Longevity Insurance

A Major Qualifying Project submitted to the Faculty of WORCESTER POLYTECHNIC INSTITUTE in partial fulfillment of the requirements for the Degree of Bachelor of Science by:

Ethan Graham
Angela Quackenbos

Submitted To:
Professor Jon Abraham
Professor Barry Posterro

Submitted On:
April 27, 2022

Table of Contents

List of Figures

List of Tables

Abstract

Planning for retirement can be a stressful time for many people when determining a sufficient amount of money to set aside for later spending. Longevity insurance provides an individual with a sense of security knowing they will have a constant stream of income throughout retirement. Our team has worked to investigate the benefits of longevity insurance through analysis on the risk of ruin for retirement accounts and through mortality rates to determine the best pricing for this product.

Chapter 1: Introduction

Insurance companies offer a wide range of products. These products all aim to offer various types of insurance to the customer by identifying a need that the customer may have. Sometimes this is life insurance, which will pay a family money if the insured passes away. This offers the insured peace of mind knowing that their family will be financially supported when they pass away. Some insurance companies insure vehicles. The price to repair a car after an accident can be very high and having insurance for such an event is valuable. The point is that insurance companies identify problems and needs of customers within a certain target market and then offer a service in return.

One problem that insurance companies have identified is that people are nervous about retirement. It is a scary concept, after all, as during retirement years people will not an income from their job. Our team investigated one of the insurance products that companies offer to help plan for retirement. The product is a type of annuity typically known as a longevity insurance. We went in depth into the product, curious to see how insurance companies go about pricing the product, and how it might help retirees feel more secure about their retirement plans.

Chapter 2: Background

2.1 - General Retirement Overview

Saving and planning for retirement is a primary concern for many people across the world, specifically within the United States. Due to the difficulty people face in working at later ages, it is important that they have another source of income to support them. There are numerous options that people have in order to sustain income later in life. Listed below are two popular options for employees:

1. **401(k)** - A 401k account is a retirement account in which an employee at a company elects to set aside a certain percentage of their paycheck to be saved in such an account. This money is not accessible until the individual has reached retirement. One benefit of a 401(k) account is its hand in reducing the taxable income of the employee. For example: if an employee earns a salary of \$100,000 per year and elects to set aside \$10,000 into a 401(k) account, the taxable portion of their income is the leftover \$90,000. Oftentimes companies are willing to match certain percentage contributions into the 401(k) account in addition to the employee's individual contribution.
2. **Pension** - A pension plan is a plan in which the employer sets aside funds for eligible employees to receive payments after they retire. The difference between a 401(k) and a pension plan is that the employee does not have to set aside any of their money in order to be eligible for this benefit. A typical pension plan is one where a retired employee receives a specified monthly payment for the remainder of their life.

As previously mentioned, there are many paths for individuals to take in order to afford retirement. Another common route for individuals is to purchase annuities.

2.2 - Overview of Annuities

Annuities are unique products offered by insurance companies. The main idea that all annuities follow is that the customer will receive payments from the company while they are alive. Although it is not the exact opposite of life insurance, they do mirror each other in that the benefit of life insurance is received when the customer passes away. Once the customer who bought an annuity passes away, the payments will cease. With this being said, there are different forms of annuities that vary based on when the potential customer wants their payments to begin.

One common annuity is an annuity-due. When purchasing this annuity, the customer can expect payments to begin immediately at time 0. Despite the name, these operate differently than annuities-immediate, where payments begin at time 1. We will further discuss annuities-immediate in the following section. In terms of total value, an annuity-due holds a larger present value as the payments begin immediately, so when comparing n payments of the two annuities it will be found that the annuity-due is larger.

In addition, another common form of annuities are deferred annuities. This will be further discussed in the following section, but as an overview it is an annuity where n payments are deferred for a specified period of time. Deferred annuities can be applied to both annuities-due and annuities-immediate. Below is a timeline to demonstrate where payments of $1 are expected to begin for the above annuities.

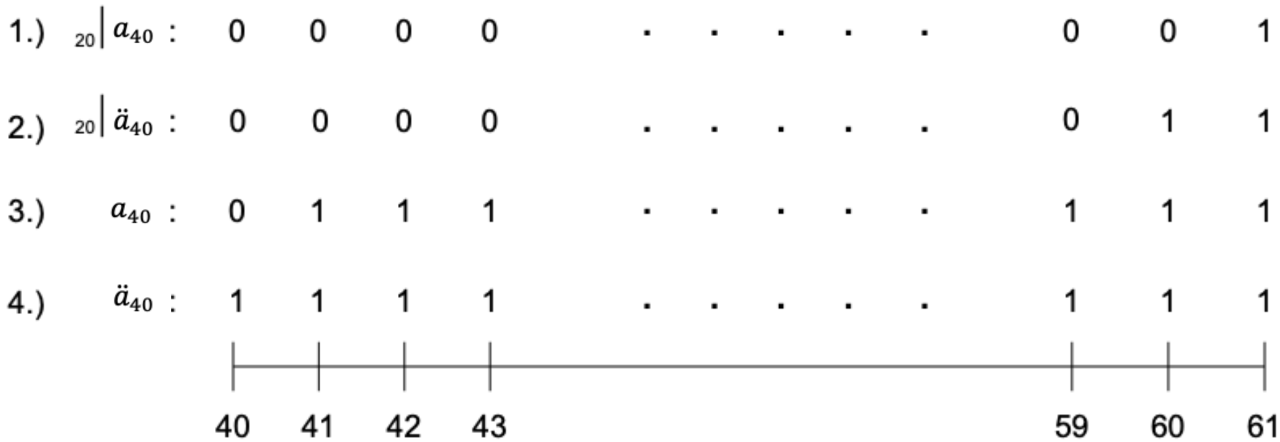

Figure 1: Timeline of payments of $1 for four whole life annuities purchased at age 40.

Figure 1 demonstrates the timeline of payments of $1 for four whole life annuities, all purchased at age 40. The first two annuities both showcase a twenty-year deferred whole life annuity, the first being an annuity-immediate with payments that begin at age 61 and the second being an annuity-due with payments that begin at 60. The third timeline portrays a whole life annuity-immediate, with payments beginning at age 41. Lastly, the fourth timeline demonstrates a whole life annuity-due, where payments begin at age 40. Because the four annuities are whole life, payments are expected to last until the annuitant dies.

2.3 - Immediate vs. Deferred Annuities

An **annuity-immediate** is an annuity in which the customer receives payments within a year of the purchase date. These payments operate on an elected schedule, whether that be monthly, quarterly, annually, etc. With annuities-immediate, the customer has the option to purchase under a fixed or variable interest rate. Under a fixed interest rate, this allows the customer a more conservative plan where they know the exact amount their payments will be each payment period. On the contrary, variable interest rates allow the customer to endure more risk in this investment as the return can be more or less than what was expected. The following is an example of an annuity-immediate:

> A person pays a one-time premium of $150,000 to an insurance company for a 10-year annuity-immediate. This customer will receive ten fixed annual payments of $19,426.

Alternatively, another form of an annuity is a **deferred annuity**. A deferred annuity is one in which a customer will begin receiving payments after a determined amount of years have passed. While operating under similar terms to annuities-immediate, it is vital to account for this deferral period when computing calculations. These annuities can be more suited for individuals who are saving for retirement. The following is an example of a deferred annuity:

> A person pays a one-time premium of $150,000 to purchase a 20-year deferred annuity that pays out for 10 years. This person will receive ten fixed annual payments beginning twenty years from now of $49,088.

In the above example, the deferred annuity was set to begin payments twenty years from the purchase age. If the customer wishes to defer these payments to an even later age, they would be interested in purchasing longevity insurance.

2.4 - Longevity Insurance

Longevity insurance is a deferred annuity where payments occur at an advanced age, such as age 80 or 85. Take into consideration an example where you plan to retire at age 60 and want annuity payments to begin at age 80. If you bought this annuity at age 45, you would have 15 working years remaining, where you would then only need to allocate your remaining retirement money for 20 years, between ages 60 and 80. After these 20 years, your annuity payments will begin. This example is portrayed in the following timeline:

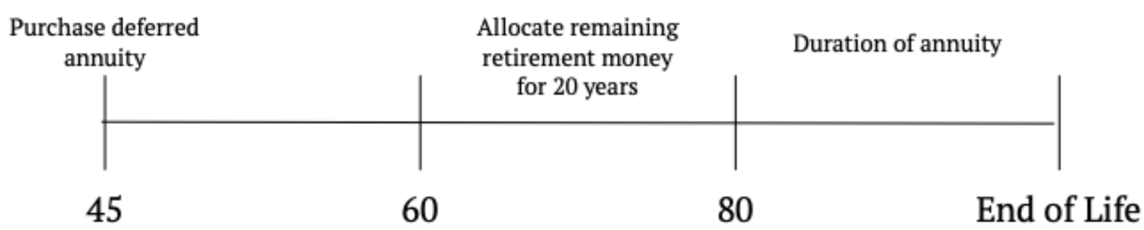

Figure 2: Timeline of longevity insurance.

Depending on the amount a potential customer invests towards purchasing this annuity, the payments can be sufficient enough by themselves for the rest of the person's life.

2.5 - Longevity Insurance in Practice

Increasing in popularity, longevity insurance has recently become available through numerous insurance companies within the United States. While it is unlikely they will be coined as specifically longevity insurance, they are marketed to potential customers as so.

Many companies promote their longevity products as deferred annuities that are designed to endure payments throughout a potential customer's lifespan. The policies can come with multiple attractive benefits such as single or joint life option and inflation protection. The ability to opt into a single life policy guarantees payments to this customer for the rest of their life. Under a joint life policy, these same payments are guaranteed for life for both individuals listed. With inflation protection, these customers are secured under a rate of return that is either at or above the current inflation rate, guarding them from the impacts increased prices of goods and services has.

Some policies also include benefits that allow the annuitant to withdraw a one-time lump-sum amount of the current present value of the annuity. This is attractive to potential customers as it provides them with a sense of security that they can have emergency access to the money they have put into their annuity in the event they need it. While it is not likely the annuitant would need to withdraw this amount, it provides added protection in the case of a catastrophic event occurring.

With these described added benefits, potential customers can find purchasing longevity insurance very appealing. In a simple sense, they are investing into their retirement with a strong sense of security that they will always have a steady source of income into their advanced ages. They also have access to the same benefits that can be applied to other annuity purchases they could make, adding a sense of familiarity into a newer idea of longevity insurance. With a better understanding of how these annuities function in practice, and the benefits they include, we will now investigate the advantages and disadvantages associated with purchasing longevity insurance.

