branch, [], success, et, fd, mpopt);
        fclose(fd);
    end
end
printpf(baseMVA, bus, gen, branch, [], success, et, 1, mpopt);

%% save solved case
if solvedcase
    savecase(solvedcase, baseMVA, bus, gen, branch);
end

%% this is just to prevent it from printing baseMVA
%% when called with no output arguments
if nargout, MVAbase = baseMVA; end

return;
```

Appendix D: Matlab model used for simple radial analysis

```
function [baseMVA, bus, gen, branch] = case1
%CASE1  Power flow data for 12 bus, 11 gen case for thesis.
%   Please see 'help caseformat' for details on the case file format.
%
%   This is the 12 bus example for thesis

%   MATPOWER

%%-----  Power Flow Data  -----%%
%% system MVA base
baseMVA = 100;

%% bus data
%   bus_i   type    Pd  Qd  Gs  Bs  area    Vm  Va  baseKV  zone
Vmax    Vmin
bus = [
    1   3   0       0       0   0   1   1    0   115     1   1.1 0.9;
    2   1   0       0       0   0   1   1    0   13.8    1   1.1 0.9;
    3   2   1.0     .30     0   0   1   1    0   13.8    1   1.1 0.9;
    4   2   1.0     .30     0   0   1   1    0   13.8    1   1.1 0.9;
    5   2   1.0     .30     0   0   1   1    0   13.8    1   1.1 0.9;
    6   2   1.0     .30     0   0   1   1    0   13.8    1   1.1 0.9;
    7   2   1.0     .30     0   0   1   1    0   13.8    1   1.1 0.9;
    8   2   1.0     .30     0   0   1   1    0   13.8    1   1.1 0.9;
    9   2   1.0     .30     0   0   1   1    0   13.8    1   1.1 0.9;
    10  2   1.0     .30     0   0   1   1    0   13.8    1   1.1 0.9;
    11  2   1.0     .30     0   0   1   1    0   13.8    1   1.1 0.9;
    12  2   1.0     .30     0   0   1   1    0   13.8    1   1.1 0.9;
];

%% generator data
%   bus Pg  Qg  Qmax    Qmin    Vg  mBase   status  Pmax    Pmin
gen = [
    12  5   0   .2  -.2     1   100 01  1.1 0;
    11  5   0   .2  -.2     1   100 0   1.1 0;
    10  5   0   .2  -.2     1   100 0   1.1 0;
    9   5   0   .2  -.2     1   100 0   1.1 0;
    8   5   0   .2  -.2     1   100 0   1.1 0;
    7   5   0   .2  -.2     1   100 0   1.1 0;
    6   5   0   .2  -.2     1   100 0   1.1 0;
    5   5   0   .2  -.2     1   100 0   1.1 0;
    4   5   0   .2  -.2     1   100 0   1.1 0;
    3   5   0   .2  -.2     1   100 0   1.1 0;
    1   0   0   100 -100    1   100 1   0   0;
];

%% branch data
%   fbus    tbus    r   x   b   rateA   rateB   rateC   ratio   angle
status
```

```
branch = [
    1   2   0.035   0.57    0.0000  250 250 250 0   0   1;
    2   3   0.08    0.17    0.0000  250 250 250 0   0   1;
    3   4   0.08    0.17    0.0000  250 250 250 0   0   1;
    4   5   0.08    0.17    0.0000  250 250 250 0   0   1;
    5   6   0.08    0.17    0.0000  250 250 250 0   0   1;
    6   7   0.08    0.17    0.0000  250 250 250 0   0   1;
    7   8   0.08    0.17    0.0000  250 250 250 0   0   1;
    8   9   0.08    0.17    0.0000  250 250 250 0   0   1;
    9   10  0.08    0.17    0.0000  250 250 250 0   0   1;
    10  11  0.08    0.17    0.0000  250 250 250 0   0   1;
    11  12  0.08    0.17    0.0000  250 250 250 0   0   1;
];

return;
```

Appendix E: Matlab model used for multi-circuit analysis

```
function [baseMVA, bus, gen, branch] = case2
%CASE2  Power flow data for 24 bus, 4 gen case for thesis.
%   Please see 'help caseformat' for details on the case file format.
%
%   This is the 24 bus example for thesis

%   MATPOWER

%%-----  Power Flow Data  -----%%
%% system MVA base
baseMVA = 100;

