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Teaching Practicum
Summary and Analysis

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Abstract

I completed a Massachusetts Department of Education teaching practicum at Concord Carlisle Regional High School, including 75 hours of observation and 150 hours of direct instruction. This consisted of teaching three classes for one semester: two sections of Geometry for freshmen and sophomores and one section of Statistics for seniors. This report summarizes and analyzes my work, my students, the school, and my overall experience and lessons learned.

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1. Background

Concord Carlisle Regional High School serves the affluent, suburban communities of Concord and Carlisle in Massachusetts for grades 9 through 12 for a total of approximately 1250 students. It is a proud school, with very high expectations of itself, its students, and its teachers. As such, CCHS has an excellent and well-deserved reputation as one of the top schools in the state, perennially ranking very well in the MCAS and sending a high number of students on to college programs. However, both Concord and Carlisle are racially and culturally quite homogenous, and this extends naturally into CCHS as well, mitigated somewhat by Concord's participation in the Metropolitan Educational Opportunity Council, or METCO, a program that arranges for racial minorities closer to Boston to travel to suburban schools for education.

The METCO program was initiated in 1966 "to provide enhanced educational opportunities for urban minority students, to help integrate suburban school districts, and to reduce segregation in city schools." [1] METCO expresses itself at CCHS in the form of several dozen minority students from greater Boston and Springfield being bussed to and from the school every day.

Concord High originally dates back to 1852. CCHS was created with neighboring Carlisle in 1960, coinciding with the completion of the campus in which the school still operates. [2] The demographics are not totally representative of either Concord or Carlisle, due to METCO: 90.1% White, 5.6% African-American, 2.8% Asian, and 1.5% Hispanic in 2004-2005. 2.0% of students speak a language other than English as a first language, and an additional 0.7% have limited English proficiency. Only 4.0% fall into the low-income category, compared to 27.7% statewide, and 10.1% are considered to be part of Special Education. Women are a slight majority, with 52.8% in 2004-2005. [3]

In 2003-2004, CCHS employed 91 total teachers, 82 of whom were considered to be teaching in Core Academic Areas. Of those in Core Academic Areas, 95.4% were identified as Highly Qualified. The student to teacher ratio was 13.2 to 1. CCHS also boasts 100% Internet accessible classrooms with a Student to Computer ratio of 4.4 to 1. For comparison, the same statistics for Doherty Memorial High are as follow: 99 total teachers, 87 of whom teach in Core Academic Areas with 92.0% identified as Highly

Qualified. The student to teacher ratio was 15.5 to 1. Meanwhile, 34.8% of students were identified as low-income and 32.1% had a first language other than English. [3]

The level of achievement and expectation matches the affluence of the base communities: For the class of 2000, 91.5% of graduating seniors moved on to a 4-year college program. Participation in the SAT-I test was 100%, compared to 42% nationally, with mean scores of 580 for Verbal and 611 for Math. [4] CCHS' MCAS results are similarly favorable. In 2004, the failure rate among those taking the 10th grade test was 0% in both subjects (English and Math), a feat matched by only 8 other schools in the state. 94% of students placed in either "Advanced" or "Proficient" in English, good for sixth in the state, while 86% of students placed in one of the same categories for Math, good for 24th in the state. [5]

Unfortunately, CCHS is an excellent example of the vicious cycle that money seems to be able to buy a good education, even in public schools. Both Concord and Carlisle exude affluence, wealth, and the very essence of upper-middle-class suburbia. In 1996, attending CCHS as a Freshman, I listened to a student answer a teacher's question of what she did the past weekend by explaining that her parents had bought for her a Porsche – despite the fact that she was not yet 16. While certainly many of the students are down to earth and many are not in fact able to purchase extraneous luxury vehicles, CCHS still provides a stark contrast to the economic landscape of the public schools in a place like Worcester. In terms of concrete numbers, the median household income of Concord in 2000 was $95,897, while the median household income of Worcester in 2000 was $35,623. Similarly, the median single-family home value in Concord was $453,400 while in Worcester the median single-family home value was $119,600. [6, 7] The ethnic, racial, and economic homogeny is broken up somewhat by the METCO program, but such is an incomplete melding; to adapt from the title of the book by Beverly Daniel Tatum, all the black kids really are sitting together in the cafeteria, both figuratively and literally.

Going into my student teaching experience, I tried to focus on being aware of these particulars of the school without being oversensitive or allowing such to get in the way of my teaching, particularly in regards to students participating in the METCO program. If one thinks of people as different then one will inevitably treat them as different, and I wanted to think of my students as different but different on the basis of being individuals rather than the color of their skin or their socioeconomic background.

2. Course Review

While at CCHS, I spent a majority of my time teaching three classes: Two sections of CP-2 Geometry for 9^{th} and 10^{th} grade and Statistics, a CP-2 level course for 12^{th} grade. There are four levels of Math at CCHS, as follows [8]:

College Preparatory III (CP-3)
Students at this level are typically able to follow a model given concrete examples and experiences, master concepts with directed practice, and rely on the teacher's assessment of understanding and performance. Instructional approaches are designed to meet the needs of the directed learner and include extending and refining types of activities/practice, explicit directions and modeling, extensive review of previous topics, direct teaching of how to use resources, direct instruction in reading the text, and extensive review of homework in class.

College Preparatory II (CP-2)
Students at this level are typically able to follow a model and reach an abstract level with guidance, learn well from periods of directed instruction in combination with in-class guided practice, select one of several models and master it, be willing to seek extra help when necessary, identify a problem in understanding or performance with guided questioning, and complete homework in a reasonable amount of time.
Instructional approaches at this level are designed to meet the needs of the guided learner who requires some direction and include reading the text with structured support to supplement lessons, considerable review of previous topics, moderate class time spent on reviewing homework built-in guided practice and guided questioning with some directed learning, and focus both on extending and refining knowledge with some performance tasks.

College Preparatory I (CP-1)
Students at this level are typically able to understand and analyze complex situations with guidance, respond well to criticism, sometimes apply concepts to model situations,

almost always answer word problems of a type previously studied, know what she/he doesn't know, have some metacognitive abilities, recall previous skills and topics, demonstrate proficiency with minimal review, understand alternative solutions when presented, understand and use several related models, be self-motivated in seeking extra help, complete homework in a reasonable amount of time, complete tests in the allotted time, and read the text to reinforce the lesson.

Instructional approaches at this level are designed to meet the needs of the guided learner and include minimal class time spent on reviewing homework, extended segments of guided instruction, instruction at a fast pace, and focus both on performance tasks and extending and refining knowledge.

Honors

Students at this level are typically able to think critically, analyze complex situations, and are comfortable with abstract concepts with an increasing level of abstraction and difficulty each year, be independently self-critical, be metacognitive, provide strong insight into algebraic thinking and visual relationships, miss a class and keep up, learn independently, learn at a fast pace, execute skills reliably, demonstrate proficiency of previous topics and skills, make use of available resources, for study groups, seek out others, complete homework in a reasonable amount of time, support answers appropriately, complete tests in the allotted time, and read the text to preview the lesson. Instructional approaches are designed to meet the needs of the independent learner and include minimal class time spent in reviewing homework, instruction at a fast pace, and are focused on performance tasks.

The Mathematics Program at CCHS, like the general program for the school, is based on preparing students for college. Since the class of 2002 the Math department has employed a four-level system designed to place students in environments most conducive to each student's particular learning style. The material and curriculum are for the most part constant across the levels, but the approach to how and speed at which material is covered varies. Naturally, this does mean that students in higher levels end up covering more material, but this is a side effect rather than being a key part of the design.