2.6 - Advantages and Disadvantages Associated With Purchase

One advantage, as mentioned briefly before, is the simplification of retirement spending. Knowing there is a steady stream of income promised at a later date, the customer will only need to allocate their remaining retirement money for a short, defined time period, if they decide that the annuity payment themselves are sufficient. For the individual's remaining years of retirement they are guaranteed a stream of income, regardless of the amount is spent from their allocated retirement money. This also allows the retiree to live at a comparable level of consumption to their working years, as they always have an income. This idea was portrayed in an earlier example.

The fact that a customer knows how much money they will be receiving once the annuity kicks in allows for a unique opportunity. A disadvantage falls in the case the customer has a chronic disease, or some other condition, that will impact their ability to live into advanced ages. The purpose of these annuities is to provide a steady source of income in your advanced years, so being unable to live until those years could turn customers away. For example, a customer develops cancer. Cancer takes the lives of many every year and is something that all people have the risk of contracting with no way of knowing if we will. If a customer purchases longevity insurance at age 45 with the intention payments will begin at age 75, the purchase only becomes profitable once the payments begin. If the customer develops cancer during the deferral period and passes away, they will no longer be the ones accessing the money but rather their beneficiaries.

Chapter 3: Analysis of Account Depletion

3.1 - Project Motivation

Our first step in completing this project was to review the research paper *Provision of Longevity Annuities* by Dale Kintzel and John A. Turner. The essence of this paper is to portray the benefits of longevity insurance in retirement, describing how they work to simplify retirement spend-down calculations and allow for potential riskier portfolios for retirees. To further describe the need for longevity insurance, the authors detail a Monte Carlo Simulation they conducted throughout their research to determine the probability of ruin for an account. In this analysis, the authors utilized historical returns found on U.S. financial market indexes and the SOA's Retirement Plan-2014 Mortality Tables to determine both return and mortality rates. Kintzel and Turner based their calculations under the assumption of a starting portfolio value of $125,000, where $93,750 is allocated to a life-cycle account for annual withdrawals and the remaining $31,250 is used to purchase the longevity insurance. With withdrawal amounts from the $93,750 ranging from $3000 to $10,000, incrementing by $500 per year, the authors portray the probability of account ruin for different payout ages (when the annuity kicks in), namely 82, 85, and 90.

In the early stages of our project, we used the authors' work as motivation to further investigate longevity insurance. Doing so sparked our curiosity in this topic in terms of how we could further develop this research, whether that be by analyzing how a longevity product could be applied to smaller audiences who have accumulated a large sum of money at a young age, or how this product may look for someone with chronic health conditions. Inevitably, we determined our best approach to furthering Kintzel and Turner's research is to explore how longevity insurance apply to a wide range of ages and how including certain riders may make them more appealing to potential customers.

3.2 - Determining Return Rates

To determine the return rates that we used to grow the money in the account over time, in this case the $93,750, we looked at the historical SPY Index accessed through Yahoo Finance. From there, we were comfortably able to identify the correct return rates to use.

3.3 - Different Withdrawal Scenarios

In alliance with the authors' original work, we chose to work with fifteen withdrawal scenarios, ranging from $3000 to $10,000 in increments of $500. A variable within these different scenarios was the starting balance of the account, which we chose to be $93,750, also in alliance with the original work of Kintzel and Turner. By keeping the beginning balance the same for all the 15 different scenarios, it allowed us to fully analyze the risk these accounts have to deplete with varying annual withdrawal amounts. The results of these calculations proved to be very valuable to us because it told us the chance that a person has of running out of money based on their withdrawal amount and the number of years they are withdrawing. Once the results are achieved, it should give us a better idea of how much someone should be withdrawing and for how long before the annuity begins.

3.4 - Account Depletion Calculations

By combining the return rates, determined through the historic SPY Index, and the varying withdrawal scenarios, we have been able to portray the fluctuating rates at which an account will deplete over time. To do so we developed an algorithm that computes 1000 trials of ending balance amounts over the different withdrawal scenarios for 110 years. For each end of year balance, the following calculations were applied:

1.) Determine the original investment, in this scenario $93,750
2.) Determine the return rate
3.) Accumulate the amount by the term. Multiply the original investment by the return rate and subtract the necessary annual withdrawal amount
4.) Find the maximum of this function, returning 0 if the max value is negative.

The following formula outlines this formula within Excel:

$$= MAX(Beginning\ Balance\ *\ (1\ +\ i)\ -\ Withdrawal, 0)$$

To properly analyze these trials, we incorporated a brief summary at the bottom of the sheet that details the number of ruined accounts each year, the amount this number differed from the previous year, and the number of ruined accounts as a percentage. This allowed us to gain a better understanding of how different withdrawal amounts impact the amount of accounts that deplete over time. From here it was that higher annual withdrawals lead to faster depletion. A portrayal of this summary is detailed in the following table:

	EOY 17	EOY 18	EOY 19	EOY 20	EOY 21
# Of ruined accounts	157	259	370	487	589
Yr to Yr difference	88	102	11	117	102
% of total accounts ruined	15.7%	25.9%	37.0%	48.7%	58.9%

Table 1: End of year account balance summaries for years 17 through 21.

The above figure details the end of year, EOY, balances for years 17 through 21. Included in each EOY summary are the amount of ruined accounts, the difference in the number of accounts in comparison to the previous year, and the number of the total amount of failed accounts as a percentage. For instance, at the end of year 18 there are 259 accounts ruined, 25.9%, where the previous year's value of 157 is subtracted from this 259 to find the year to year difference of 102.

To provide a visual representation of the risks associated with different withdrawal amounts we graphed the ending balances of an account for ages 82, 85, and 90 in accordance with the authors' original work. To develop this graph, we extracted the appropriate percentages of ruin for the respective three ages from our collective data set. These values were separated by the withdrawal scenario as a way to better represent the impact each withdrawal scenario has on the probability an account will ruin. The following two figures depict both our extraction and graph in relation to this work:

Withdrawals	% Ruin by 82	% Ruin by 85	% Ruin by 90
3,000	0.00%	0.00%	0.00%
3,500	0.00%	0.00%	0.70%
4,000	0.00%	0.50%	3.10%
4,500	0.50%	1.70%	14.50%
5,000	1.70%	10.10%	37.20%
5,500	8.90%	28.70%	65.20%
6,000	25.60%	52.60%	87.70%
6,500	48.70%	79.30%	97.20%
7,000	72.20%	93.90%	99.60%
7,500	89.90%	98.60%	99.80%
8,000	97.50%	99.70%	100.00%
8,500	99.50%	100.00%	100.00%
9,000	100.00%	100.00%	100.00%
9,500	100.00%	100.00%	100.00%
10,000	100.00%	100.00%	100.00%

Table 2: Risk of ruin in an account with starting balance $93,750 for ages 82, 85, and 90 under various withdrawal scenarios.

Table 2 outlines the probability of an account failing by ages 82, 85, and 90 based on different withdrawal scenarios for amounts $3,000 to $10,000 in increments of $500. For example, an individual has a 25.6% chance of their account failing by age 82 if they were to withdraw $6,000 annually.

Figure 3: Risk of ruin in an account by ages 82, 85, and 90 portrayal.

Figure 3 takes the data found in Table 2 to visualize the gravity these risks of ruin hold. It is evident from the graph that regardless of the age of ruin the three scenarios follow a similar trend, where the risk of ruin with smaller withdrawal amounts is essentially 0%, curves sharply upwards as this withdrawal amount increases, and levels out once the account has a 100% chance of ruin.

The combination of the fifteen annual withdrawal scenarios and determined return rates allowed us to successfully mimic the authors' original work. We will explore the differences in our work versus that of Kintzel and Turner in the following section.

3.5 - Comparison to the Authors' Work

As we tested the different withdrawal scenarios, we determined the closest comparison we could make to the authors' work was by using 30% of the original return rates. To compare the true ratio of differences between the two pieces of work, we compiled the ending percentages of ruined accounts at the end of 20, 23, and 28 years across the original paper and our developed spreadsheet. We separated this by the authors' work, our work, and the difference between ours and the authors'. The following table depicts the differences between the two.

Difference			
	20 yr	23 yrs	28 yrs
1	0.00%	0.00%	0.00%
2	0.00%	0.00%	0.58%
3	0.00%	0.49%	2.96%
4	0.24%	-0.08%	3.65%
5	-0.54%	0.84%	4.97%
6	-1.05%	1.28%	5.22%
7	-1.69%	-0.14%	5.58%
8	-2.60%	3.35%	3.21%
9	-1.60%	-0.08%	1.19%
10	1.02%	1.72%	-0.16%
11	1.38%	0.46%	0.06%
12	0.58%	0.16%	0.01%
13	0.26%	0.02%	0.00%
14	0.05%	0.00%	0.00%
15	0.02%	0.00%	0.00%

Table 3: Percentage difference of ruined accounts from our findings to the authors' calculations.

The difference between our work and the authors' work is very small. This was the closest match we were able to develop using 30% of the original return rates. Seeing a difference between the two pieces of work this small ensured us that our calculations were correct throughout our spreadsheet, providing us a good basis for the remainder of our project.

Chapter 4: Analysis with Mortality Tables

4.1 - Retirement Plans (RP)-2014 Tables

As a method to gather data in regard to mortality rates for employees within the United States, both male and female, we utilized the RP-2014 mortality tables. Within the RP-2014 mortality tables, we gained access to information that included mortality rates of employees and healthy annuitants for different categories: the total dataset, blue collar, white collar, bottom quartile, top quartile, and juvenile. Inevitably, we chose to utilize the total dataset as that would provide us with a wider range of values to incorporate into our calculations.

From these values we crafted an overall table that detailed both probabilities of living and dying. The two most common, yet useful, probabilities are:

1. $\boldsymbol{q_x}$ **-** This calculation assumes that the person is alive at age x. The calculation gives the probability that someone who is alive at age x will be dead by age x+1.
2. $\boldsymbol{p_x}$ **-** This calculation assumes the person is alive at age x. The calculation gives the probability that someone who is alive at age x will also be alive at age x+1.

We chose ages 1-17 to hold a probability of zero under the assumption that everyone will live to this age. From there, we inputted the values for ages 18-64 as the mortality rates of employees and for ages 65-120 as the mortality rates of healthy annuitants. By separating the gathered mortality rates into both employees and healthy annuitants, we were able to encompass a typical trajectory for working individuals within the United States, where most retire around age 65.