%% bus data
%   bus_i   type    Pd  Qd  Gs  Bs  area    Vm  Va  baseKV  zone
Vmax    Vmin
bus = [
    1   3   0       0       0   0   1   1   0   115     1   1.1 0.9;
    2   2   0       0       0   0   1   1   0   13.8    1   1.1 0.9;
    3   2   0       0       0   0   1   1   0   13.8    1   1.1 0.9;
    4   2   0       0       0   0   1   1   0   13.8    1   1.1 0.9;
    5   1   0       0       0   0   1   1   0   13.8    1   1.1 0.9;
    6   1   0       0       0   0   1   1   0   13.8    1   1.1 0.9;
    7   1   0       0       0   0   1   1   0   13.8    1   1.1 0.9;
    8   1   0       0       0   0   1   1   0   13.8    1   1.1 0.9;
    9   1   0       0       0   0   1   1   0   13.8    1   1.1 0.9;
    10  1   0       0       0   0   1   1   0   13.8    1   1.1 0.9;
    11  1   0       0       0   0   1   1   0   13.8    1   1.1 0.9;
    12  1   0       0       0   0   1   1   0   13.8    1   1.1 0.9;
    13  1   0       0       0   0   1   1   0   13.8    1   1.1 0.9;
    14  1   0       0       0   0   1   1   0   13.8    1   1.1 0.9;
    15  1   3.000   1.000   0   0   1   1   0   13.8    1   1.1 0.9;
    16  1   3.000   1.000   0   0   1   1   0   13.8    1   1.1 0.9;
    17  1   3.000   1.000   0   0   1   1   0   13.8    1   1.1 0.9;
    18  1   3.000   1.000   0   0   1   1   0   13.8    1   1.1 0.9;
    19  1   3.000   1.000   0   0   1   1   0   13.8    1   1.1 0.9;
    20  1   3.000   1.000   0   0   1   1   0   13.8    1   1.1 0.9;
    21  1   3.000   1.000   0   0   1   1   0   13.8    1   1.1 0.9;
    22  1   3.000   1.000   0   0   1   1   0   13.8    1   1.1 0.9;
    23  1   0       0       0   0   1   1   0   13.8    1   1.1 0.9;
    24  1   0       0       0   0   1   1   0   115     1   1.1 0.9;
];

%% generator data
%   bus Pg  Qg  Qmax    Qmin    Vg  mBase   status  Pmax    Pmin
gen = [
    4   1   0   .2  -.2     1   100 0   1.1 0;
    3   6   0   .2  -.2     1   100 1   1.1 0;
    2   1   0   .2  -.2     1   100 0   1.1 0;
    1   0   0   100 -100    1   100 1   0   0;
];

%% branch data
```

```
%   fbus    tbus    r   x   b   rateA   rateB   rateC   ratio   angle
status
branch = [
    1   24  0.0100  0.100   0.0000  250 250 250 0   0   1;
    24  23  0.035   0.57    0.0000  250 250 250 0   0   1;
    23  2   0.380   1.33    0.0000  250 250 250 0   0   1;
    23  5   0.1468  0.3353  0.0000  250 250 250 0   0   1;
    5   6   0.1468  0.3353  0.0000  250 250 250 0   0   1;
    6   7   0.1468  0.3353  0.0000  250 250 250 0   0   1;
    7   13  0.1468  0.3353  0.0000  250 250 250 0   0   1;
    13  3   0.380   1.33    0.0000  250 250 250 0   0   1;
    7   17  0.38    1.33    0.0000  250 250 250 0   0   1;
    6   16  0.38    1.33    0.0000  250 250 250 0   0   1;
    5   15  0.38    1.33    0.0000  250 250 250 0   0   1;
    23  9   0.1468  0.3353  0.0000  250 250 250 0   0   1;
    9   10  0.1468  0.3353  0.0000  250 250 250 0   0   1;
    10  11  0.1468  0.3353  0.0000  250 250 250 0   0   1;
    11  21  0.38    1.33    0.0000  250 250 250 0   0   1;
    10  20  0.38    1.33    0.0000  250 250 250 0   0   1;
    9   19  0.38    1.33    0.0000  250 250 250 0   0   1;
    23  12  0.1468  0.3353  0.0000  250 250 250 0   0   1;
    12  14  0.1468  0.3353  0.0000  250 250 250 0   0   1;
    14  8   0.1468  0.3353  0.0000  250 250 250 0   0   1;
    8   18  0.38    1.33    0.0000  250 250 250 0   0   1;
    14  4   0.380   1.33    0.0000  250 250 250 0   0   1;
    12  22  0.38    1.33    0.0000  250 250 250 0   0   1;
];

return;
```

Appendix F: Sample Matlab power flow output

```
================================================================================
|     System Summary                                                           |
================================================================================

How many?                How much?              P (MW)            Q (MVAr)
---------------------    -------------------    -------------    -----------------
Buses              12     Total Gen Capacity      11.0        -102.0 to 102.0
Generators         11     On-line Capacity         1.1        -100.2 to 100.2
Committed Gens      2     Generation (actual)     10.3             3.9
Loads              10     Load                    10.0             3.0
  Fixed            10       Fixed                 10.0             3.0
  Dispatchable      0       Dispatchable           0.0 of 0.0      0.0
Shunts              0     Shunt (inj)              0.0             0.0
Branches           11     Losses (I^2 * Z)         0.35            0.95
Transformers        0     Branch Charging (inj)     -              0.0
Inter-ties          0     Total Inter-tie Flow     0.0             0.0
Areas               1

                          Minimum                      Maximum
                 -------------------------  --------------------------------
Voltage Magnitude   0.900 p.u. @ bus 12         1.000 p.u. @ bus 1
Voltage Angle      -6.42 deg   @ bus 12         0.00 deg   @ bus 1
P Losses (I^2*R)             -                  0.08 MW    @ line 3-4
Q Losses (I^2*X)             -                  0.24 MVAr  @ line 1-2