During my experience I also noticed a distinct perception among the teachers of what a typical student in each level was, and at times it was difficult to determine if teachers were teaching a particular way because another way would truly be outside of the grasp of the students or if it was outside the student's grasp because the teachers expected it to be. I have great faith in the experience of those that I worked with, but some small part of me still nagged that it didn't always feel right to not demand students constantly taking initiative and going above and beyond. Of course, having been an honors-level student myself at CCHS it was a different and new experience to see and teach directed at students whose learning styles varied so much from my own.

A few classes fell outside of the four classifications, but I noticed that in almost every case there was an implied level. For example, the Senior Calculus class (not AP) did not have a CP designation but was considered to be "CP-1 Level". This was true of the Statistics section that I taught as well; it was "CP-2" level.

The basic course threads for all levels start with Algebra I, then move on to Geometry, Algebra II, Trigonometry and Precalculus, and finally for many students ending with one of Calculus, Statistics, or Advanced Topics.

The course threads started for most students in middle school, where the standard is to take Algebra I in 8th grade. The 8th grade math teacher then recommends a level to start in High School, and for many students they will stay on that level for the full four years. Occasionally students will switch down one level (Honors to CP-1, CP-1 to CP-2, etc) or switch up a level, although the latter is far less common. Students who struggled greatly in Algebra in Middle School, who took Pre-Algebra in 8th grade, or come in from another school system that had not taught Algebra prior to High School enter either CP Algebra I or Algebra I Part 1, each being a full-year course. CP Algebra I generally leads to either CP-2 or CP-3 Geometry while Algebra I Part 1 leads to Algebra I Part 2.

I taught two sections of CP-2 Geometry; approximately half of my students in those classes were sophomores and half were freshmen.

The class descriptions are taken from the CCHS Mathematics Department program and course descriptions [8]; all Learning Standards quotations are taken from the Massachusetts DOE Curriculum Frameworks. [9]

College Preparatory II Geometry Full Year – 5.00 Credits

For students in grades 9 & 10

Prerequisites: Placement in this course based on performance in previous course and other assessments.

Description: This course consists of all the topics generally included in a Euclidean Geometry course such as geometric definitions and symbols, angles, triangles and congruencies, geometric inequalities, parallel lines in a plane, quadrilaterals, triangle similarity, areas of polygons, circles and spheres, solids and volumes, and some geometry in three dimensions. It emphasizes the algebraic skills needed to solve geometric problems.

Evaluation: Students will be evaluated based on their level of achievement of the learning standards as demonstrated through homework, class participation, quizzes, tests, and semester examinations.

The following Learning Standards from the Curriculum Frameworks were emphasized as part of CP-2 Geometry:

From the Geometry strand:

10.G.1 Identify figures using properties of sides, angles, and diagonals. Identify the figures' type(s) of symmetry.

10.G.2 Draw congruent and similar figures using a compass, straightedge, protractor, and other tools such as computer software. Make conjectures about methods of construction. Justify the conjectures by logical arguments.

10.G.3 Recognize and solve problems involving angles formed by transversals of coplanar lines. Identify and determine the measure of central and inscribed angles and their associated minor and major arcs. Recognize and solve problems associated with radii, chords, and arcs within or on the same circle.

10.G.4 Apply congruence and similarity correspondences (e.g., ΔABC ΔXYZ) and properties of the figures to find missing parts of geometric figures, and provide logical justification.

10.G.5 Solve simple triangle problems using the triangle angle sum property and/or the Pythagorean Theorem.

10.G.6 Use the properties of special triangles (e.g., isosceles, equilateral, 30°-60°-90º, 45°-45°-90°) to solve problems.

10.G.7 Using rectangular coordinates, calculate midpoints of segments, slopes of lines and segments, and distances between two points, and apply the results to the solutions of problems.

10.G.8 Find linear equations that represent lines either perpendicular or parallel to a given line and through a point, e.g., by using the "point-slope" form of the equation.

12.G.1 Define the sine, cosine, and tangent of an acute angle. Apply to the solution of problems.

From the Measurement strand:

10.M.1 Calculate perimeter, circumference, and area of common geometric figures such as parallelograms, trapezoids, circles, and triangles.

10.M.3 Relate changes in the measurement of one attribute of an object to changes in other attributes, e.g., how changing the radius or height of a cylinder affects its surface area or volume.

The following Learning Standards were part of Algebra I, but were reviewed in CP-2 Geometry on a regular basis in preparation for both the MCAS and Algebra II, as well as occasionally being directly connected to a given example:

From the Patterns, Relations, and Algebra strand:

10.P.2 Demonstrate an understanding of the relationship between various representations of a line. Determine a line's slope and x- and y-intercepts from its graph or from a linear equation that represents the line. Find a linear equation describing a line from a graph or a geometric description of the line, e.g., by using the "point-slope" or "slope y-intercept" formulas. Explain the significance of a positive, negative, zero, or undefined slope.

10.P.3 Add, subtract, and multiply polynomials. Divide polynomials by monomials.

10.P.4 Demonstrate facility in symbolic manipulation of polynomial and rational expressions by rearranging and collecting terms; factoring (e.g., $a^2 - b^2 = (a + b)(a - b)$, $x^2 + 10x + 21 = (x + 3)(x + 7)$, $5x^4 + 10x^3 - 5x2 = 5x2 (x2 + 2x - 1)$); identifying and canceling common factors in rational expressions; and applying the properties of positive integer exponents.

10.P.5 Find solutions to quadratic equations (with real roots) by factoring, completing the square, or using the quadratic formula. Demonstrate an understanding of the equivalence of the methods.

10.P.6 Solve equations and inequalities including those involving absolute value of linear expressions (e.g., $|x - 2| > 5$) and apply to the solution of problems.

10.P.8 Solve everyday problems that can be modeled using systems of linear equations or inequalities. Apply algebraic and graphical methods to the solution. Use technology when appropriate. Include mixture, rate, and work problems.

From the Number Sense strand:

10.N.1 Identify and use the properties of operations on real numbers, including the associative, commutative, and distributive properties; the existence of the identity and inverse elements for addition and multiplication; the existence of nth roots of positive real numbers for any positive integer n; and the inverse relationship between taking the nth root of and the nth power of a positive real number.

10.N.2 Simplify numerical expressions, including those involving positive integer exponents or the absolute value, e.g., $3(2^4 - 1) = 45$, $4|3 - 5| + 6 = 14$; apply such simplifications in the solution of problems.

10.N.3 Find the approximate value for solutions to problems involving square roots and cube roots without the use of a calculator, e.g., .

10.N.4 Use estimation to judge the reasonableness of results of computations and of solutions to problems involving real numbers.

It should be noted that while in theory the topics above were covered or at least started in Algebra I, the level of mastery by the students in my CP-2 Geometry class was mixed at best. I would estimate that even though CP-2 Geometry was a course in Geometry, around one third to one half of the class while I was teaching was spent on the Learning Standards that more closely matched Algebraic topics. At times this felt remedial and slow, but under the direction of my cooperating teacher for CP-2 Geometry I tried to strike a good balance between pure Geometry and Algebraic ideas, integrating the two and showing where they related whenever possible. I felt this approach was especially good for showing that often there is more than one way to approach a problem and if a student did not understand or follow one method there might be another just as valid method available.

As previously mentioned, my third class, Statistics, was a CP-2 level course for seniors. The CP-2 leveling was implicit, not explicit, and students at the CP-1 or even Honors level were not discouraged from taking it if they did not want to take Calculus.

Statistics Full Year – 5.00 Credits

For students in grade 12

Prerequisites: Successful completion of a pre-calculus or introduction to mathematical analysis course.