4.2 - Mortality Table Goals

By crafting the table described above, we were able to see what would happen in theory if people bought the annuity product. Before having the probabilities of people living and dying at certain ages, it would be impossible to model what the pricing of this annuity would be. In Chapter 3 of our paper, we discussed the growth and decline of people holding their own money in a certain amount of time. The theory behind this did not require mortality probabilities, as we only calculated how much money they had as a beginning balance and whether this was sufficient for the needed amount of time.

The purpose of this approach was to calculate the prices this annuity product would cost potential customers. This work is separate from Chapter 3, as we are assuming a customer will not run out of money as long as they are alive at the time the annuity payments kick in. In order to get accurate calculations, we have identified five variables:

1. **Current Age** - the age the customer purchases the annuity
2. **Start Age** - the age the customer starts receiving payments from the annuity purchased at the current age
3. **Return Rate** - the interest rate
4. **Inflation Rate** - the inflation rate
5. **Annuity PMT** - the annual payments that the customer will receive once the annuity kicks in at the start age

We constructed our Microsoft Excel sheet in a way where we can change the input value of these variables at any time, allowing us to view the results based on different scenarios.

4.3 - Calculations

In addition to the mortality tables that we structured in Section 4.1, there were further calculations we needed in order to get the desired results. The first was ${}_tp_x$ **,** which is the probability of someone living t years given an x starting age. This variable is very useful for us as it gave us the specific probability of a customer living to a certain age, say the age the annuity payments begin at, given a purchase age. The next is ${}_{t|}q_x$, which is the probability of someone living t years, and then dying at age x+1. An example is ${}_{4|}q_{50}$, the chance that someone age 50 lives through age 54 and then dies before age 55. We also calculated the present value factor, depicted by the symbol v^t. Because v^t is related to the return rate, it gave us an idea of how interest is compounded over a certain period of time. It should also be noted that we set up the annuity payments in a way where inflation is only applied after the first payment is made, and continues for the ones thereafter. This means that during the deferral period no inflation is applied.

4.4 - Present Value

One of the core concepts in Actuarial Mathematics is the idea of present value. It is defined as "the current value of a future sum of money or stream of cash flows given a specified rate of return," (Fernando). As mentioned in Section 4.2, we have a variable named *Annuity PMT*. *Annuity PMT* allows us to input and test different annuity payment amounts. However, the calculations that are dependent on the annuity payment will follow the same procedure regardless of the value that is inputted. One of these calculations is the present value of the annuity. Calculating the present value of the annuity at specified times gives us the answer as to how much this product would currently be worth at that point in time. The high level concept to calculate the present value is by taking the given annuity payment and adjusting it by the return rate.

Keep in mind that once the customer who purchased the annuity dies, the payments to them will cease. Thus, an important aspect of calculating the present value is to account for the mortality of the customer. A high level concept is how the longer someone lives, the lower the present value would be for that particular year since their chance of dying is increasing as they age. Our Excel spreadsheet is able to calculate the present value of the annuity up until age 120, the age we assume the population will not surpass. The answer our Excel spreadsheet gives us when taking the present value of the annuity from the customer's purchase age until age 120 is also how much the annuity is work, the breakeven price to sell the annuity.

4.5 - Inclusion of Death Benefit Rider

A path we decided to go down was to include a useful tool known as the death benefit rider. It is common that even in annuities, where payments cease when the customer dies, companies include products that pay the premium back to the customer in the event they die during the deferral period. A simple example is as follows: A customer age 45 purchases longevity insurance with a death benefit rider that will begin payments at age 75. The customer then passes away at age 67. In this scenario since he has a death benefit rider included, the premium that he paid at age 45 is payable as a death benefit.

It is clear that the cost of the annuity will increase if we include a death benefit rider. So the premium that the customer gets back in the event that they die in the deferral period takes into account the rider. The death benefit rider affects the price of the annuity only during the deferral period because once the annuity payments begin, the death benefit rider is no longer in effect. The amount in which the death benefit rider affects the price of the annuity is fully reliant on the deferral period. By this, we mean that the length of the deferral period and the age of the customer during this period will dictate how much this rider will increase in price.

Chapter 5: Findings

In an effort to portray the data we have collected through these mortality calculations, we have stored this information into a pivot table to later develop a pivot chart. This has provided us a way to illustrate three factors of the data: the expected present value of the annuity, the expected present value of the death benefit, and the standard deviation of the total expected premium. In each scenario we have assumed a $5,000 annuity payment that accumulates given the applied inflation rate.

By outlining the present values of both the annuity and death benefit, we have been able to better understand trends within this data collection. Graphing the standard deviation of the expected premium has made it possible for us to better understand the variability of this data collection. The following sections will discuss our findings within the developed graphs.

Section 5.1 - Expected Present Value of an Annuity

As previously mentioned, to better understand the expected present values an annuity will endure over time, we have stored this information within a pivot chart. These present values were grouped by return rate, 0%, 2%, 4%, and 6%, as well as payment start ages, 75, 80, and 85. In order to better see the impact of inflation on these present values, we chose to filter the graphs by the various rates, 0%, 1%, 2%, and 3%, and 4%. The following subsections will detail key discoveries we have made within the graphs, based on the two extremes of inflation: 0% and 4%.

<u>0% Inflation:</u>

The following three figures are graphs, beginning on the next page, that we have deemed most valuable to our analysis:

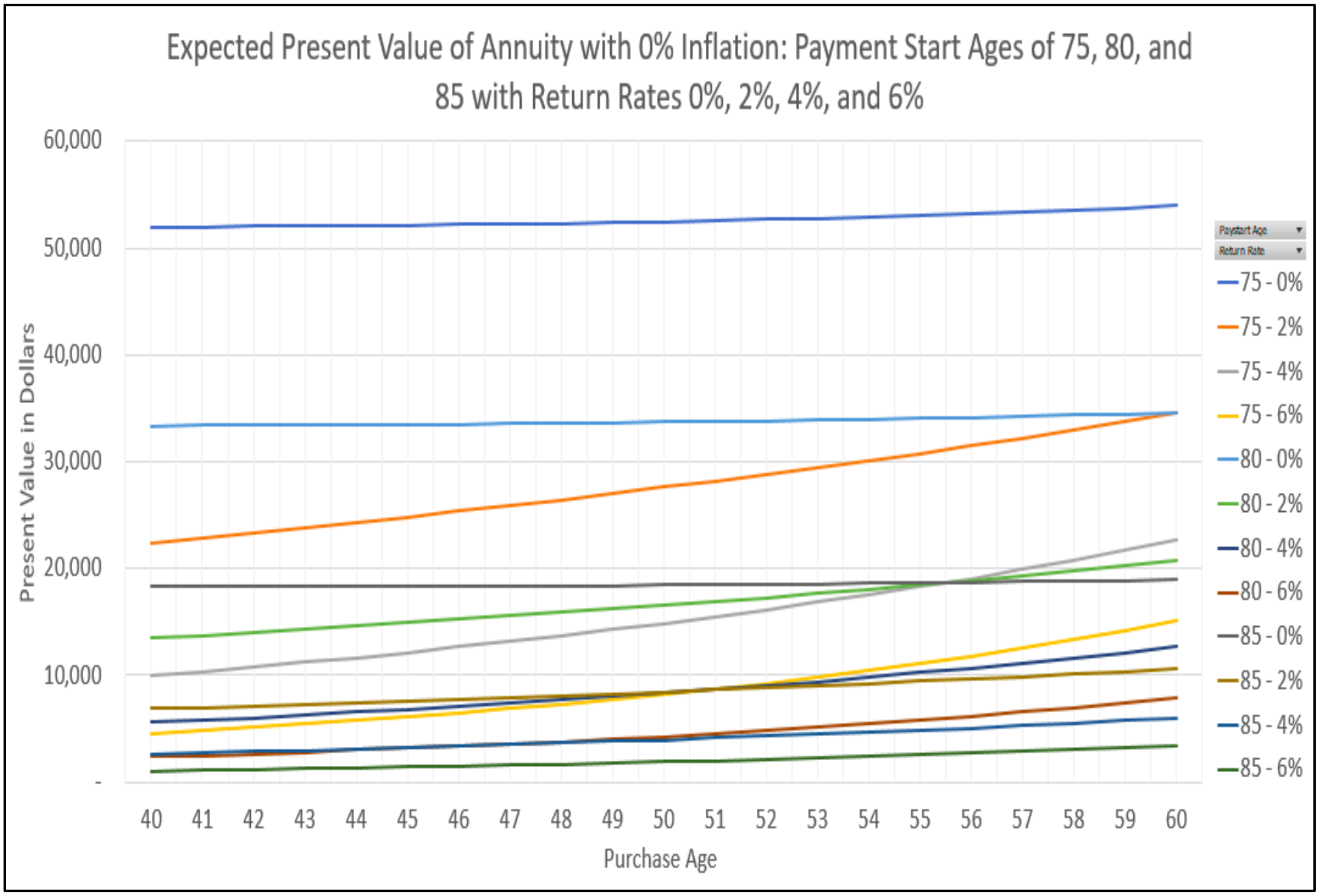

Figure 4: Expected present value of annuity under 0% inflation with different pay start ages and return rates.

Figure 4 outlines the expected present value for annuities with payment starts ages 75, 80, and 85 under returns rates of 0%, 2%, 4%, and 6%. Because this figure is portrayed under a 0% inflation rate, it is important to note the patterns that arise. It is evident that the present value of annuities with a return rate and inflation rate of 0% grow on a relatively constant basis, as each present value is weighted only by the probability of death by the purchase age. When compared to the expected present values of annuities with return rates greater than 0% this figure portrays how these values tend to intersect at a point in time with those of 0% return and inevitably surpass them in expected present value.