================================================================================
|     Bus Data                                                                 |
================================================================================
 Bus      Voltage          Generation             Load
  #   Mag(pu) Ang(deg)   P (MW)   Q (MVAr)   P (MW)   Q (MVAr)
----- ------- --------  --------  --------  --------  --------
    1  1.000    0.000     5.35      3.75       -         -
    2  0.977   -1.710      -         -         -         -
    3  0.967   -2.090     5.00      0.20      1.00      0.30
    4  0.953   -2.907      -         -        1.00      0.30
    5  0.941   -3.653      -         -        1.00      0.30
    6  0.931   -4.321      -         -        1.00      0.30
    7  0.922   -4.905      -         -        1.00      0.30
    8  0.915   -5.401      -         -        1.00      0.30
    9  0.909   -5.804      -         -        1.00      0.30
   10  0.905   -6.109      -         -        1.00      0.30
   11  0.902   -6.314      -         -        1.00      0.30
   12  0.900   -6.417      -         -        1.00      0.30
```

	--------	--------	--------	--------
Total:	10.35	3.95	10.00	3.00

==
| Branch Data |
==

Brnch #	From Bus	To Bus	From Bus Injection P (MW)	From Bus Injection Q (MVAr)	To Bus Injection P (MW)	To Bus Injection Q (MVAr)	Loss (I^2 * Z) P (MW)	Loss (I^2 * Z) Q (MVAr)
1	1	2	5.35	3.75	-5.33	-3.51	0.015	0.24
2	2	3	5.33	3.51	-5.30	-3.43	0.034	0.07
3	3	4	9.30	3.33	-9.21	-3.16	0.084	0.18
4	4	5	8.21	2.86	-8.15	-2.71	0.067	0.14
5	5	6	7.15	2.41	-7.10	-2.31	0.051	0.11
6	6	7	6.10	2.01	-6.06	-1.92	0.038	0.08
7	7	8	5.06	1.62	-5.03	-1.57	0.027	0.06
8	8	9	4.03	1.27	-4.02	-1.23	0.017	0.04
9	9	10	3.02	0.93	-3.01	-0.91	0.010	0.02
10	10	11	2.01	0.61	-2.00	-0.60	0.004	0.01
11	11	12	1.00	0.30	-1.00	-0.30	0.001	0.00
						Total:	0.347	0.95

==
| System Summary |
==

How many?		How much?	P (MW)	Q (MVAr)
Buses	12	Total Gen Capacity	11.0	-102.0 to 102.0
Generators	11	On-line Capacity	0.0	-100.0 to 100.0
Committed Gens	1	Generation (actual)	9.1	1.9
Loads	10	Load	9.0	1.2
Fixed	10	Fixed	9.0	1.2
Dispatchable	0	Dispatchable	0.0 of 0.0	0.0
Shunts	0	Shunt (inj)	0.0	0.0
Branches	11	Losses (I^2 * Z)	0.14	0.74
Transformers	0	Branch Charging (inj)	-	0.0
Inter-ties	0	Total Inter-tie Flow	0.0	0.0
Areas	1			

	Minimum	Maximum
Voltage Magnitude	0.954 p.u. @ bus 12	1.000 p.u. @ bus 1
Voltage Angle	-5.50 deg @ bus 12	0.00 deg @ bus 1

P Losses (I^2*R) - 0.07 MW @ line 2-3
Q Losses (I^2*X) - 0.50 MVAr @ line 1-2

==

| Bus Data |

==

Bus #	Voltage Mag(pu)	Voltage Ang(deg)	Generation P (MW)	Generation Q (MVAr)	Load P (MW)	Load Q (MVAr)
1	1.000	0.000	9.14	1.94	-	-
2	0.987	-2.987	-	-	-	-
3	0.977	-3.839	-	-	5.40	0.30
4	0.973	-4.164	-	-	0.40	0.10
5	0.969	-4.456	-	-	0.40	0.10
6	0.965	-4.713	-	-	0.40	0.10
7	0.962	-4.936	-	-	0.40	0.10
8	0.959	-5.122	-	-	0.40	0.10
9	0.957	-5.272	-	-	0.40	0.10
10	0.956	-5.385	-	-	0.40	0.10
11	0.955	-5.460	-	-	0.40	0.10
12	0.954	-5.498	-	-	0.40	0.10
		Total:	9.14	1.94	9.00	1.20

==

| Branch Data |

==

Brnch #	From Bus	To Bus	From Bus Injection P (MW)	From Bus Injection Q (MVAr)	To Bus Injection P (MW)	To Bus Injection Q (MVAr)	Loss (I^2 * Z) P (MW)	Loss (I^2 * Z) Q (MVAr)
1	1	2	9.14	1.94	-9.11	-1.44	0.031	0.50
2	2	3	9.11	1.44	-9.04	-1.29	0.070	0.15
3	3	4	3.64	0.99	-3.63	-0.96	0.012	0.03
4	4	5	3.23	0.86	-3.22	-0.84	0.009	0.02
5	5	6	2.82	0.74	-2.81	-0.73	0.007	0.02
6	6	7	2.41	0.63	-2.41	-0.62	0.005	0.01
7	7	8	2.01	0.52	-2.00	-0.51	0.004	0.01
8	8	9	1.60	0.41	-1.60	-0.40	0.002	0.01