Description: This course addresses the field of statistics from a non-calculus –based perspective. It allows students to develop both an intuitive and rigorous understanding of statistical concepts and applications. This course will increase students' ability to model quantitatively real-life situations, interpret and analyze data, and make necessary inferences. Students will develop competency with several appropriate technologies, including the TI-83, Excel, and other statistics software applications. Topics covered include descriptive statistics, probability, discrete probability p-distribution, normal probability distributions, confidence intervals, hypothesis testing with one sample, hypothesis testing with two samples, correlation and regression, chi-square tests and f-distribution, non parametric test.

Evaluation: Evaluation will be based on homework, completion of problem sets and take home questions, class participation, quizzes, tests, and semester examinations.

The following Learning Standards were concentrated on in Statistics:

From the Data Analysis, Statistics, and Probability strand:

12.D.1 Design surveys and apply random sampling techniques to avoid bias in the data collection.

12.D.2 Select an appropriate graphical representation for a set of data and use appropriate statistics (e.g., quartile or percentile distribution) to communicate information about the data.

12.D.3 Apply regression results and curve fitting to make predictions from data.

12.D.4 Apply uniform, normal, and binomial distributions to the solutions of problems.

12.D.5 Describe a set of frequency distribution data by spread (i.e., variance and standard deviation), skewness, symmetry, number of modes, or other characteristics. Use these concepts in everyday applications.

12.D.7 Compare the results of simulations (e.g., random number tables, random functions, and area models) with predicted probabilities.

Statistics was somewhat unusual in the sense that many of the topics covered were more at a college-level of inferential Statistics, especially during the second semester when I was teaching the course. Advanced topics included confidence intervals and hypothesis testing, both on several different types of distributions (Normal, T-distribution, Chi-squared).

Most of the material that was expected to have been covered previously came in Precalculus, including the following:

10.D.1 Select, create, and interpret an appropriate graphical representation (e.g., scatterplot, table, stem-and-leaf plots, box-and-whisker plots, circle graph, line graph, and line plot) for a set of data and use appropriate statistics (e.g., mean, median, range, and mode) to communicate information about the data. Use these notions to compare different sets of data.

10.D.2 Approximate a line of best fit (trend line) given a set of data (e.g., scatterplot). Use technology when appropriate.

10.D.3 Describe and explain how the relative sizes of a sample and the population affect the validity of predictions from a set of data.

12.D.6 Use combinatorics (e.g., "fundamental counting principle," permutations, and combinations) to solve problems, in particular, to compute probabilities of compound events. Use technology as appropriate.

Unlike with CP-2 Geometry, Statistics as it is taught at CCHS falls somewhat outside the standard Curriculum Frameworks strands and doesn't rely quite as much upon previous instruction, although that is more a reflection upon how clearly CP-2 Geometry was part of a sequential chain than any particular isolation of Statistics.

3. Development of Course Materials

3.1 – Statistics

The flow and curriculum of Statistics was fairly well regimented. Virtually all of the homeworks were pre-determined by my cooperating teacher, and she had a fairly inflexible idea of about how quickly the class would go through the material (as lined out by the homeworks). However, this was based on her fairly detailed experience and for the most part was not difficult to follow. It did feel like I was restricted at times, though, and it certainly taught me that in the future establishment of my own curriculum and getting a sense for myself of what can be changed when would be nice. I should also note that there was another teacher teaching the same course (different sections), so a good deal of the rigidity came from not wanting to get too far ahead or behind the other sections. In retrospect the system was not stressful; the piercing, disapproving look that was elicited whenever there was the suggestion that there might be some variation or that I felt unsure of something was very stressful.

The pre-determined homework schedule did not include materials that I developed over the course of the semester, being pre-determined. Occasionally when we got behind by a day or otherwise split things up I would include a worksheet I had developed. Generally, though, assignments were out of the book or rarely worksheets that had been developed already. These assignments are not included in discussion here as I was not involved in their development. I will comment generally on what was expected in the homeworks, though, as I did check on a daily basis – homework was worth 2 points daily, compared to 30 – 50 points for a quiz and 100+ points for a test.

Writing was not important except where it was part of the mathematical assignment; that is, where an English explanation was a key skill. High quality or correctness were not important either; I checked for key intermediate results and general evidence that each problem had been worked on, such as a graph of the appropriate distribution with key values labeled. Incomplete or late homework received half credit (1 point). That quality and correctness were not important was for two reasons: First, the theory was that homework was for the students' practice and not meant as an assessment.

My cooperating teacher's belief was that if a student was faking the homework and faking it well enough to fool us it wasn't worth making a stink: we would find out at test time who had been taking the homework seriously or not. Second, for time reasons: It took enough time as it was to go around the room and check, and while I did my best to establish warm-ups and instruct the students to confer with each other about the homework, it was difficult to keep everyone on task while I was concentrating on checking homeworks.

Prior to actually taking over the class, I developed a 1 – lecture lesson on the binomial probability distribution as part of ID3100. Fortunately, I was able to go in and actually give the lesson when appropriate for the Statistics class.

The formal lesson is S-0 in Appendix A1.

The applicable framework is 12.D.4: Apply uniform, normal, and binomial distributions to the solutions of problems.

While developing the lesson, I originally thought to incorporate a physical element, flipping pennies with a partner and going from there. The theory that getting students physically involved can help keep them mentally involved was my primary thought. However, near the eve of the class I decided to go with a mental experiment, considering the strikeout rate of then newly acquired Red Sox pitcher Curt Schilling.

"I scrapped the idea of doing a physical experiment and went to the mental experiment for a few reasons. First, I felt I could connect with many of the students in the class with the sports example much better than with pennies. Second, the goal was to get the class to develop the formula on their own. While pennies or dice would have been a good experiment, they would not have assisted in the goal of understanding why or how the formula described binomial probabilities. With the mental exercise, students are given a better chance of understanding and remembering the formula than if they were simply told what it was and that it described some data, even if they had generated the data themselves."

I was very pleased with how the lesson actually played out. When I began writing the introduction on the board as students were filtering in before the bell, I heard a student say something along the lines of "Any time a class starts with the Red Sox you know it's gonna be good". In addition, this was the first time I had given direct instruction to the whole class, and the "new, young, interesting student teacher" factor helped as well.

S-1 and S-2 in Appendix A1

These lessons were on the Student t distribution and how we could still construct confidence intervals for smaller sample sizes. This is one of the topics in inferential Statistics that fell outside the normal purview of the Curriculum Frameworks, but the most applicable framework would probably be 12.D.4.

A key point to the lessons was starting to establish a decision-making process to decide what method (Z, Student t, etc) was appropriate to answer or study a given statistical query of some sort. The idea would be to make sure we always knew what assumptions we were making. Also, I stressed the how and why of the differences between Student t and the normal distribution ~ I wanted students to have an understanding of the distribution itself and why it was useful and different rather than simply giving them another table and telling them how to apply it. Certainly, not 100% of the students would reach the higher level of understanding, but approaching the question holistically still hopefully gave a better understanding than would have been conferred had the approach to Student t been purely for application.

I also stressed how similar the process was to previous constructions of confidence intervals, trying to draw parallels and highlight areas that were general to (and would be applicable again) inferential statistics, such as the concept of x bar +/- E.

I introduced the history of the Student t distribution as well. Being a high school course I felt my hyping up the historical relation to beer for the purpose of eliciting more interest would have been out of line, but many of the students got a good chuckle out of it anyway without any prompting. That said, given the choice I would have preferred none of the students finding that particular aspect related to their lives but unfortunately that was not the case.

These lessons were somewhat of a variation from the norm for me, as I did not use a concrete, consistent example as a baseline to study from. Especially with the CP-2 level class, the concrete examples were generally more effective (such as the Curt Schilling example for binomial probabilities) than abstract examples. The thought was to introduce the distribution, get the students familiar with the table, and then attack examples from there. I wonder in retrospect if at least having a concrete example that was presented and then gone back to as appropriate would have been more effective, though I did not see any signs that the route I did take was ineffective in the long term.