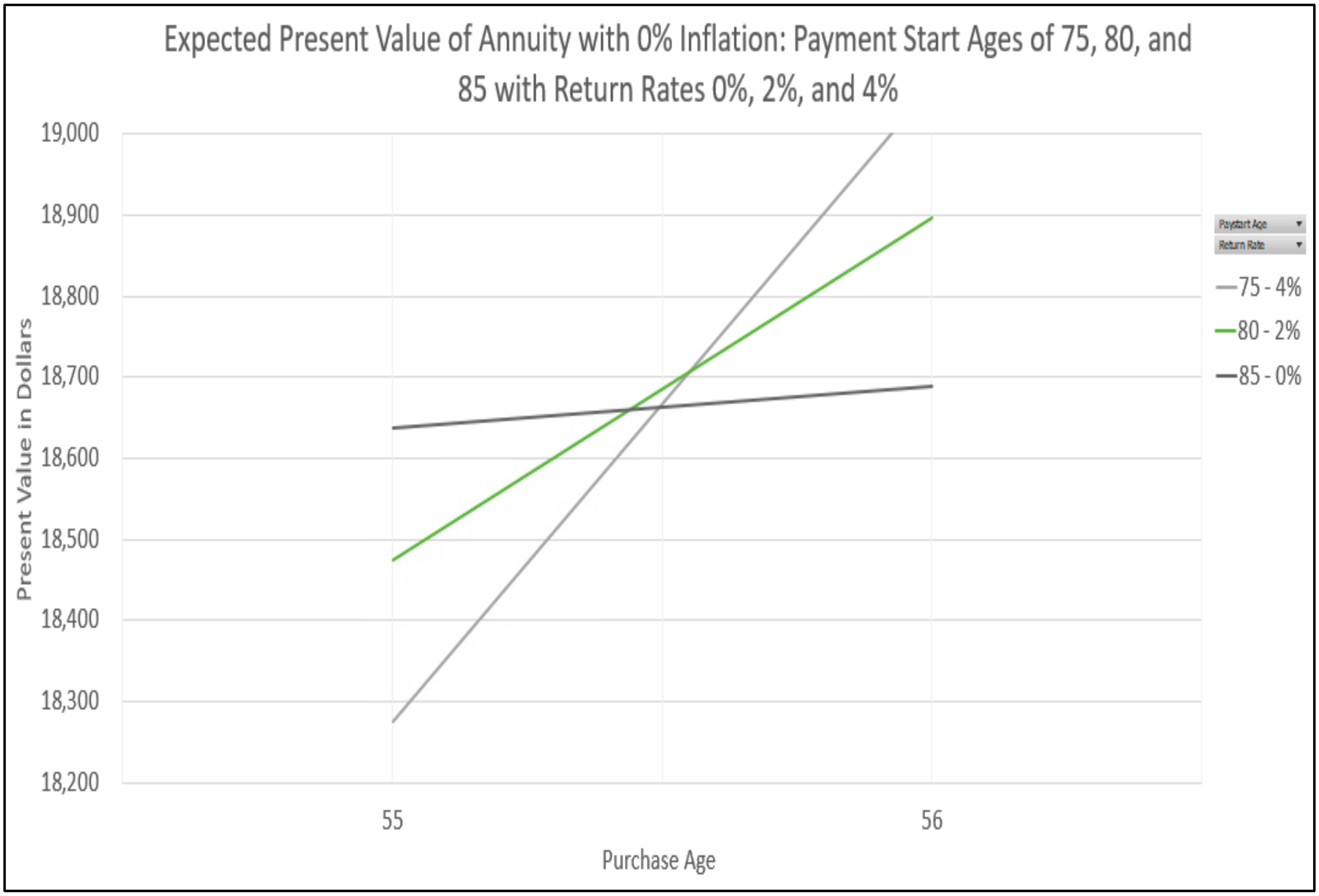

Figure 5: Relationship between present values of annuities with pay start ages 75, 80, and 85 under 4%, 2%, and 0% return.

Figure 5 illustrates a relationship between present values of three annuities found in Figure 4: a payment start age of 75 under a 4% return rate, a payment start age of 80 under a 2% return rate, and a payment start age of 85 under a 0% return rate. Upon first inspection in Figure 4, these three expected present values appear to intersect at the same point in time between purchase age 55 and 56, but enlarging this shows they do not. It is unlikely for this total intersection to have occurred as each expected present value is weighted by a different return rate, which changes the value of their discount factors. Regardless of rates, from this figure we are able to determine that at some arbitrary time between age 55 and 56 these three present values do intersect, causing the trajectory of their expected present values to reverse.

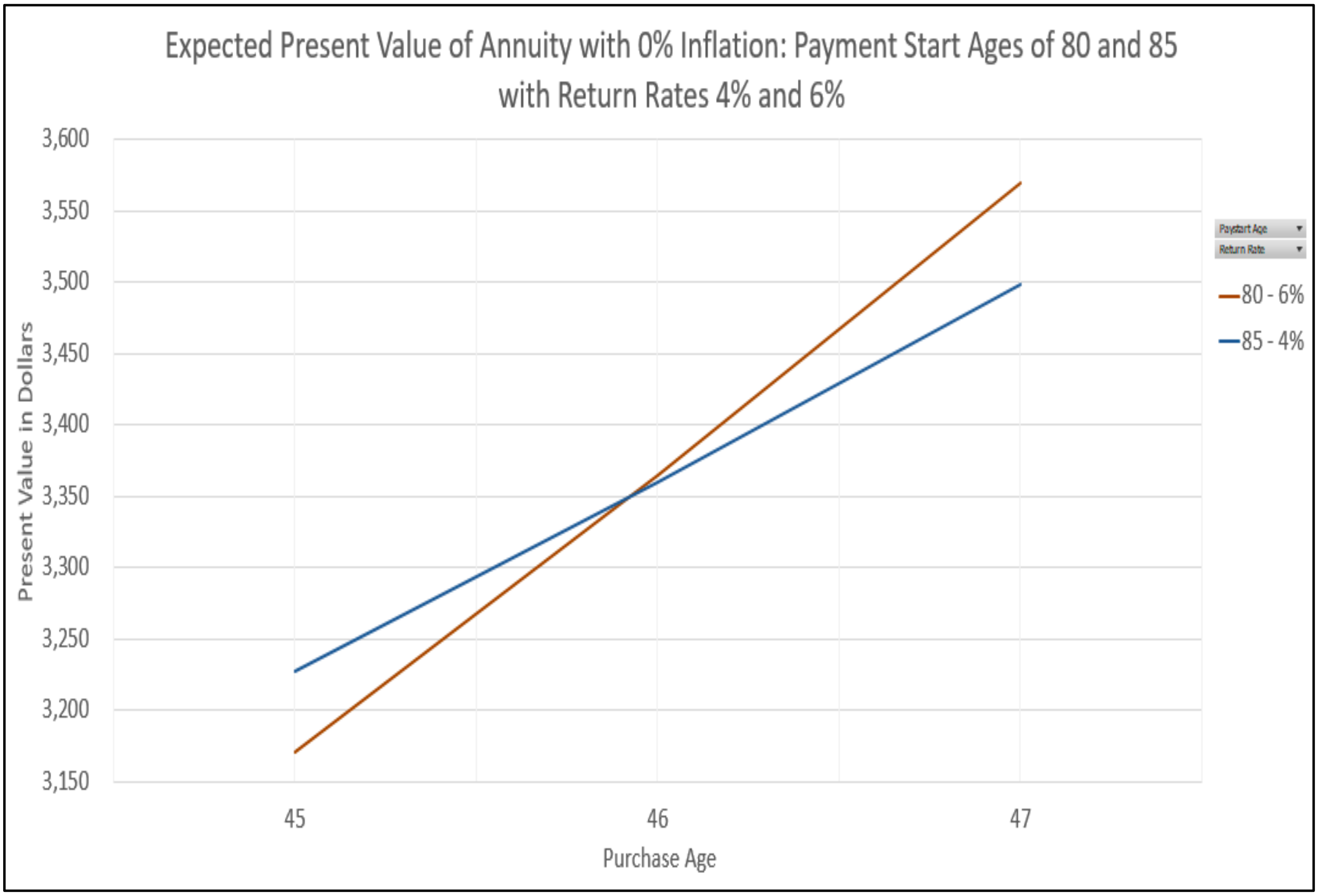

Figure 6: Relationship between present values of annuities with pay start ages 80 and 85 under 6% and 4% return.

Figure 6 outlines two present values from Figure 4, an annuity with a payment start age of 80 under a 6% return rate and an annuity with a payment start age of 85 under a 4% return rate. This is an important portion of Figure 4 to illustrate, as at a point in time between purchase age 45 and 46, the expected present values of these two annuities intersect, causing the expected value of an annuity with a payment start age 80 under a 6% return rate to exceed that of an annuity with a start age of 85 under a 4% return rate.

4% Inflation:

As a way to portray the impact 4% inflation has on the expected present value of these same scenarios, the following two figures will outline the same points in time that were extracted within Figures 5 and 6.

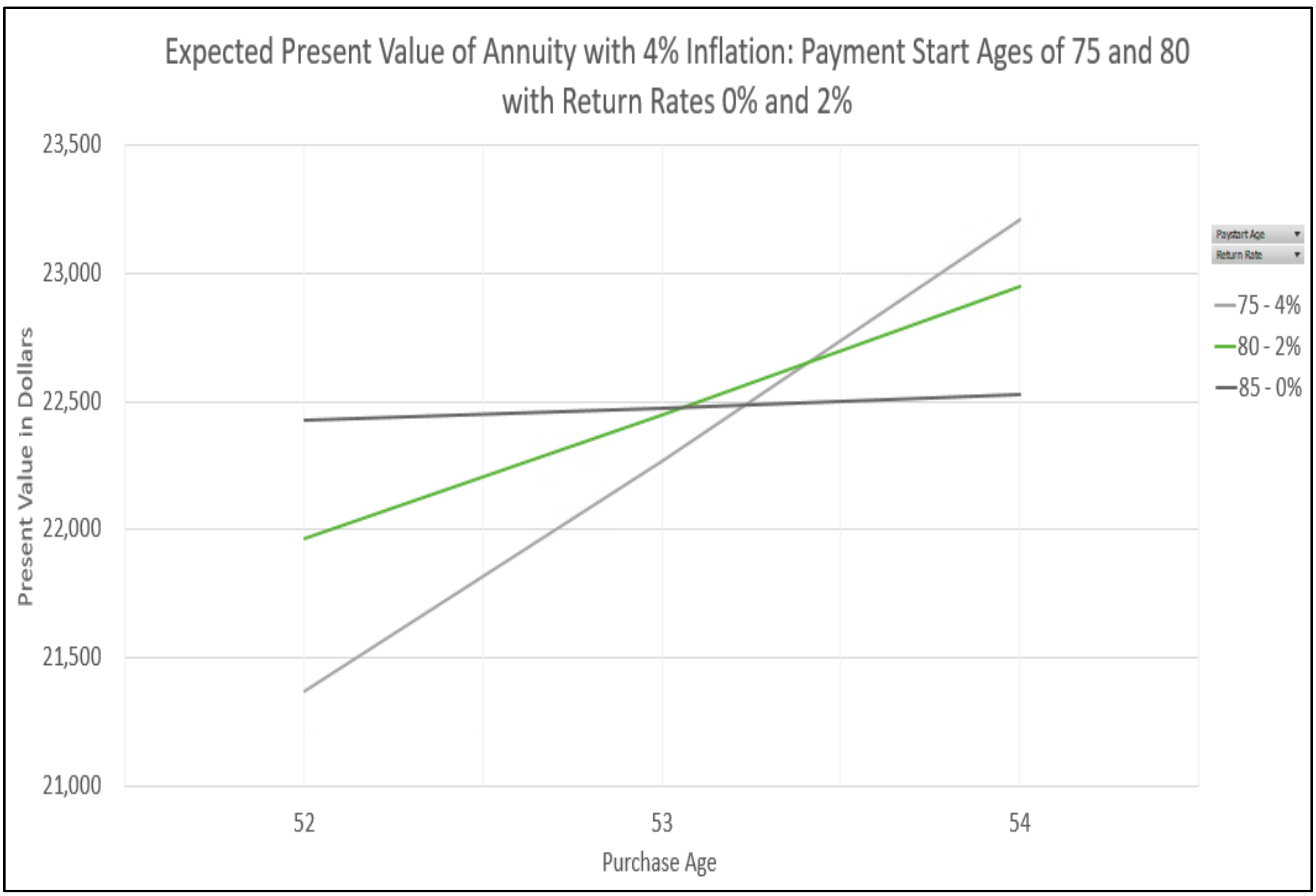

Figure 7: Relationship between present values of annuities with pay start ages 75, 80, and 85 under 4%, 2%, and 0% return.