9	9	10	1.20	0.30	-1.20	-0.30	0.001	0.00
10	10	11	0.80	0.20	-0.80	-0.20	0.001	0.00
11	11	12	0.40	0.10	-0.40	-0.10	0.000	0.00
						Total:	0.143	0.74

Appendix G: Cost calculation input data

24 hour load loss and energy costs used in cost examples

Base load shape

	% Pk	load	loss	run loss	$/Mwh	load $	loss $	
1	0.675	16.20	626	626	38	$615.60	$23.79	
2	0.610	14.64	496	1122	37	$541.68	$18.35	
3	0.598	14.35	475	1597	37	$531.02	$17.58	
4	0.594	14.26	467	2064	36	$513.22	$16.81	
5	0.615	14.76	506	2570	37	$546.12	$18.72	
6	0.689	16.54	657	3227	37	$611.83	$24.31	
7	0.721	17.30	731	3958	40	$692.16	$29.24	
8	0.746	17.90	793	4751	41	$734.06	$32.51	
9	0.786	18.86	900	5651	41	$773.42	$36.90	
10	0.811	19.46	972	6623	44	$856.42	$42.77	
11	0.826	19.82	1017	7640	44	$872.26	$44.75	
12	0.829	19.90	1027	8667	45	$895.32	$46.22	
13	0.842	20.21	1068	9735	47	$949.78	$50.20	
14	0.867	20.81	1150	10885	47	$977.98	$54.05	
15	0.898	21.55	1259	12144	52	$1,120.70	$65.47	
16	0.931	22.34	1386	13530	53	$1,184.23	$73.46	
17	0.960	23.04	1506	15036	60	$1,382.40	$90.36	
18	0.982	23.57	1604	16640	61	$1,437.65	$97.84	
19	1.000	24.00	1688	18328	67	$1,608.00	$113.10	
20	0.985	23.64	1617	19945	67	$1,583.88	$108.34	
21	0.957	22.97	1493	21438	63	$1,446.98	$94.06	
22	0.884	21.22	1209	22647	52	$1,103.69	$62.87	
23	0.791	18.98	914	23561	41	$778.08	$37.47	
24	0.698	16.75	677	24238	39	$653.33	$26.40	
								Tot $
	Total	463.08	24.238			22409.81	1225.56	$23,635.37
	Avg	19.2951	1.009917					

shift 1 mw for 3 hours, 5 hr .6 mw charge 100% eff

	% Pk	load	loss	run loss	$/Mwh	load $	loss $
1	**0.700**	16.80	682	682	38	$638.40	$25.92
2	**0.635**	15.24	544	1226	37	$563.88	$20.13
3	**0.623**	14.95	521	1747	37	$553.15	$19.28
4	**0.619**	14.86	513	2260	36	$534.96	$18.47
5	**0.640**	15.36	554	2814	37	$568.32	$20.50
6	0.689	16.54	657	3471	37	$611.83	$24.31
7	0.721	17.30	731	4202	40	$692.16	$29.24
8	0.746	17.90	793	4995	41	$734.06	$32.51
9	0.786	18.86	900	5895	41	$773.42	$36.90
10	0.811	19.46	972	6867	44	$856.42	$42.77
11	0.826	19.82	1017	7884	44	$872.26	$44.75
12	0.829	19.90	1027	8911	45	$895.32	$46.22

13	0.842	20.21	1068	9979	47	$949.78	$50.20	
14	0.867	20.81	1150	11129	47	$977.98	$54.05	
15	0.898	21.55	1259	12388	52	$1,120.70	$65.47	
16	0.931	22.34	1386	13774	53	$1,184.23	$73.46	
17	0.960	23.04	1506	15280	60	$1,382.40	$90.36	
18	**0.940**	22.57	1422	16702	61	$1,376.77	$86.74	
19	**0.958**	23.00	1497	18199	67	$1,541.00	$100.30	
20	**0.943**	22.64	1434	19633	67	$1,516.88	$96.08	
21	0.957	22.97	1493	21126	63	$1,446.98	$94.06	
22	0.884	21.22	1209	22335	52	$1,103.69	$62.87	
23	0.791	18.98	914	23249	41	$778.08	$37.47	
24	0.698	16.75	677	23926	39	$653.33	$26.40	
								Tot $
	Total	463.09	23.926			22326.00	1198.44	$23,524.44
	Avg	19.29527	0.996917					

no2 bat 1 mw for 3 hours, 5 hr .6 mw charge 100% eff

	% Pk	load	loss	run loss	$/Mwh	load $	loss $	
1	**0.675**	16.80	644	644	38	$638.40	$24.47	
2	**0.610**	15.24	511	1155	37	$563.88	$18.91	
3	**0.598**	14.95	489	1644	37	$553.15	$18.09	
4	**0.594**	14.86	482	2126	36	$534.96	$17.35	
5	**0.615**	15.36	521	2647	37	$568.32	$19.28	
6	0.689	16.54	657	3304	37	$611.83	$24.31	
7	0.721	17.30	731	4035	40	$692.16	$29.24	
8	0.746	17.90	793	4828	41	$734.06	$32.51	
9	0.786	18.86	900	5728	41	$773.42	$36.90	
10	0.811	19.46	972	6700	44	$856.42	$42.77	
11	0.826	19.82	1017	7717	44	$872.26	$44.75	
12	0.829	19.90	1027	8744	45	$895.32	$46.22	
13	0.842	20.21	1068	9812	47	$949.78	$50.20	
14	0.867	20.81	1150	10962	47	$977.98	$54.05	
15	0.898	21.55	1259	12221	52	$1,120.70	$65.47	
16	0.931	22.34	1386	13607	53	$1,184.23	$73.46	