The warm-up table lookups in S-2 deserve particular note as well. Here I was trying to develop and practice the particular skill of looking up t-values in the Student t table. While an inherent understanding of the distribution is an excellent goal, the students still needed the skill of the table lookup.

S-3 in Appendix A1

This lesson (which followed those in S-1 and S-2) used the approach of starting with a concrete example and going from there. Here, I started with an example that seemed somewhat familiar to the students but asked them to work backwards and look at the problem in a different light. I prefer this as it helps to tie one concept to the next in a nice, natural manner. However, I had to be careful not to make the differences and amount of inference too complex as that would tend to elicit blank stares and complaints of "Mr. Case, I don't know how to do this." On the other hand, though, not ever expecting students to go above and beyond would not be very helpful for their long-term growth as students. Finding a balance is the tricky part.

S-4 in Appendix A1

This worksheet was used as a warm up. I liked this piece because it tied together several different concepts and worked on several skills at once, or at least within the context of a single example. While not precisely authentic assessment, it was closer to authentic assessment than the typical book homework problems. The skills used were confidence intervals for both the Student t and normal distributions and estimating minimum sample size to construct confidence intervals for population mean and population standard deviation.

S-5 in Appendix A1

This worksheet was also used as a warm up. It was a change from the typical warm-ups which were more practice of skills and process. I tried to construct the worksheet holistically so it would be more authentic; that is, solving and considering a whole problem rather than merely practicing or demonstrating a skill. Asking the students to work out what was relevant and what was not relevant was meant as a critical thinking exercise – another way to approach thinking about a problem, one that requires slightly more thought and understanding that simply applying the process that had been taught. After the students are asked to construct the interval, the key element is interpreting the results. What would you do in real life if this were what you got? Again, it is a critical thinking / authentic question rather than a purely mechanical one. Also, it alludes to the basic concept of hypothesis testing, which was the next major topic.

S-6 in Appendix A1

This sheet, as titled, was used as practice for the skill of interpreting the results of a statistical hypothesis test. A considerable amount of time and effort was spent on bridging the gap between how the math works and what the math really means in understandable English terms, and this skill was a key part of this as it related to hypothesis testing. We, being my cooperating teacher, myself, and the book, all agreed that simply determining the mathematical result and either rejecting the null hypothesis or not is insufficient to truly understand what the purpose and true result of a hypothesis test are. While the skill being practiced is somewhat abstract, this worksheet is still one of the more pointed and specific worksheets I developed for Statistics. While the skill would certainly be a very, very key part of an authentic assessment, the problems on the worksheet are not examples of possible authentic assessment at all.

S-7 and S-8 in Appendix A1

These two worksheets each concentrated on different aspects of practicing the skills related to hypothesis testing. The first worksheet, S-9, concentrated on the process, asking the students to report each step along the way and giving a more step-by-step approach to practice. The second worksheet, S-10, was a slightly more holistic practice tool, not asking the students too many details about the intermediate results and also incorporating the key skill of interpreting the results in English. In general, I tried to find a balance of practice between very focused skill practice, such as S-8, and more holistic practice, such as S-10, with S-9 being somewhere in the middle, with a slight tilt towards the more holistic and authentic approaches. Boiling down a problem to just the math or a very specific skill helps to weed out distraction, but when all is said and done more realistic examples that allow students to go through a process from start to finish including questioning assumptions and making decisions about what method to use are in theory both more interesting and more useful in the long run as teaching tools.

S-9 in Appendix A1

I established this project from scratch as a wrap-up of the Confidence Interval section. I liked it for several reasons. First, it broke up the monotony in a classroom that was accustomed to a very regular routine, giving the students something physical, different, and potentially 'fun'. M&M's were chosen because they were lighthearted, easy props, and I was counting on the very concept of doing math by "playing with candy" being appealing to the students. In addition, I was able to be a little more humorous and creative, writing my own introduction with a more personal and comfortable feel to it. Again, the goal was to help relax the students and remind them that there was math everywhere and that it doesn't always need to be purely calculators and notebooks and textbooks all the time. Also, it gave an excellent premise for a very holistic set of practice problems and what I believe was a fairly authentic assessment. I should note, though, that to truly be an authentic assessment more freedom should have been given to the students in choosing what methods to use and less direct instruction on process spelled out as such. Finally, it was at last an excellent example to offer a truly challenging above-and-beyond exercise of the sort that I had been craving to challenge the students with but had felt restricted by the syllabus and my cooperating teacher's regimented, regular system. Even as a CP-2 level course there were still several students who found exploration and challenging themselves to be interesting and worthy, and the extra credit at the end was to give them an opportunity to grow and exercise that part of their minds since the regular class did not do so as often as I perhaps would have liked. I also liked the non-extra credit critical thinking exercises of question #19, which asked the student to go beyond the math and think about how the statistics might be used and when one interval would be appropriate or preferred over another.

3.2 – Geometry

The flow of CP-2 Geometry was much more fluid. My cooperating teacher for CP-2 Geometry went much more by experience and feel rather than having a clear, set syllabus right from the beginning of the year. As such I was given much more flexibility and freedom in my lessons as well, but he still had a very good idea of when certain topics should be covered and what the balance of algebraic topics and geometric topics should be. We constantly evaluated and re-evaluated where the students were and adjusted the schedule accordingly, reviewing topics that needed more review. In addition, my cooperating teacher liked to break up the monotony of the class with fun little mental exercises and group activities on a fairly regular basis. Very occasionally these activities would not be directly related to the math at hand; activities that were directly related such as a math scavenger hunt, what he dubbed "pass 'em", and the like were favorites. For example, in the scavenger hunt-like activity, a number of practice problems would be placed around and immediately outside the room and students, in pairs or small groups, would need to find and solve all of the problems before reconvening and going over them as a class.

In terms of how I actually presented new material to the class my standard was fairly similar to Statistics: I would present a new idea and engage the class in a discussion and exploration of the topic, using practice worksheets and homework to practice and reinforce the concepts. Homework came from the book about four-fifths of the time, per following the lead of my cooperating teacher. My selection process was to select problems which seemed the most "authentic" while at the same time mixing in some straight-up practice / plug-and-chug problems. When possible I selected one or two problems that were above and beyond and required more active thinking; results were mixed, as the tendency of the class would be to give up to a problem if the solution or process was not immediately apparent or if the process seemed too difficult. I tried to disabuse the students of this idea as best I could over the time I was teaching, but I have doubts about how effective my efforts were at least in the short run. This really has its roots in the age-old problem of getting students engaged for more than just getting the grade. In CP-2 Geometry, per the lead of my cooperating teacher, I tried to mix in more of the more enjoyable activities and teamwork exercises such as pass 'em, but it was a tricky balance. The difficulties included keeping everyone in each group involved, keeping the class focused and on task, and keeping the students from falling into the trap of asking "Mr. Case, how do I do this?" or worse "Mr. Case, I don't know how to do this" at the first sign of trouble.

When checking homework, I did not concern myself overly much with correctness or quality; per my cooperating teacher the requirements were that the students exhibit effort on each problem, even if they're not sure how to do it. Writing was not important in the daily homeworks, but occasionally my cooperating teacher had Problem Sets. In the Problem Sets and more formal projects that went beyond daily homework, writing and presentation was a considerable factor.

G-1 in Appendix A1

Applicable Framework: 10.G.4 : Apply congruence and similarity correspondences and properties of the figures to find missing parts of geometric figures, and provide logical justification.