Figure 7 portrays the same expected present values as the annuities in Figure 5, where there is a similar relationship in regard to where these three present values intersect with one another. The main difference when accounting for 4% inflation is the increase in dollar amount of these present values. This increase in value is due to the weight the inflation rate holds on each calculated present value, thus growing the total expected value.

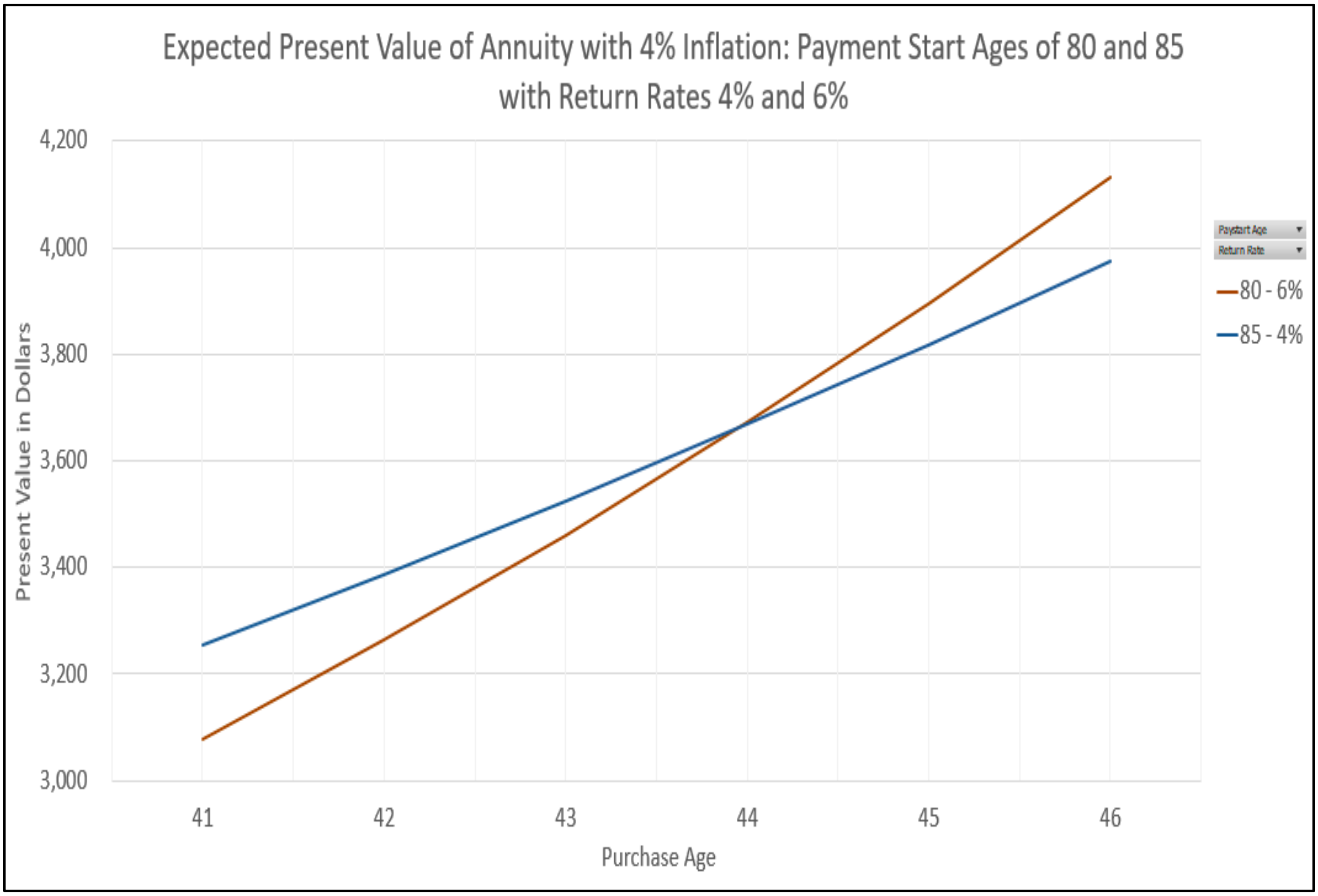

Figure 8: Relationship between present values of annuities with pay start ages 80 and 85 under 6% and 4% return.

Figure 8 illustrates the same present values as Figure 6 did, one annuity with a payment start age of 80 under a 6% return rate and one with a payment start age of 85 under a 4% return rate, this time under a 4% inflation rate. Under a higher inflation rate, these two intersect at an earlier point in time, between purchase ages 43 and 44 as opposed to age 44 to 45. This intersection also occurs at a higher expected present value due to inflation.

The relationships we have portrayed comparing the present values of annuities under both a 0% and 4% inflation rate are important for potential customers as this can help them understand which annuity purchase would best fit their needs. Regardless of the current inflation rate, annuities purchased with a higher return rate will determine a lower present value overall. But when comparing these high rate annuities with an earlier payment start age to those of similar value at a lower rate with later payment start ages it can appear they can produce a higher present value. This is due to the earlier payment ages as the annuity is expected to withstand a larger duration of time. The best suiting annuity will be best determined by the amount of money they are willing to invest towards the annuity. Once this amount is determined the customer could use the above figures to determine which scenario will suit their needs for retirement.

Section 5.2 - Expected Present Value of Insurance Benefit

Another important factor that plays into the calculation of the total premium is the cost of the insurance death benefit. In order to visualize how the expected present value of the death benefit alters in different scenarios, we have graphed our collected data in a similar manner to those of the present values for annuities. By graphing this information under the same conditions we can see if these two variables behave in a similar manner, regardless of the actual price these expected present values hold.

The following figures will be portrayed under one condition: 0% inflation. It is unnecessary to graph these variables under different inflation rates as inflation has no impact on the expected value of the death benefit. Thus, we have chosen to set inflation at 0% in order to simplify analysis on these graphs.

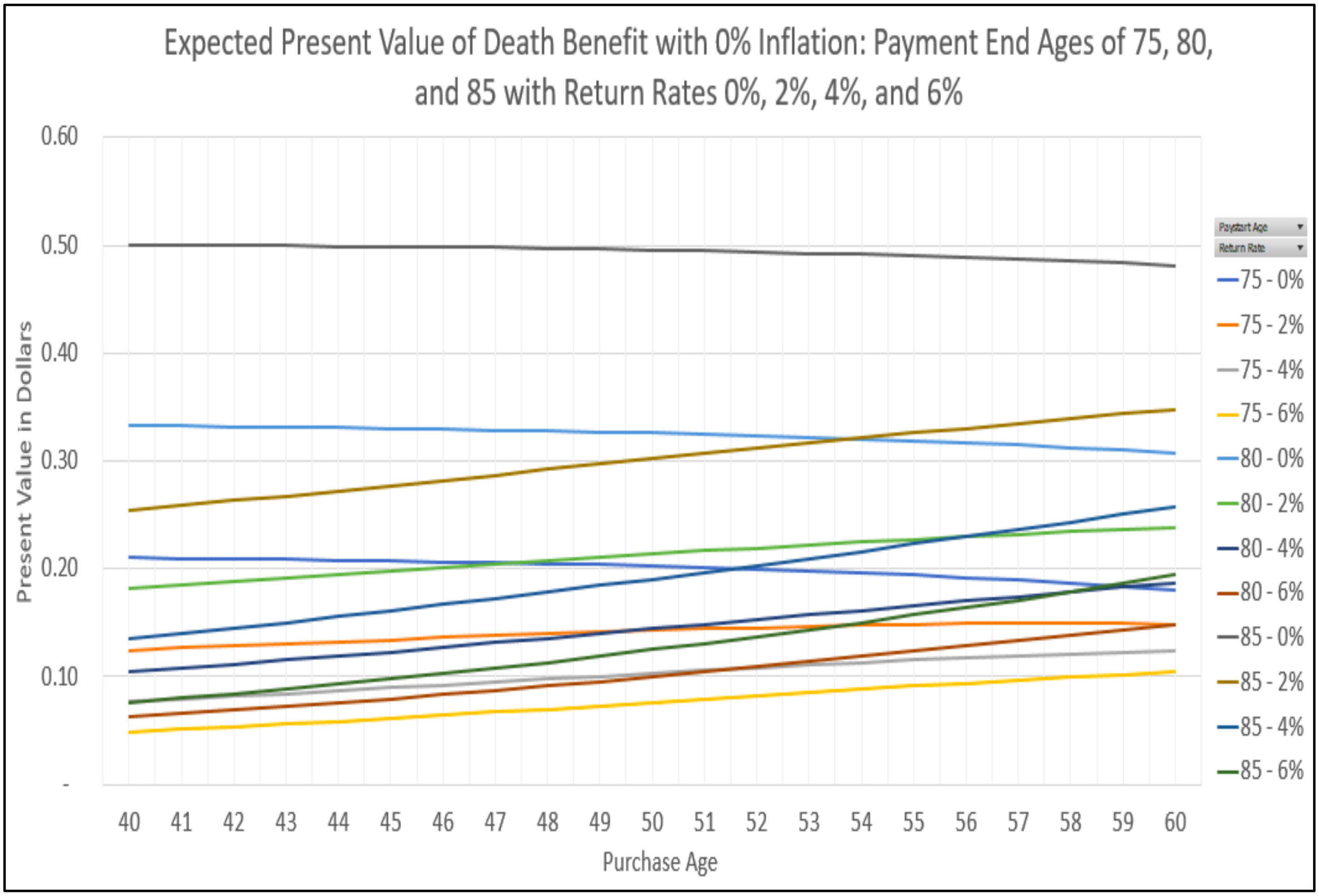

Figure 9: Expected present value of death benefit under 0% inflation with different payment end ages and return rates.