17	0.960	23.04	1506	15113	60	$1,382.40	$90.36	
18	**0.982**	22.57	1561	16674	61	$1,376.77	$95.22	
19	**1.000**	23.00	1643	18317	67	$1,541.00	$110.08	
20	**0.985**	22.64	1574	19891	67	$1,516.88	$105.46	
21	0.957	22.97	1493	21384	63	$1,446.98	$94.06	
22	0.884	21.22	1209	22593	52	$1,103.69	$62.87	
23	0.791	18.98	914	23507	41	$778.08	$37.47	
24	0.698	16.75	677	24184	39	$653.33	$26.40	
								Tot $
	Total	463.09	24.184			22326.00	1219.89	$23,545.89
	Avg	19.29527	1.007667					

no3 bat 1 mw for 3 hours, 5 hr .6 mw charge 100% eff

	% Pk	load	loss	run loss	$/Mwh	load $	loss $

1	**0.675**	16.80	682	682	38	$638.40	$25.92	
2	**0.610**	15.24	544	1226	37	$563.88	$20.13	
3	**0.598**	14.95	521	1747	37	$553.15	$19.28	
4	**0.594**	14.86	513	2260	36	$534.96	$18.47	
5	**0.615**	15.36	554	**2814**	37	$568.32	$20.50	
6	0.689	16.54	657	3471	37	$611.83	$24.31	
7	0.721	17.30	731	4202	40	$692.16	$29.24	
8	0.746	17.90	793	4995	41	$734.06	$32.51	
9	0.786	18.86	900	5895	41	$773.42	$36.90	
10	0.811	19.46	972	6867	44	$856.42	$42.77	
11	0.826	19.82	1017	7884	44	$872.26	$44.75	
12	0.829	19.90	1027	8911	45	$895.32	$46.22	
13	0.842	20.21	1068	9979	47	$949.78	$50.20	
14	0.867	20.81	1150	11129	47	$977.98	$54.05	
15	0.898	21.55	1259	12388	52	$1,120.70	$65.47	
16	0.931	22.34	1386	13774	53	$1,184.23	$73.46	
17	0.960	23.04	1506	15280	60	$1,382.40	$90.36	
18	**0.982**	22.57	1471	16751	61	$1,376.77	$89.73	
19	**1.000**	23.00	1549	18300	67	$1,541.00	$103.78	
20	**0.985**	22.64	1484	19784	67	$1,516.88	$99.43	
21	0.957	22.97	1493	21277	63	$1,446.98	$94.06	
22	0.884	21.22	1209	22486	52	$1,103.69	$62.87	
23	0.791	18.98	914	23400	41	$778.08	$37.47	
24	0.698	16.75	677	24077	39	$653.33	$26.40	
								Tot $
	Total	463.09	24.077			22326.00	1208.26	$23,534.26
	Avg	19.29527	1.003208					

no4 bat 1 mw for 3 hours, 5 hr .6 mw charge 100% eff

	% Pk	load	loss	run loss	$/Mwh	load $	loss $
1	**0.675**	16.80	662	662	38	$638.40	$25.16
2	**0.610**	15.24	527	1189	37	$563.88	$19.50
3	**0.598**	14.95	504	1693	37	$553.15	$18.65
4	**0.594**	14.86	497	2190	36	$534.96	$17.89
5	**0.615**	15.36	537	2727	37	$568.32	$19.87
6	0.689	16.54	657	3384	37	$611.83	$24.31
7	0.721	17.30	731	4115	40	$692.16	$29.24
8	0.746	17.90	793	4908	41	$734.06	$32.51
9	0.786	18.86	900	5808	41	$773.42	$36.90
10	0.811	19.46	972	6780	44	$856.42	$42.77
11	0.826	19.82	1017	7797	44	$872.26	$44.75
12	0.829	19.90	1027	8824	45	$895.32	$46.22
13	0.842	20.21	1068	9892	47	$949.78	$50.20
14	0.867	20.81	1150	11042	47	$977.98	$54.05
15	0.898	21.55	1259	12301	52	$1,120.70	$65.47
16	0.931	22.34	1386	13687	53	$1,184.23	$73.46
17	0.960	23.04	1506	15193	60	$1,382.40	$90.36
18	**0.982**	22.57	1520	16713	61	$1,376.77	$92.72
19	**1.000**	23.00	1600	18313	67	$1,541.00	$107.20

20	**0.985**	22.64	1533	19846	67	$1,516.88	$102.71	
21	0.957	22.97	1493	21339	63	$1,446.98	$94.06	
22	0.884	21.22	1209	22548	52	$1,103.69	$62.87	
23	0.791	18.98	914	23462	41	$778.08	$37.47	
24	0.698	16.75	677	24139	39	$653.33	$26.40	
								Tot $
	Total	463.09	24.139			22326.00	1214.72	$23,540.73
	Avg	19.29527	1.005792					

all bat .33 mw each for 3 hours, 5 hr .2 mw charge 100% eff

	% Pk	load	loss	run loss	$/Mwh	load $	loss $	
1	**0.675**	16.80	663	663	38	$638.40	$25.19	
2	**0.610**	15.24	528	1191	37	$563.88	$19.54	
3	**0.598**	14.95	505	1696	37	$553.15	$18.69	
4	**0.594**	14.86	498	2194	36	$534.96	$17.93	
5	**0.615**	15.36	537	2731	37	$568.32	$19.87	
6	0.689	16.54	657	3388	37	$611.83	$24.31	
7	0.721	17.30	731	4119	40	$692.16	$29.24	
8	0.746	17.90	793	4912	41	$734.06	$32.51	
9	0.786	18.86	900	5812	41	$773.42	$36.90	