This worksheet was used to practice the "Altitude-Hypotenuse Theorem", which was a specific application of similar triangles and ratios. Like most in-class practice materials for CP-2 Geometry, this was intended to be worked on in small groups. I also used a very slight variation of the worksheet as part of a homework assignment. While use of the A-H formulae is what the worksheet was designed for and is the most effective and efficient method, basic knowledge of ratios and similar triangles can be used to solve the problems.

I used AppleWorks' "Drawing" feature to develop virtually all of my materials for CP-2 Geometry. This is what my cooperating teacher used and provided good tools to establish diagrams for the highly visual topics in geometry. In addition, I was not beholden to a word-processor's formatting or spacing, and was able to experiment with different arrangements of pictures, writing, and questions.

I should also mention that two digits that are spaced such as the "4 2" on this worksheet, this usually meant that I would draw in a square root before copying. In this case, "4 2" became "four times the square root of two".

G-2 in Appendix A1

Applicable Framework: 10.M.1 : Calculate perimeter, circumference, and area of common geometric figures such as parallelograms, trapezoids, circles, and triangles.

This worksheet is an example of my attempts to mix both symbolic representations and concrete examples. As a CP-2 level class with many of the students possessing poor algebra skills or even a general dislike or fear of algebra, the abstract concepts of how algebraic formulae apply to geometry could be difficult for some. As such I tried to often couple work on an abstract concept with clear, very concrete examples or wording the abstract concept in a more concrete manner.

G-3 in Appendix A1

Applicable Frameworks: 10.G.4 : Apply congruence and similarity correspondences and properties of the figures to find missing parts of geometric figures, and provide logical justification; 10.G.5 : Solve simple triangle problems using the triangle angle sum property and/or the Pythagorean theorem; 10.G.6 : Use the properties of special triangles to solve problems.

This is an example of the "pass 'em" game / exercise. The six questions would be separated and distributed among the 6 groups. Each group was then given a set amount of time to record and work on the problem. After the set amount of time, the teacher would call "Pass 'em!" and each group would pass the problem they had been working on to the next group. Once each group had received the problem that had been the start, a little time was given for groups to finish problems that they had not yet completed and then answers were gone over as a class. Most of the time score would be kept, and occasionally I would offer a simple reward such as each group member getting a piece of candy from a stash to the group or groups with the most correct answers. Pass 'em was one of the favorite activities of the two CP-2 Geometry sections, although this was more pronounced in one section than the other.

The Crayon Project
G-4 in Appendix A1

Applicable Framework: 12.G.1 : Define the sine, cosine, and tangent of an acute angle. Apply to the solution of problems.

In the courtyard in the center of CCHS is a large chimney that several years ago was painted to look like a giant purple crayon. This project was intended to use the crayon to get the students interested as well as getting them outside and working with their hands on something concrete, as well as a clear-cut example of one possible application of the basic trigonometric functions.

However, the Crayon project proved very difficult to manage, time and material-wise. Two simple improvements would be making sure every group had a long enough measuring tape and giving a clearer walk-through prior to beginning the activity. It was also difficult maintaining focus outside the classroom environment; this also happened with the scavenger hunt activity. Part of the problem was that I was not able to keep an eye on everyone at the same time and part of the problem was that being active and outside the normal routine of the classroom naturally elicits less focus and more of a feeling of freedom. I am not sure I have solutions as to particulars for changes to approach that would have immediately impacted this except for more clear direction, as mentioned above. On some level I believe mastery of this can only come through experience, and even then the best teachers still find it a challenge, or so I imagine.

G-5 in Appendix A1

This exercise was designed to coincide with a "pizza lunch" that one section of CP-2 Geometry had requested and been granted by my cooperating teacher and myself. It was much wordier and relied on the students following along on their own, as I did not go through it at length with the class as the topics were a bit outside the purview of what we had been working on at the time. It would likely have been a much more successful activity in the long run for an Honors-level course wherein students tend to be more independent and are more willing to learn on their own.

G-6, G-7 in Appendix A1

Applicable Frameworks: 12.G.1 : Define the sine, cosine, and tangent of an acute angle. Apply to the solution of problems. [G-9] 10.G.6 : Use the properties of special triangles to solve problems. [G-8]

G-8 and G-9 are two examples of practice and review. By this stage the students have already learned the appropriate skills, or should have, and practice is for reinforcement. The problems in an assessment would not come from nowhere; every problem would resemble something that had been practiced in a manner somewhat like G-8 or G-9. For the CP-2 level class, this is what was expected.

G-8, G-9 in Appendix A1

Applicable Framework: 12.G.1 : Define the sine, cosine, and tangent of an acute angle. Apply to the solution of problems.

These "Learning Guides" as I called exploratory worksheets from time to time came at similar times, while we explored the concepts of the basic trigonometric functions. One of the mnemonics taught was SOHCAHTOA, or Sine-Opposite-Hypotenuse, Cosine-Adjacent-Hypotenuse, Tangent-Opposite-Adjacent. At first telling the class that SOHCAHTOA was a great Native American chief who was a master of trigonometry was a fun story, and the students enjoyed greeting it with playful (and rightful) skepticism.

The key behind my thinking for the Learning Guides was to be sure to make things very concrete and step-by-step for the CP-2 level course. Demonstrating how sine and cosine change with a change in angle was probably just a little too abstract a connection for some to make, but even so I still felt it an important foreshadowing of future learning in some trigonometry course in the future. I didn't want to railroad myself just because these students had been designated as CP-2, and I did my best to make every reasonable attempt to communicate the more subtle yet wonderful aspects of mathematics in general and geometry in particular. In the end, though, teaching the more concrete skills was still the focus of the class and of my teaching efforts, including my development of materials.

4. Analysis of Students

4.1 – Statistics

When I formally took over Statistics at the beginning of the CCHS' second semester, my section of Statistics stood at 22 students. One student had dropped the class immediately after the midyear, and several more had graduated a semester early. In any case, the 22 provided a fairly "normal" cross-section of CCHS' senior student body. As I would have expected from most CP-2 level sections, there were no real prima donnas or students who went out of their way to be the best. That's not to say there were not students who desired to do well, and there were even a few students who could have been lightly labeled teacher's pets while I was at the helm, but no one was falling over him or herself to answer all the in-class questions before everyone else.

Two of the more interesting cases in the class were what I would call class clowns. One was perhaps the most naturally bright of all the students; he was also taking calculus concurrently, and on some level he felt the CP-2 level of Statistics to be beneath him. When he paid attention and I was able to engage him his observations were usually very good, and he probably got a lot more out of the class than he wanted to admit to himself. Getting him to pay attention was a struggle at times, though. Worse was his propensity to distract others when he didn't feel like paying attention. Pursuant to the policies set by my cooperating teacher I did my best to keep things under control and put an end to chattering or worse, such as throwing a rubber ball around, and make it well known that such behavior was unacceptable.

The other "class clown" was an even tougher case. The material was not beneath him; in fact, he believed it to be challenging. The problem was he did not want to put the work in. Try as I might to engage him, and I was sometimes successful, he was not able to maintain consistency. He would get up the motivation to pay attention for a section or two and study, do well on a quiz, then would fall into bad habits again and get behind. Once he was behind I think the very concept of catching up overwhelmed him; he wanted an easy way out, and there wasn't one. I spent more personal attention on this student than any other, including outside class. In the end, though, I just couldn't get him to get over the "second semester senior syndrome" and accept that he was most definitely bright enough to get it with a little bit of work and that there was no easy way out. A few weeks before my time at CCHS ended, he dropped the class once he had bombed enough assessments to make it no longer worth staying in the class. Something else very important to his case that I should mention is that he had been having problems throughout his CCHS career and was part of a program for students who struggled staying focused and even attending class. On a daily basis he would have me sign a sheet that said he had attended class and whether or not he had done his homework. I wonder now what he's doing with his life; I do not believe he intended to go to college, and sadly I think that may be because he believed he wasn't smart enough. He provided a tremendous learning experience for me in any case, and I will remember him and my efforts to reach him for a long time to come.