Figure 9 outlines the overall expected present value of the death benefit under various scenarios: benefits ranging from purchase age to ending payment ages of 75, 80, and 85 under return rates 0%, 2%, 4%, and 6%. From this figure it is clear there are many points in time in which the values of these benefits intersect with one another, changing the trajectory of their total values. Benefits under a 0% return rate tend to slope downwards, as their value is only weighted by the probability of death, which accumulates to a smaller present value. This decline occurs as the purchase age approaches the annuity payment start age. On the contrary, benefits under a return rate greater than 0% slope upwards over time, as the return rate is now a factor weighing into their overall expected present value.

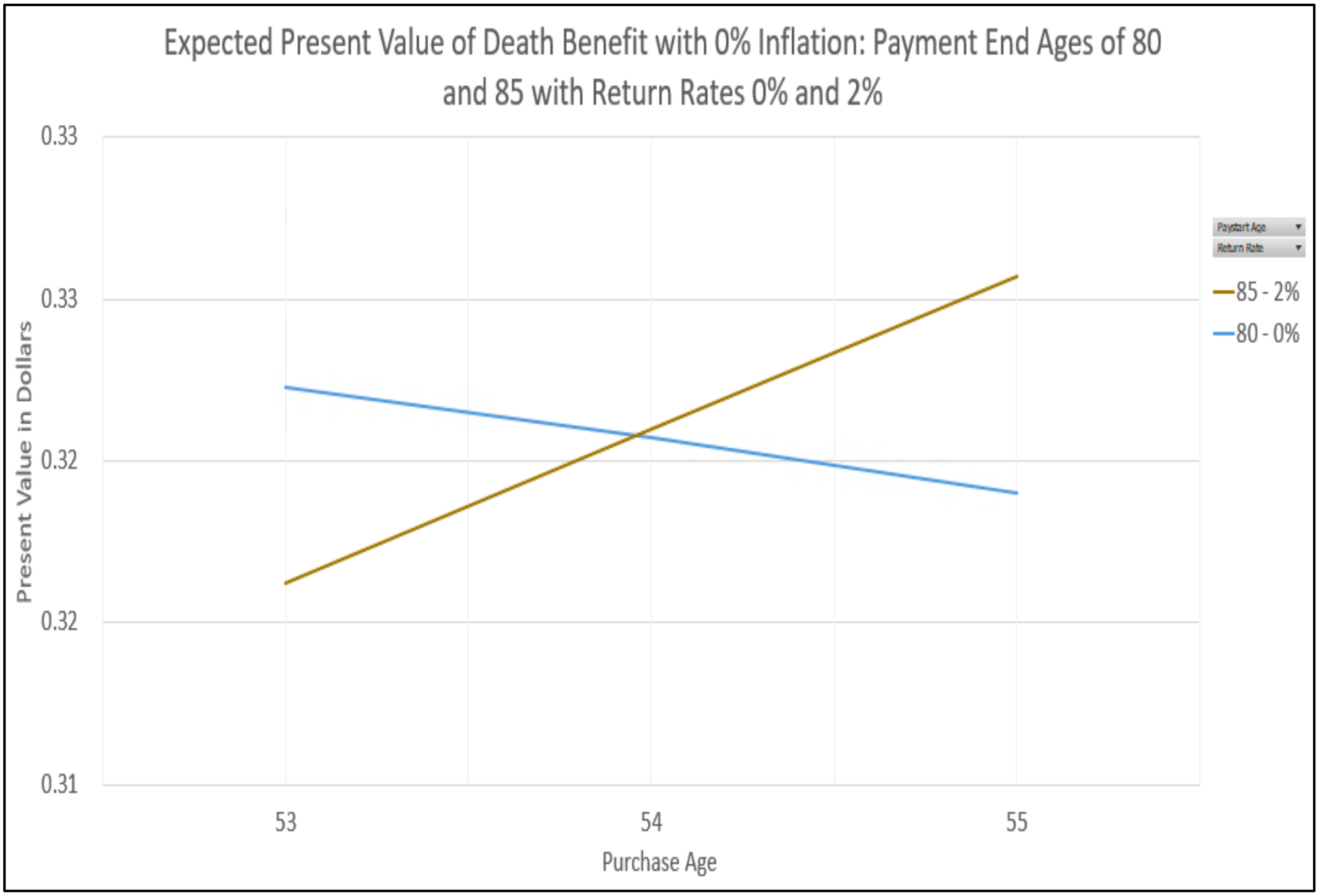

Figure 10: Relationship between present values of death benefits with end ages 80 and 85 under 0% and 2% return.

Figure 10 enlarges an intersection that has occurred between two present values of death benefits, one with a payment end age of 85 under a 2% return rate and one with an end age of 80 under a 0% return rate. At a point in time between purchase ages 53 and 54 the expected present values of these two benefits reach the same value, leading them to reverse in terms of which holds the highest present value. As previously mentioned, death benefits under a 0% return rate tend to decline in present value over time, which is portrayed within this figure.

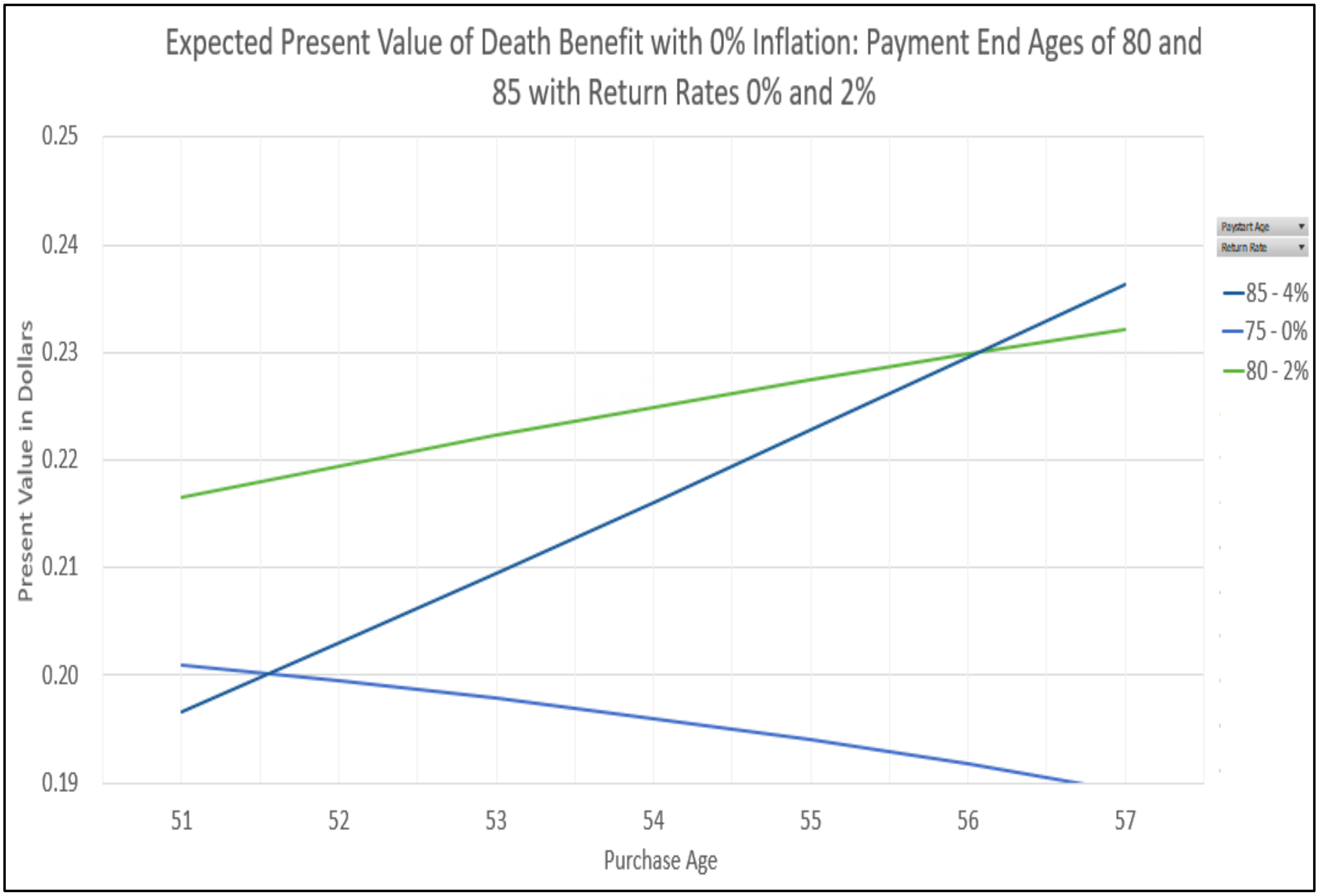

Figure 11: Relationship between present values of death benefits with end ages 75, 80, and 85 under 0%, 2%, and 4% return.

Figure 11 illustrates another interesting intersection from Figure 9, including present values of death benefits with payment end ages of 75, 80, and 85 under return rates 0%, 2%, and 4%, respectively. From this figure, we are able to see how at a point in time between purchase age 51 and 52 two present values, one with a payment end age of 75 under 0% return and one with end age 85 under 4% return, intersect allowing the expected present value of end age 85 to increase rapidly in value. This benefit then intersects one with an end age of 80 under 2% return, continuing to increase in expected present value.

Similar to the present values of annuities, each payment end age scenario tends to follow a similar trajectory regardless of the return rate, just at different values. The potential customer will once again need to choose a death benefit option that best suits their needs at the desired purchase age. Factors that will determine the benefit's expected present value include when the payment end age is and the size of the return rate.

Section 5.3 - Standard Deviation of the Total Expected Premium

The above figures served to portray key relationships between annuities and death benefits at varying purchase and payment ages. Because we were able to build a strong understanding of the projected growths for these separate products, it was important to investigate how these impact the total insurance premium as they are both necessary in determining the expected cost of the premium.