10	0.811	19.46	972	6784	44	$856.42	$42.77	
11	0.826	19.82	1017	7801	44	$872.26	$44.75	
12	0.829	19.90	1027	8828	45	$895.32	$46.22	
13	0.842	20.21	1068	9896	47	$949.78	$50.20	
14	0.867	20.81	1150	11046	47	$977.98	$54.05	
15	0.898	21.55	1259	12305	52	$1,120.70	$65.47	
16	0.931	22.34	1386	13691	53	$1,184.23	$73.46	
17	0.960	23.04	1506	15197	60	$1,382.40	$90.36	
18	**0.982**	22.57	1484	16681	61	$1,376.77	$90.52	
19	**1.000**	23.00	1562	18243	67	$1,541.00	$104.65	
20	**0.985**	22.64	1497	19740	67	$1,516.88	$100.30	
21	0.957	22.97	1493	21233	63	$1,446.98	$94.06	
22	0.884	21.22	1209	22442	52	$1,103.69	$62.87	
23	0.791	18.98	914	23356	41	$778.08	$37.47	
24	0.698	16.75	677	24033	39	$653.33	$26.40	
								Tot $
	Total	463.09	24.033			22326.00	1207.72	$23,533.72
	Avg	19.29527	1.001375					

shift 3 mw for 3 hours, 5 hr 1.8 mw charge 100%
eff

	% Pk	load	loss	run loss	$/Mwh	load $	loss $
1	**0.750**	18.00	803	803	38	$684.00	$30.51
2	**0.685**	16.44	648	1451	37	$608.28	$23.98
3	**0.673**	16.15	622	2073	37	$597.55	$23.01
4	**0.669**	16.06	614	2687	36	$578.16	$22.10
5	**0.690**	16.56	659	3346	37	$612.72	$24.38

6	0.689	16.54	657	4003	37	$611.83	$24.31	
7	0.721	17.30	731	4734	40	$692.16	$29.24	
8	0.746	17.90	793	5527	41	$734.06	$32.51	
9	0.786	18.86	900	6427	41	$773.42	$36.90	
10	0.811	19.46	972	7399	44	$856.42	$42.77	
11	0.826	19.82	1017	8416	44	$872.26	$44.75	
12	0.829	19.90	1027	9443	45	$895.32	$46.22	
13	0.842	20.21	1068	10511	47	$949.78	$50.20	
14	0.867	20.81	1150	11661	47	$977.98	$54.05	
15	0.898	21.55	1259	12920	52	$1,120.70	$65.47	
16	0.931	22.34	1386	14306	53	$1,184.23	$73.46	
17	0.960	23.04	1506	15812	60	$1,382.40	$90.36	
18	**0.857**	20.57	1117	16929	61	$1,254.77	$68.14	
19	**0.875**	21.00	1178	18107	67	$1,407.00	$78.93	
20	**0.860**	20.64	1126	19233	67	$1,382.88	$75.44	
21	0.957	22.97	1493	20726	63	$1,446.98	$94.06	
22	0.884	21.22	1209	21935	52	$1,103.69	$62.87	
23	0.791	18.98	914	22849	41	$778.08	$37.47	
24	0.698	16.75	677	23526	39	$653.33	$26.40	
								Tot $
	Total	463.09	23.526			22158.00	1157.53	$23,315.53
	Avg	19.29527	0.98025					

no2 bat 3 mw for 3 hours, 5 hr 1.8 mw charge 100% eff

	% Pk	load	loss	run loss	$/Mwh	load $	loss $
1	**0.675**	18.00	682	682	38	$684.00	$25.92
2	**0.610**	16.44	546	1228	37	$608.28	$20.20
3	**0.598**	16.15	523	1751	37	$597.55	$19.35
4	**0.594**	16.06	516	2267	36	$578.16	$18.58
5	**0.615**	16.56	556	2823	37	$612.72	$20.57
6	0.689	16.54	657	3480	37	$611.83	$24.31
7	0.721	17.30	731	4211	40	$692.16	$29.24
8	0.746	17.90	793	5004	41	$734.06	$32.51
9	0.786	18.86	900	5904	41	$773.42	$36.90
10	0.811	19.46	972	6876	44	$856.42	$42.77
11	0.826	19.82	1017	7893	44	$872.26	$44.75
12	0.829	19.90	1027	8920	45	$895.32	$46.22
13	0.842	20.21	1068	9988	47	$949.78	$50.20
14	0.867	20.81	1150	11138	47	$977.98	$54.05
15	0.898	21.55	1259	12397	52	$1,120.70	$65.47
16	0.931	22.34	1386	13783	53	$1,184.23	$73.46
17	0.960	23.04	1506	15289	60	$1,382.40	$90.36
18	**0.982**	20.57	1535	16824	61	$1,254.77	$93.64
19	**1.000**	21.00	1615	18439	67	$1,407.00	$108.21
20	**0.985**	20.64	1548	19987	67	$1,382.88	$103.72
21	0.957	22.97	1493	21480	63	$1,446.98	$94.06
22	0.884	21.22	1209	22689	52	$1,103.69	$62.87
23	0.791	18.98	914	23603	41	$778.08	$37.47
24	0.698	16.75	677	24280	39	$653.33	$26.40

								Tot $
	Total	463.09	24.280			22158.00	1221.20	$23,379.20
	Avg	19.29527	1.011667					

no3 bat 3 mw for 3 hours, 5 hr 1.8 mw charge 100% eff

	% Pk	load	loss	run loss	$/Mwh	load $	loss $	
1	**0.675**	18.00	809	809	38	$684.00	$30.74	
2	**0.610**	16.44	658	1467	37	$608.28	$24.35	