Beyond the two main instigators behavior was generally very good. Certainly there were the students who could not resist chatting with their neighbors, but I got better as the semester went on at catching that sort of thing. It seemed that as long as I was able to keep the instigators from instigating poor behavior, either by keeping a sharp eye on them, by carefully choosing seating arrangements, or a combination, I was able to keep the class relatively quiet and on track.

Of note is that there was only one METCO student in the class. She was a small Hispanic woman who put a lot of effort into her work most of the time but would get frustrated easily when she didn't get something right away. As a result her assessments were inconsistent, mixing excellent with poor from time to time. Her work did pay off, though, as she had a very solid B by the time I finished.

There was at least one student with an IEP (Individualized Education Process) that involved being allowed extended time on assessments when required. I am not very familiar with the IEP or special education process; I was only made aware when the IEP had a direct effect on my teaching. In this case the IEP was very rarely invoked.

In terms of learning styles, the class was a CP-2 level class and not without reason. With possibly the lone exception of the bright student who felt statistics was beneath him, the class responded very well to concrete examples and was able to reach abstract conclusions through steps and patience. Purely abstract discussions were halting and slow at best, but by mixing in a healthy dose of concrete examples or connections we were able to make considerable progress as a class in terms of many of the tricky concepts that are the basis of inferential statistics.

Attendance for the most part was good. There were one or two students who tended to be absent more than their fair share; a function of "senioritis" more likely than not. My cooperating teacher was better than I at shaming students who purely cut class, and the policies of the class were unforgiving for unexcused absences. The particular policy as far as assessments is covered in Chapter 5. I made myself available outside of class to help students catch up if they missed time for legitimate reasons, but no particular effort was made to catch up during class time as the inflexible schedule of Statistics rolled on. I do not disagree with this theory; in truth, I feel I am at my best in one-on-one or small group teaching situations, when I can give large amounts of personal attention to each student present and really engage them fully.

If I had to summarize my H-Block Statistics section with a phrase, it would be "typical CP-2 level CCHS seniors". We had jocks (soccer, lacrosse, and hockey); we had drama kids; we had the quiet, sci-fi reading student who started slowly but who I think I was able to reach quite well by the end of the semester. I suppose I do not have much of an experience base to draw conclusions from, but in retrospect I was pretty pleased with how I was able to manage the senioritis in most cases.

My approach to the classroom was generally that of a teacher-led discussion. I made particular effort to make sure every student had a chance to talk and be called on every few days if not daily, and I considered that to be part of my approach to general classroom management as well. I joked with the class, I made eye contact, I smiled, and I generally made an effort to engage the class not only as a whole entity but also giving and sometimes demanding personal contact and engagement with as many students as possible. I did my best to be fair, impartial, and treat every student with respect at all times. I think I was helped in a large way by my cooperating teacher's methods and, as she put it, the class being "well trained", but whatever the reason I was pleased with my overall management of the class and how well I was able to get to know and engage the students within.

4.2 – Geometry

I taught two sections of CP-2 Geometry, starting with F-Block and later taking on the E-Block section as well. The only important difference between the blocks is that E-Block met four times a week for 56 minutes per class, like H-Block, while F-Block met five times a week for 45 minutes per class. Each section had approximately 22-23 students; there was more mobility for the freshmen sections than there was for Statistics, though still at a rate much lower than I imagine occurs at the Worcester Public Schools.

F-Block's personality as a class was dominated by several energetic young girls. They loved chatting, gossip, and let a hint of drama into the class from time to time. My cooperating teacher, who has been for years a favorite of gossipy female freshmen and sophomores, was loved by this crowd. His secret seemed to be an uncanny ability to listen and give advice, and the girls, like those who came before them, trusted him implicitly and were among those who would frequent the room before and after school to study, talk to my cooperating teacher, or just hang out. They regarded me as something new and exciting, and I think especially early on I was afforded more attention from this crowd than might have been expected simply because I was a young, new, charismatic student teacher.

F-Block only had 2 METCO students, one of whom played freshman baseball and thus called my cooperating teacher coach since he was the varsity baseball coach and oversaw freshman baseball as well. One of the brighter CP-2 Geometry students that I taught was part of F-Block as well, but he fit the mold of a student who was bright enough to get by with paying minimal attention and doing minimal amounts of work so his results were inconsistent and on the whole nothing particularly extraordinary. On the whole, though, the class meant well and was essentially good kids who were still really learning how to be students and people. They would get stuck because they lacked confidence rather than lacking any motivation or desire for the most part.

While I knew of no particular parental issues, one student's situation sticks out in my mind. Her father is a prominent Boston sportswriter; nowadays I see him on Fox Sports Net as a guest commentator from time to time and I am reminded of my student.

She was not unfriendly but at times tended to be a loner, sometimes staying after to study by herself, and my cooperating teacher suggested that she may not feel like she got enough attention at home. She was the sort of student who preferred doodling, art, and the nifty ring tones of her cell phone and who likely did not understand or relate to her father's passion for sports. Academically she was typical of the class in that she meant well and did her best to stay focused but was still a kid with a wandering mind at heart and would get frustrated when answers were more difficult to reach.

The character of E-Block was most notable for the half dozen METCO students, by far the largest proportion in any of my classes. This included two very large African American young men and one of the most dedicated students in any of my sections, an African native who went by the nickname "Brown". One of the challenges for me was trying to not even be aware of the fact that there were METCO students, not in the sense that I wanted to be less aware but that I didn't want it to affect how I treated my students in any way and still do my best to relate even though my own socioeconomic background was generally disparate to the students participating in METCO.

E-Block was a little more forthcoming in class discussions as a group, but also a little more prone to losing focus as a group as well. While I certainly had to keep chatting under wraps in F-Block, it was more of a daily challenge in E-Block. On the plus side as a group E-Block responded well to direct engagement as a class better than F-Block, which seemed a little more keen on swaying discussions to non-math related topics when possible.

Another notable group learning style present was that of quiet, do-the-work, and don't-take-a-lot-of-attention. Several members of E-Block exhibited this learning style very acutely, most notably several girls. When the class was asked to go over the homework in groups, most would comply, but these girls were by far the most consistent and focused, going through each problem and checking the answers exactly as had been instructed. They were not always extremely voluntary participants in class discussions, but I still made a point to engage them and they did quite well.

Both sections of CP-2 Geometry were set up in groups of 3-4, which were changed every quarter or so, and arranged logically but not in a stiff, rectangular or regimented way. This gave the class a feel more like a forum and also facilitated active and regular group work, including one or two group assessments. As discussed in Chapter 3, many of the course materials developed were developed with small groups in mind, and the daily routine always included some sort of group work or group discussion. Keeping the groups focused and the participation within each group balanced was tricky at times, but the emphasis on teamwork had a generally very positive effect. In contrast, Statistics was set up as three columns of pairs of desks, giving it a much more orderly feel that befit the style and methods of my cooperating teacher for that course. My cooperating teacher for CP-2 Geometry was much more laid back, and it showed in how he set up and managed his classes.

The method I chose to employ was very similar to that of Statistics, with most classes consisting of teacher-led discussion to start off topics and review topics with a healthy dose of practice and problem solving, especially in groups, mixed in. For CP-2 Geometry I also mixed in a number of more active activities like those discussed in Chapter 3 such as pass 'em and math scavenger hunt. When possible I included props and tried to give the students a chance to use their hands. For example, when teaching the Altitude-Hypotenuse Theorem I had each student cut out triangles as appropriate to demonstrate which were similar and how; some students kept their cut outs in their folders and used them as reference whenever A-H cropped up.