To do this, we evaluated the standard deviation of the total expected premium. By investigating the standard deviation, it gave us the opportunity to better understand the variability the collected data holds amongst itself as opposed to solely the expected cost. The expected cost is not as valuable to our analysis as it will fluctuate depending on the proposed initial value of the annuity payments. Again, we have portrayed our findings in the figures below based on the two inflation extremes.

0% Inflation:

The following figure portrays the standard deviation of the total expected premium based on differing scenarios:

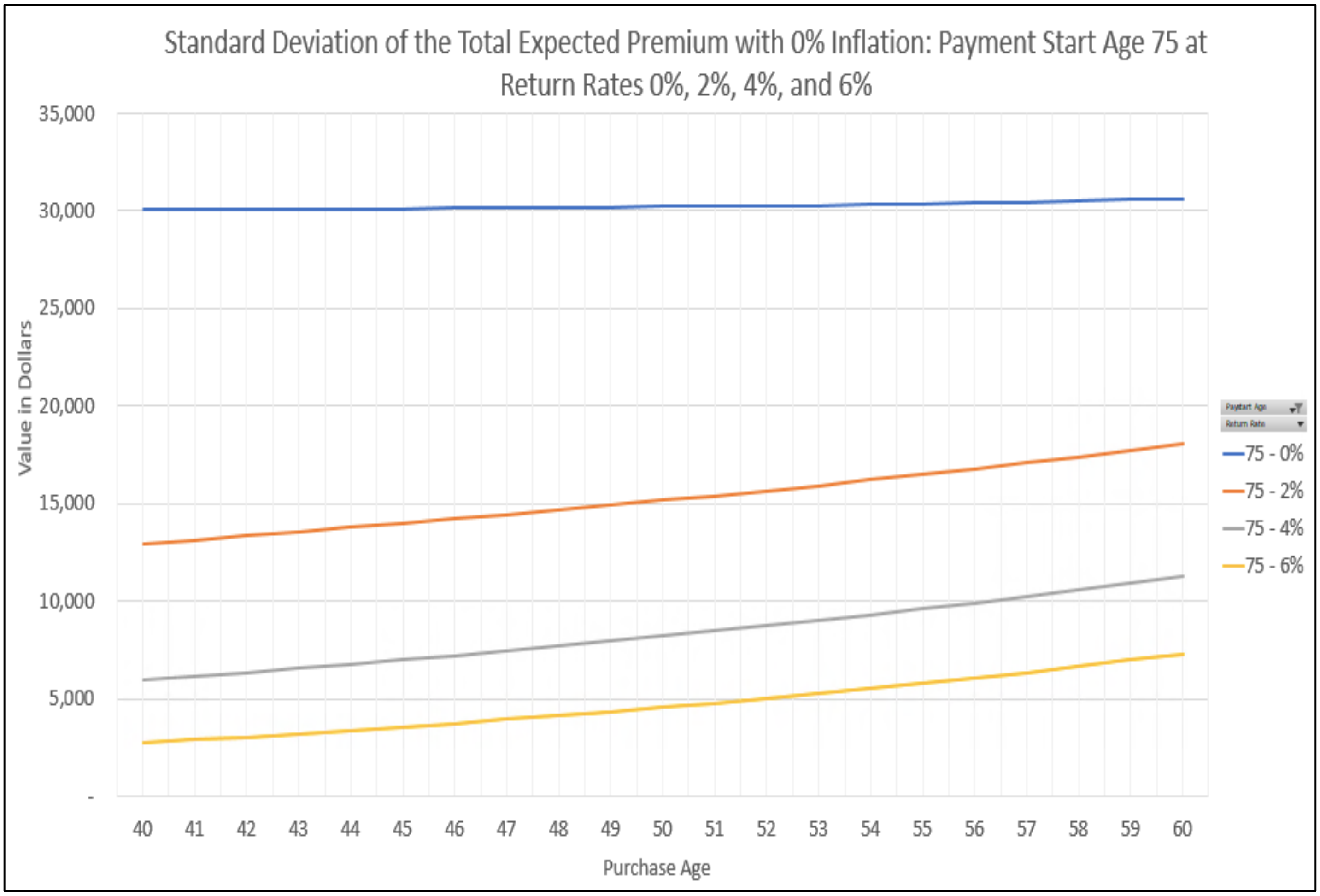

Figure 12: Standard deviation of the total expected premium for payment start age 75 under 0% inflation.

Figure 12 outlines the standard deviation of the total expected premium cost for a payment start age of 75 under return rates 0%, 2%, 4%, and 6%. From this figure it is clear that annuity payments that begin at age 75 with a 0% return rate have little variability in terms of the standard deviation, as payments are only weighted by the probability of death. With this little variability it can be understood that the total premium is not expected to vary much from the expected value. For return rates 2%, 4%, and 6% there appears to be a steady incline in value of the standard deviation.

4% Inflation:

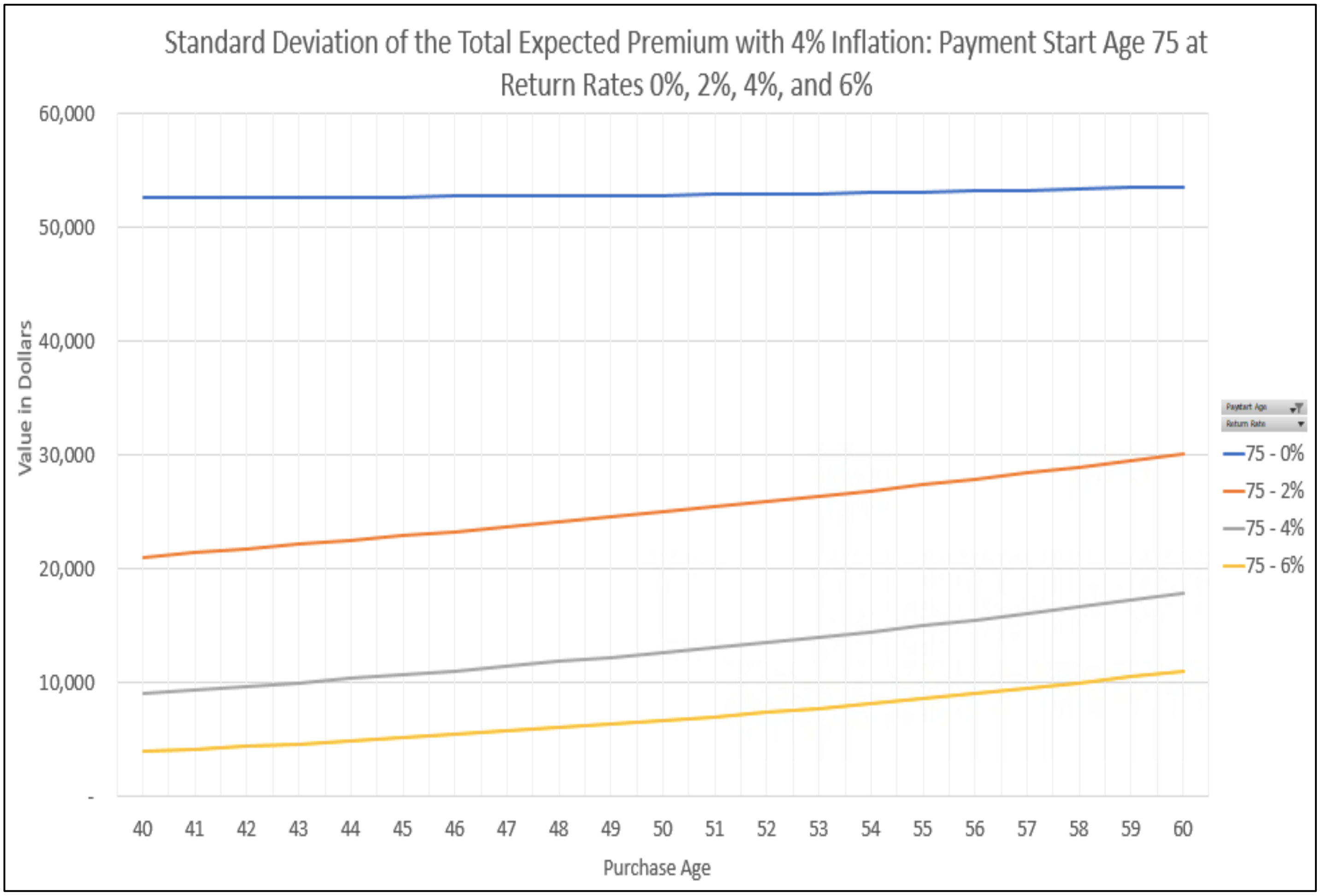

Figure 13: Standard deviation of the total expected premium for payment start age 75 under 4% inflation.

Figure 13 highlights the same information Figure 12 did, this time under 4% inflation. From this figure we are able to see that the trajectory of the standard deviation for the total expected premium does not change with an increase in inflation. This is important to note as this provides further validation that the standard deviation does not vary a significant amount from what the expected total premium has been projected to be.

Chapter 6 - Conclusion

Through our analysis of the risk of ruin an annuitant's account will face under incrementing withdrawal and interest rate scenarios and our work in understanding the impact mortality rates hold on the cost of insurance policies, we have come to conclude the following information:

1. Assuming a set investment amount into the annuity purchase, an account will deplete at a higher rate given a large withdrawal amount each period, specifically if this account is projected to last until a very advanced age
2. Accounts have a lower chance of failing when interest rates are higher
3. Due to inflation having no impact on the present value of the death benefit, it is best to purchase under a return rate greater than 0% as the present value is expected to increase when the purchase age approaches the last benefit payment
4. Regardless of inflation, return rates, or payment start ages the total cost of the premium does not vary from what it's expected total is to be

It is important to note that the above conclusions are hindered on the potential customer purchasing longevity insurance. Because an annuitant's account is projected to deplete sooner when withdrawing a larger sum of money, it is of their best interest to withdraw relatively small amounts each period if they anticipate to live until a very advanced age, say age 90. When they are purchasing this annuity, and if they have the opportunity to purchase it under a higher interest rate, the probability their account will survive until an advanced age is higher. This is due to the behavior of the interest rate in calculating the ending balance for any given year as it serves as an accumulation factor. Thus, with a higher interest rate each ending balance will increase, eliminating the chance of early depletion.