3	**0.598**	16.15	633	2100	37	$597.55	$23.42	
4	**0.594**	16.06	624	2724	36	$578.16	$22.46	
5	**0.615**	16.56	669	3393	37	$612.72	$24.75	
6	0.689	16.54	657	4050	37	$611.83	$24.31	
7	0.721	17.30	731	4781	40	$692.16	$29.24	
8	0.746	17.90	793	5574	41	$734.06	$32.51	
9	0.786	18.86	900	6474	41	$773.42	$36.90	
10	0.811	19.46	972	7446	44	$856.42	$42.77	
11	0.826	19.82	1017	8463	44	$872.26	$44.75	
12	0.829	19.90	1027	9490	45	$895.32	$46.22	
13	0.842	20.21	1068	10558	47	$949.78	$50.20	
14	0.867	20.81	1150	11708	47	$977.98	$54.05	
15	0.898	21.55	1259	12967	52	$1,120.70	$65.47	
16	0.931	22.34	1386	14353	53	$1,184.23	$73.46	
17	0.960	23.04	1506	15859	60	$1,382.40	$90.36	
18	**0.982**	20.57	1359	17218	61	$1,254.77	$82.90	
19	**1.000**	21.00	1430	18648	67	$1,407.00	$95.81	
20	**0.985**	20.64	1371	20019	67	$1,382.88	$91.86	
21	0.957	22.97	1493	21512	63	$1,446.98	$94.06	
22	0.884	21.22	1209	22721	52	$1,103.69	$62.87	
23	0.791	18.98	914	23635	41	$778.08	$37.47	
24	0.698	16.75	677	24312	39	$653.33	$26.40	
								Tot $
	Total	463.09	24.312			22158.00	1207.32	$23,365.32
	Avg	19.29527	1.013					

no4 bat 3 mw for 3 hours, 5 hr 1.8 mw charge 100% eff

	% Pk	load	loss	run loss	$/Mwh	load $	loss $
1	**0.675**	18.00	741	741	38	$684.00	$28.16
2	**0.610**	16.44	599	1340	37	$608.28	$22.16
3	**0.598**	16.15	575	1915	37	$597.55	$21.28
4	**0.594**	16.06	567	2482	36	$578.16	$20.41
5	**0.615**	16.56	609	3091	37	$612.72	$22.53
6	0.689	16.54	657	3748	37	$611.83	$24.31
7	0.721	17.30	731	4479	40	$692.16	$29.24
8	0.746	17.90	793	5272	41	$734.06	$32.51
9	0.786	18.86	900	6172	41	$773.42	$36.90
10	0.811	19.46	972	7144	44	$856.42	$42.77
11	0.826	19.82	1017	8161	44	$872.26	$44.75
12	0.829	19.90	1027	9188	45	$895.32	$46.22

13	0.842	20.21	1068	10256	47	$949.78	$50.20	
14	0.867	20.81	1150	11406	47	$977.98	$54.05	
15	0.898	21.55	1259	12665	52	$1,120.70	$65.47	
16	0.931	22.34	1386	14051	53	$1,184.23	$73.46	
17	0.960	23.04	1506	15557	60	$1,382.40	$90.36	
18	**0.982**	20.57	1452	17009	61	$1,254.77	$88.57	
19	**1.000**	21.00	1528	18537	67	$1,407.00	$102.38	
20	**0.985**	20.64	1465	20002	67	$1,382.88	$98.16	
21	0.957	22.97	1493	21495	63	$1,446.98	$94.06	
22	0.884	21.22	1209	22704	52	$1,103.69	$62.87	
23	0.791	18.98	914	23618	41	$778.08	$37.47	
24	0.698	16.75	677	24295	39	$653.33	$26.40	
								Tot $
	Total	463.09	24.295			22158.00	1214.67	$23,372.68
	Avg	19.29527	1.012292					

all bat 1 mw each for 3 hours, 5 hr .6 mw charge 100% eff

	% Pk	load	loss	run loss	$/Mwh	load $	loss $	
1	**0.675**	18.00	726	726	38	$684.00	$27.59	
2	**0.610**	16.44	584	1310	37	$608.28	$21.61	
3	**0.598**	16.15	559	1869	37	$597.55	$20.68	
4	**0.594**	16.06	552	2421	36	$578.16	$19.87	
5	**0.615**	16.56	594	3015	37	$612.72	$21.98	
6	0.689	16.54	657	3672	37	$611.83	$24.31	
7	0.721	17.30	731	4403	40	$692.16	$29.24	
8	0.746	17.90	793	5196	41	$734.06	$32.51	
9	0.786	18.86	900	6096	41	$773.42	$36.90	
10	0.811	19.46	972	7068	44	$856.42	$42.77	
11	0.826	19.82	1017	8085	44	$872.26	$44.75	
12	0.829	19.90	1027	9112	45	$895.32	$46.22	
13	0.842	20.21	1068	10180	47	$949.78	$50.20	
14	0.867	20.81	1150	11330	47	$977.98	$54.05	
15	0.898	21.55	1259	12589	52	$1,120.70	$65.47	
16	0.931	22.34	1386	13975	53	$1,184.23	$73.46	
17	0.960	23.04	1506	15481	60	$1,382.40	$90.36	
18	**0.982**	20.57	1369	16850	61	$1,254.77	$83.51	
19	**1.000**	21.00	1441	18291	67	$1,407.00	$96.55	
20	**0.985**	20.64	1381	19672	67	$1,382.88	$92.53	
21	0.957	22.97	1493	21165	63	$1,446.98	$94.06	
22	0.884	21.22	1209	22374	52	$1,103.69	$62.87	
23	0.791	18.98	914	23288	41	$778.08	$37.47	
24	0.698	16.75	677	23965	39	$653.33	$26.40	