Catching up was considerably easier in CP-2 Geometry than in Statistics. We constantly reviewed topics, tests and quizzes could be retaken, and students were actively encouraged to come in outside of class hours to get extra help, review, or to catch up, and many did indeed take advantage of this. As mentioned before, some students would even come in just to chat with my cooperating teacher, and this just helped the welcoming, open-door feel that hopefully gave students an idea that coming in for extra help and review really was something positive.

I had one student drop the course and one student join the course halfway through, dropping down from CP-1 Geometry. Both events happened early on in the semester; it certainly helped that the student who joined had already been taking a higher level geometry course and was generally very bright. She still came in for extra help from time to time and ended up doing very well. Had a student joined who had not taken any geometry, such as if from another school, it would have been more of a challenge but my response would have been the same: I would have offered my assistance for however long it took to get caught up outside of class, and if necessary I would have insisted that the student take me up on the offer. I imagine that in a school with a much higher mobility rate such personalized attention for every case is difficult at best and impossible for many situations.

Chapter 5

5.1 - Statistics

In Statistics, with the class so well laid out, there was not that much creativity required or even, I believe, desired in terms of developing standard evaluations such as quizzes and tests. My cooperating teacher liked to give a quiz every 1 to 3 sections and a test at the conclusion of each chapter, and as with the other parts of her routines I followed her lead. Quizzes were designed to take anywhere from 25 minutes to the full 56 minute class period. Tests were universally designed to take the full class time. In terms of how each evaluation counted towards each student's grade, a quiz would range from 30 – 50 points while a test would generally be 100+ points. Homework also counted 2 points per day and the project described in Chapter 3, S-Project in Appendix A1, was worth 40 points. At the end of each term, points earned over points possible was calculated for each student and grades ranging from F to A+ were assigned based off the raw percentage. No curve or scaling was employed, though admittedly the grades were decently well distributed and no curving was needed.

Later on in the semester I started recording times that the students would complete each assessment, both for curiosity's sake and to help me in evaluating my own assessments as well as helping to better estimate time allotted for future assessments. I found my "aim" got much better as the semester rolled on, although as my cooperating teacher pointed out there was usually a student or two who would take the full allotted time no matter what. Also, there was one student who would often need to reinvent the wheel and teach himself the math as an assessment was going on after he had ignored the homework and lectures for a section or two. While he was bright enough to get by this way he occasionally had very poor results and took considerably more time than he did when he had actually done the homework and paid attention.

Not all of the materials presented here are presented as complete works, and a few contain typos that needed to be corrected at test time. Such does not strike me as having a great impact on the learning of the students, though, so I did not stress about it too much if I missed a minor error in proofreading an assessment.

In Statistics, calculators were generally not only allowed and encouraged but were in fact required. My cooperating teacher believed that teaching the students the skill to use their calculators effectively was as important as any of the pure math skills developed, a sentiment I would tend to agree with. Using the tables was also a skill taught and assessed, and when either method could be used I generally specified which was expected.

First Test developed, Chapter 5
S-1 in Appendix A12

The probability density function for problem #1 was drawn onto the master copy using a straightedge prior to being copied. This was straightforward as the distribution was a uniform distribution. Question #6 was a bit of a reach, and Question #7 was poorly organized and caused some confusion.

Quizzes, Chapter 6
S-2 through S-4 in Appendix A12

Quiz 1, for Sections 6.2 – 6.4, lost some formatting in being transferred from a Mac, where it was developed, to a PC, where this paper is being written. I generally avoided having questions be split up by page breaks. Unfortunately several of the other quizzes also exhibit this loss of formatting.

Question #6 turned out to be a very difficult question. It was intended to elicit thought about questioning assumptions and choosing which method was appropriate, but instead caused considerable confusion even among those students who gave a good-faith effort to take the problem seriously even with the confusion. In retrospect I should have been clearer from the outset of the question what was expected and eschewed the space-saving technique of clumping three sub-questions together. Really this was an early attempt at giving more life to the question and more freedom and thus more difficulty to the students, but I fumbled the exchange.

When developing material, both for practice or lecture and for assessment, I employed three main methods for generating problems and examples. My preferred method was gathering real data on my own, such as with the Curt Schilling example of S-0 in Appendix A1. Most commonly, though, I borrowed problems or at least set-ups from one of the several statistics books that CCHS had on hand. Sometimes, though, I could not find any example problems or real world problems that were suitable and in these cases I would simply make up a situation that seemed plausible. Quiz 2, question #1 was one of my favorite quiz questions as the data was quite real and so very applicable to what were we doing. Also, I favored giving sub-questions such as part f) which required a touch of critical thinking, and this was a good one in my opinion.

These three quizzes give a good contrast in length, as well. Quiz 1 was one of the longest quizzes given, while quizzes 2 and 3 were on the shorter side.

Test, Chapter 6
S-5 in Appendix A12

Quizzes, Chapter 7
S-6 through S-9 in Appendix A12

When a student missed an assessment with an excused absence, they would be allowed to make up the assessment within a day of returning to school. I was generally more lenient about allowing students to choose a time that would work for them than my cooperating teacher wanted me to be. In any case, if the assessment had already been returned and sometimes regardless of if the assessment had been returned or not a second version of a quiz or test was needed. Here I was fortunate to have three good options. First, I was able to take the previous year's quiz and use that; second, I could ask the other teacher who was teaching the course for her version; third, I could develop my own second version. Really, the only practice that was being warded off when I just changed numbers around was memorizing and passing answers. Truth be told, if a student wanted to ask a friend what was on the quiz and how to do the questions, then that wouldn't bother me overly much as it would really just amount to assisted studying and not really cheating. My assessments were almost always comprehensive, so little would be gained by knowing what wouldn't be directly assessed as almost everything was directly assessed, at least for quizzes.

Missing an assessment because of an unexcused absence resulted in a grade of zero for that assessment with no questions asked.

Question #5 on Quiz 4, while a somewhat silly example that I used variations of a few times, is an example of a question that's closer to authentic assessment. Quiz 4 gives a pretty good contrast between Question #5 and Questions #1 and #2, which were clearly skill-based and not authentic assessment.

Test, Chapter 7
S-10 in Appendix A12

This document purports itself as a chapter 6 test, but that was a typo – it is indeed the chapter 7 test.

Quizzes 6 and 7, S-Q6 and S-Q7, provide a contrast for how questions, while essentially asking the same thing, can be different just by mean of how they are presented or how the results are asked for. For me personally, I prefer the less regimented style of quiz 6, but for the CP-2 level class the extra structure exhibited in quiz 7 proved to be a little more effective. This was also the format I used for the chapter 7 test, S-T3. I should be clear that I did not develop the format but instead borrowed it from materials that had been developed for the class in the past by my cooperating teacher. Regardless, it is interesting how the additional structure had any effect at all. The lesson to be learned from a teacher's perspective is that presentation is everything and every little detail counts. That being said I think I tended to obsess too much about the little details at times and stressed myself out unnecessarily as a result.

MCAS did not play any role whatsoever in Statistics. Being a senior-level (12^{th} grade) course, each student had already taken and passed the MCAS, which for High Schoolers is first given in 10^{th} grade.

5.2 - Geometry

In Geometry, there was a little more freedom to be a little creative with my assessments than there was in Statistics. However, the basic recipe was the same: take example and practice problems, give some slight variation, and assess the skills that were directly taught and practice. I did include authentic assessment, but I had to set such up by spending time practicing similar problems and working on general word problem and problem solving techniques. My cooperating teacher did not seem to recommend trying to challenge the students for a lot of critical thought on quiz or test assessments, and I can understand why; I can envision terrified stares and possibly even tears in some cases. Truth be told, though, I cannot disagree that the most fair assessment is one that assesses what was taught and not the innate cleverness of a student. In that vein I did not feel I was doing a disservice limiting myself as I did. It is likely that it is just another example of my Honors-trained thought needing to be reworked to be appropriate for CP-2 level.