Additionally, the customer can expect the present value of their annuity to be at its greatest when purchased at an age closest to when the payments begin, as it allows the annuity more time to accumulate until the annuitant is deceased. This makes sense as the present value represents how much is necessary now to fund these payments, so it should be greater given it will have a longer duration. If the potential customer does not have a large amount to invest into this annuity at the time of purchase, we recommend them to purchase an annuity with payments that begin at an advanced age. This will allow them to still enjoy the benefits of financial security later in retirement without needing to spend a mass amount up front with earlier payouts. This will also help alleviate the risks of ruin their account will face because annuity payments will begin at an advanced age as opposed to the individual's retirement age.

When considering various inflation and return rates, as well as ages for annuity payments to start and death benefits to expire, it is evident that these do not increase the variability of the expected total cost of the premium. With this in mind, we can conclude that the potential customer should choose the insurance policy that will best fit their needs both at the time of purchase and into retirement.

In conclusion, we have been able to determine that longevity insurance is beneficial for individuals planning for retirement. They help the individual feel a sense of financial security throughout retirement as they will always have a steady stream of income, which can allow them to take more risks in other aspects of life, like when investing in stocks and bonds.

References

Blueprint Income, LLC. (2021, December 17). *The Official Guide to Longevity Annuities*. Blueprint Income. Retrieved January 24, 2021, from https://www.blueprintincome.com/resources/longevity-annuities/

Cussen, M. P. (2022, February 8). *The basics of a 401(k) retirement plan*. Investopedia. Retrieved April 24, 2022, from https://www.investopedia.com/articles/retirement/08/401k-info.asp

Fernando, J. (2022, February 1). *How to calculate present value (PV), and why investors need to know it*. Investopedia. Retrieved April 24, 2022, from https://www.investopedia.com/terms/p/presentvalue.asp#:~:text=Present%20value%20(PV)%20is%20the,of%20the%20future%20cash%20flows

Kintzel, D., & Turner, J. A. (2020). Provision of longevity insurance annuities. *Financial Analysts Journal*, *76*(4), 119–133. https://doi.org/10.1080/0015198x.2020.1809903

Society of Actuaries. (2014, October). RP-2014 Mortality Tables Report. *Society of Actuaries*. Retrieved 2021, from https://www.soa.org/globalassets/assets/files/research/exp-study/research-2014-rp-report.pdf.

Team, T. I. (2022, February 8). *Pension plan*. Investopedia. Retrieved April 24, 2022, from https://www.investopedia.com/terms/p/pensionplan.asp

Appendix A

RP-2014 Mortality Tables

Released by the Society of Actuaries, the RP-2014 Mortality Tables are determined through data collected on 10.5 million life years of exposure and over 220,000 deaths. These tables help to form a basis regarding the scale of retirement obligations within the United States. The 2014 edition of mortality tables outlines the following information for both males and females:

- Employee mortality rates for ages 18-80 applicable to:
    - Total Dataset
    - Blue Collar
    - White Collar
    - Bottom Quartile
    - Top Quartile
- Healthy annuitant mortality rates for ages 50-120 applicable to:
    - Total Dataset
    - Blue Collar
    - White Collar
    - Bottom Quartile
    - Top Quartile
- Disabled retiree mortality rates for ages 18-120 applicable to:
    - Total Dataset

Where the subsections are defined as:

Total Dataset – All available data for nondisabled individuals

Blue Collar – All available data for employees who are categorized as blue collar workers (i.e., manufacturing, construction, etc.)

White Collar – All available data for employees who are categorized as white collar workers (i.e., lawyers, accountants, etc.)

Bottom Quartile – All available data for individuals who earn less than the average, hence bottom quartile. These differentiate based on need within the table; employee mortality rates are based on salary and healthy annuitant rates are based on benefit amounts

Top Quartile – all available data for individuals earning more than the average. These differentiate in the same way data on the bottom quartile does

The following figure is a small excerpt of the described mortality rates in terms the total dataset, highlighting mortality rates for employees and disabled retirees from ages 18-27, for both males and females:

	Total Dataset; Males			Total Dataset; Females		
Age	Employee	Healthy Annuitant	Disabled Retiree	Employee	Healthy Annuitant	Disabled Retiree
18	0.000328		0.005744	0.000157		0.002162
19	0.000369		0.006462	0.000162		0.002231
20	0.000406		0.007110	0.000162		0.002231
21	0.000449		0.007863	0.000162		0.002231
22	0.000488		0.008546	0.000162		0.002231
23	0.000509		0.008914	0.000166		0.002286
24	0.000516		0.009036	0.000169		0.002328
25	0.000484		0.008476	0.000173		0.002383
26	0.000462		0.008090	0.000179		0.002465
27	0.000449		0.007863	0.000187		0.002576

RP-2014 Mortality Tables of the total data set for males and females.

Appendix B

Expected Value Calculations within Excel

As defined, the expected value of an annuity is determined using the following equation:

$$E[Y] = \sum_{t=0}^{\infty} ({}_{t|}q_x)(\ddot{a}_{\overline{t+1}|})$$

Where t is the deferral period, q_x is the percentage of people alive at age x who will die by age x+1, and the annuity-due angle (t+1) is the whole life annuity-due, assuming the person will be alive for t+1 years.

In order to find this expected value for an annuity within Excel, we formulated a sheet to calculate necessary values, such as q_x, p_x, ${}_tp_x$, ${}_{t|}q_x$, and v^t, in order to determine and accumulate the expect present value of an annuity for different scenarios. Included is an example of the information this sheet contains:

	A	B	C	D	E	F	G	H	I	J
1										
2	Current Age	Paystart Age	Return Rate	Inflation	Annuity PMT	Total Cost (P)				
3	60	75	0.06	0.02	5000	19076.65846				
4										
5										
6	t	x	qx	px	tpx	t|qx	v^t	pmt	Pay Status Annuity	PV Y
7	0	60	0.00469	0.99531	1.00000	0.00469	1.00000	0	0	-
8	1	61	0.00524	0.99476	0.99531	0.00522	0.94340	0	0	-
9	2	62	0.00587	0.99413	0.99010	0.00581	0.89000	0	0	-
10	3	63	0.00658	0.99342	0.98429	0.00647	0.83962	0	0	-

Excel sheet to calculate present value of annuity, insurance death benefit, and total expected premium.

This sheet calculates the present value of Y and the present value of Y^2, which we have used to find the necessary E[Y], $E[Y^2]$, Var[Y], and SD[Y]. As an example, take our calculations for the E[Y]: to do so, we took the sum of the product of the present value of Y variables with their respective ${}_{t|}q_x$ probabilities. We utilized the Excel function *sumproduct*, which returns the sum of the products of the corresponding arrays. Based on the above figure, this formula would be as follows:

$$= SUMPRODUCT(F7{:}F127, J7{:}J127)$$

This same process was applied when determining the expected present values of the insurance death benefit and of the total premium cost.

Data Collection in VBA

After collecting the necessary expected values and standard deviations, it was vital for our team to develop a macro that would run hundreds of different combination scenarios surrounding different payment start ages, current ages, return rates, and inflation rates.

Our first step in doing so was developing a *modeloffice* sheet. This sheet was utilized to store the different permutations we hoped to achieve within our macro. This sheet outlined every possible scenario we could collect data on. Below is a portion of the described sheet:

current age	start age	rates	inflation	annuity
40	75	0%	0%	5000
40	75	0%	1%	5000
40	75	0%	2%	5000
40	75	0%	3%	5000
40	75	0%	4%	5000
40	75	2%	0%	5000
40	75	2%	1%	5000
40	75	2%	2%	5000
40	75	2%	3%	5000
40	75	2%	4%	5000
40	75	4%	0%	5000
40	75	4%	1%	5000
40	75	4%	2%	5000
40	75	4%	3%	5000

Using this sheet formed the basis for our code. We instructed VBA to assign each row of these values to our calculation sheet, as described in Appendix B.1, thus calculating each possible scenario, then copy and pasted each of the necessary values into our *Results* sheet. Having this *Results* sheet made it simpler for us to store this information into a Pivot Chart as a way to portray the results, and determine which combination of annuities and death benefits provided the most beneficial results. The described VBA code can be found on the next two pages.

```
For i = 4 To 1263
    ' assigning values from model office names
    currentAge = Worksheets("modeloffice").Cells(i, 1)
    startingAge = Worksheets("modeloffice").Cells(i, 2)
    interestRate = Worksheets("modeloffice").Cells(i, 3)
    inflation = Worksheets("modeloffice").Cells(i, 4)
    ' benefit = Worksheets("modeloffice").Cells(i, 5)
    Payment = Worksheets("modeloffice").Cells(i, 5)

    ' now assigning cells in RV sheet these values
    Range("returnRate") = interestRate
    Range("currentAge") = currentAge
    Range("paystart") = startingAge
    Range("inflation") = inflation
    ' Range("Premium") = benefit
    Range("pmt") = Payment

    ' reading these values from RV sheet
    E_Y = Range("EY").Value
    E_Y2 = Range("EY_2").Value
    Var_Y = Range("VAR_Y").Value
    SD_Y = Range("SD_Y").Value

    E_Z = Range("EZ").Value
    E_Z2 = Range("EZ_2").Value
    Var_Z = Range("VAR_Z").Value
    SD_Z = Range("SD_Z").Value

    E_W = Range("EW").Value
    E_W2 = Range("EW_2").Value
    Var_W = Range("VAR_W").Value
    SD_W = Range("SD_W").Value
```

```
    ' pasting all of these values into Results 2
    'Worksheets("Results").Cells(i, 1) = benefit
    Worksheets("Results").Cells(i, 1) = Payment
    Worksheets("Results").Cells(i, 2) = inflation
    Worksheets("Results").Cells(i, 3) = interestRate
    Worksheets("Results").Cells(i, 4) = currentAge
    Worksheets("Results").Cells(i, 5) = startingAge
    Worksheets("Results").Cells(i, 6) = E_Y
    Worksheets("Results").Cells(i, 7) = E_Y2
    Worksheets("Results").Cells(i, 8) = Var_Y
    Worksheets("Results").Cells(i, 9) = SD_Y
    Worksheets("Results").Cells(i, 10) = E_Z
    Worksheets("Results").Cells(i, 11) = E_Z2
    Worksheets("Results").Cells(i, 12) = Var_Z
    Worksheets("Results").Cells(i, 13) = SD_Z
    Worksheets("Results").Cells(i, 14) = E_W
    Worksheets("Results").Cells(i, 15) = E_W2
    Worksheets("Results").Cells(i, 16) = Var_W
    Worksheets("Results").Cells(i, 17) = SD_W

Next i
```