								Tot $
	Total	463.09	23.965			22158.00	1195.34	$23,353.34
	Avg	19.29527	0.998542					

Appendix H: Plant factor data

U.S. Electric net summer generating capacity [2]

year	kw capacity	mwh generated	MW capacity	MWh capacity	Plant factor
1949	63,400,000	296,124,289	63,400	555,384,000	53.3%
1950	69,200,000	334,087,601	69,200	606,192,000	55.1%
1951	75,500,000	375,298,355	75,500	661,380,000	56.7%
1952	83,200,000	403,829,413	83,200	728,832,000	55.4%
1953	93,300,000	447,048,563	93,300	817,308,000	54.7%
1954	100,000,000	476,257,618	100,000	876,000,000	54.4%
1955	114,200,000	550,298,862	114,200	1,000,392,000	55.0%
1956	119,700,000	603,875,763	119,700	1,048,572,000	57.6%
1957	131,100,000	634,642,367	131,100	1,148,436,000	55.3%
1958	143,300,000	648,450,862	143,300	1,255,308,000	51.7%
1959	155,900,000	713,378,831	155,900	1,365,684,000	52.2%
1960	167,100,000	759,155,788	167,100	1,463,796,000	51.9%
1961	179,000,000	797,124,391	179,000	1,568,040,000	50.8%
1962	192,100,000	857,943,656	192,100	1,682,796,000	51.0%
1963	209,700,000	920,028,271	209,700	1,836,972,000	50.1%
1964	223,700,000	987,218,326	223,700	1,959,612,000	50.4%
1965	234,800,000	1058385671	234,800	2,056,848,000	51.5%
1966	247,500,000	1147531895	247,500	2,168,100,000	52.9%
1967	266,700,000	1217795688	266,700	2,336,292,000	52.1%
1968	284,000,000	1332825601	284,000	2,487,840,000	53.6%
1969	309,800,000	1445458056	309,800	2,713,848,000	53.3%
1970	336,400,000	1535111467	336,400	2,946,864,000	52.1%
1971	366,400,000	1615853616	366,400	3,209,664,000	50.3%
1972	396,000,000	1752978413	396,000	3,468,960,000	50.5%
1973	439,800,000	1864056631	439,800	3,852,648,000	48.4%
1974	468,500,000	1870319405	468,500	4,104,060,000	45.6%
1975	491,300,000	1920754569	491,300	4,303,788,000	44.6%
1976	517,200,000	2040913681	517,200	4,530,672,000	45.0%
1977	535,900,000	2127447487	535,900	4,694,484,000	45.3%
1978	552,100,000	2209376911	552,100	4,836,396,000	45.7%
1979	565,500,000	2250665025	565,500	4,953,780,000	45.4%
1980	578,600,000	2289600364	578,600	5,068,536,000	45.2%
1981	598,300,000	2297973339	598,300	5,241,108,000	43.8%
1982	613,700,000	2244372488	613,700	5,376,012,000	41.7%
1983	621,100,000	2313445685	621,100	5,440,836,000	42.5%
1984	635,100,000	2419465368	635,100	5,563,476,000	43.5%
1985	655,200,000	2473002122	655,200	5,739,552,000	43.1%
1986	664,800,000	2490470952	664,800	5,823,648,000	42.8%
1987	674,100,000	2575287666	674,100	5,905,116,000	43.6%

1988	677,700,000	2707411177	677,700	5,936,652,000	45.6%
1989	721,797,008	2967146087	721,797	6,322,941,790	46.9%
1990	734,121,844	3037827337	734,122	6,430,907,353	47.2%
1991	739,870,417	3073798885	739,870	6,481,264,853	47.4%
1992	746,506,755	3083882204	746,507	6,539,399,174	47.2%
1993	754,581,735	3197191096	754,582	6,610,135,999	48.4%
1994	763,966,856	3247522388	763,967	6,692,349,659	48.5%
1995	769,463,315	3353487362	769,463	6,740,498,639	49.8%
1996	775,889,530	3444187621	775,890	6,796,792,283	50.7%
1997	778,649,296	3492172283	778,649	6,820,967,833	51.2%
1998	775,868,273	3620295498	775,868	6,796,606,071	53.3%
1999	785,926,835	3694809810	785,927	6,884,719,075	53.7%
2000	811,719,238	3802105043	811,719	7,110,660,525	53.5%
2001	848,253,890	3736643653	848,254	7,430,704,076	50.3%
2002	905,301,090	3858452252	905,301	7,930,437,548	48.7%
2003	948,446,470	3883185205	948,446	8,308,391,077	46.7%
2004	962,941,950	3970555261	962,942	8,435,371,482	47.1%
2005	978,019,850	4055422750	978,020	8,567,453,886	47.3%
2006	986,214,930	4064702228	986,215	8,639,242,787	47.0%
2007	994,886,600	4156744724	994,887	8,715,206,616	47.7%