Like for Statistics, CCHS did have several geometry books I was able to borrow examples from, but with geometry being both more pure math and also more visual coming up with problems on my own was nice and easy and was my first option rather than third. I was surprised, though, by the relative lack of coordination with the other CP-2 Geometry teachers. I taught only two of four sections, with two other teachers taking one section each. I did not generally borrow materials from them or vice versa, and no particular effort was made to stay perfectly in track with each other. I did appreciate the freedom to plan my lessons, but the end result is that there was probably a lot more variation in what the students learned and what was focused on than for the Statistics sections. That's not to say there was not coordination and communication, just that compared to the strong coupling of the Statistics course CP-2 Geometry seemed much more disparate.

My understanding is that there was a lot more coordination on MCAS-specific topics, with the other classes doing regular algebra reviews and MCAS practice as well. Most of the direct communication and coordination was handled by my cooperating teacher and passed to me through him, for example there was a large MCAS review packet handed out by my cooperating teacher to the sophomores in the class that I believe was used by the other teachers in their sections as well. For that matter, several of the algebra review quizzes were developed by my cooperating teacher, and he may very well have been taking them from years past, a source which presumably the other teachers had access to.

G-1 in Appendix A12

This quiz was on Special Right Triangles as well as Pythagorean Triples. The second page is a second version of the same quiz; students who missed a quiz were always given a chance to make it up. In addition, my cooperating teacher's policy was that quizzes and tests could be retaken to try for a better grade. This is in stark contrast to the far more strict policy of my cooperating teacher for Statistics – my cooperating teacher for CP-2 Geometry just came across as a lot 'nicer' and more lenient. I feel it was good for me to directly experience both extremes in successful situations, reinforcing that there are pros and cons to each and either policy can be part of a quality classroom experience.

G-2 in Appendix A12

This test gives a good example of how challenging I tried to make my tests for CP-2 Geometry. As mentioned above, I avoided questions and problem types that the students had not seen before. However, I also did not make the questions that were included all cupcakes. I very deliberately had some problems that were presented algebraically, some that were presented only visually, some that were presented with a little bit of algebraic notation but mostly visually, and finally the word problems. I wanted to evaluate how the students were with the math skills, and I felt that by mixing presentations I would get a broader view of how a particular student handled the math rather than simply how comfortable they were with a particular presentation. In addition, by challenging the students to not simply memorize and get used to one form over and over again I hoped to expand their ability to break down problems in general into the math, a general skill that has importance well beyond mere geometry.

I should note that the critical thinking question, #10, was a very slight exception to the rule of not giving problems of a sort that had not been seen before. We had indeed gone over at length the difference between a ratio and a proportion, and even discussed the differences, but I do not recall ever placing that question on a practice sheet. Many students did quite well with the question, though. That the students were able to answer the question with English rather than math may have made some more comfortable than they would have been with an application challenge question.

G-3 in Appendix A12

I have included two versions of the same test. The assessment is fairly unremarkable in terms of particular creativity or variation. Questions #8 and #10 are fairly good examples of efforts to get the students to think in terms of the abstract 'x' and get away from the purely concrete real number applications. However, both were deliberately included near real number applications of each special right triangle. This goes back to my underlying theory of taking small steps and working on just establishing those connections for the long haul even if there is not immediate evidence of results.

In CP-2 Geometry, MCAS did play a strong role in the class and its assessments. As mentioned previously, about half of the students were sophomores and took the test while they were in CP-2 Geometry. I did not find myself going out of my way to adjust my assessments on a regular basis to match MCAS; the most obvious effort was the mixing of algebraic problems, purely geometric problems, and word problems in assessment situations just to get the students used to seeing different kinds of problems and presentations as there would be on the MCAS. As for the material itself, I trusted in my normal teaching and the teaching of those who came before me for these students to be sufficient for the most part when reinforced with our regular algebra reviews and quizzes. As a final preparation the students were given an MCAS review packet, which we spent a little time going over, and highly encouraged questions and review with us outside of class. By the state's basic rubric, something must have gone well as every student who took the test passed.

6. Conclusion

My teaching practicum was one of the most tremendous learning experiences of my young life and career. It was both one of the most stressful and yet one of the most fulfilling, a combination that would likely continue to follow any teaching experience. Thinking back on my time at CCHS, it really was special to me how I was able to reach the students, and those who I was not able to fully reach at least walked away a little better at Statistics or Geometry. Joe put it best: I was not teaching math; I was teaching kids. Especially with the two younger classes, the teaching that was most important was not just the math but also communication, problem solving, and how to act and respect one another. Little else that I do in life will bring the joy and fulfillment of teaching.

However, the stress may even be a more lasting memory for me. Only a daily basis I was struggling to always be at my best, and when a lesson plan was sub par I beat myself up over it excessively. I felt a considerable amount of pressure from my cooperating teachers, especially June, although I know on some level part of it came from my pressuring myself even more. The problem was that I did not feel up to the task, and in retrospect feel that much more was expected of me in terms of being classroom ready than I was prepared for. I would be told I needed to put more effort into my lesson plans, but all that would end up accomplishing would be me banging my head on the wall, stressed out, trying to come up with better, more creative ideas. I had no real experience or tricks to employ in terms of how I could put more effort into my lesson planning, and by the end of the practicum I was quite exhausted and burnt out even though I had only been teaching for a few months. Teaching every day felt like a 24-7 job, a job which I didn't feel very good at a lot of the time.

This is said, perhaps experiencing the stress of teaching for the first time is what the practicum is all about, or at least perhaps that's what mine was all about. My natural classroom skills are there; I feel at home in front of a class, especially when I have indeed come up with a creative approach or two. As I continue to explore teaching in the future, I have my practicum experience in my back pocket. Stressful as it was it certainly gave me plenty of food for thought for the future. I do wish I had been better able to cut through the stress and being burnt out and apply the lessons I was learning more immediately, but I gave it my all and did my best and at least I will be that much more prepared for my next teaching experience. I will be better prepared to manage my time, foresee the flow and timing of classes, and plan accordingly. Even outside of the academic world the lessons I learned during my semester of teaching will serve me well.

A few other things come to mind as I reflect. CCHS' situation could certainly be called lucky; how would I do at a school that wasn't so affluent and stable? In Geometry, MCAS was a concern but I did not have to mutilate my lessons on a daily basis, yet I read stories about how teachers at schools with weaker economics have to constantly change everything in the face of the terrible MCAS beast. Could I make an impact? Somewhere long down the road could I somehow influence a public school in Worcester or Boston to reach the academic standards I've been used to at CCHS? I'd like to think so, but at the same time I recognize I am but one man and can only do the best I can for the students who I end up teaching. As I have learned that teaching is a tradeoff between stress and fulfillment, working in a tougher situation such as in an inner city school would amplify that well beyond what I experienced at CCHS.

Perhaps I would be better as a teacher of Computer Science, a subject which comes even more naturally to me? Perhaps I would be better teaching at the college or university level, taking out some of the stress of dealing with young teenagers but also taking away some of the reward? These are thoughts that sometimes cross my mind and I intend to explore as I move on with my life and career, but this is not to say I have ruled anything out, least of all teaching high school math just as I got a taste of in my practicum.

As a final thought, I feel I can be proud of much of what I did at CCHS but also know that I have plenty to work on as a teacher and even have a few instances where I am rightfully ashamed, such as sleeping through my alarm one morning. It's unfortunate that when I think of my time at CCHS I am first reminded of the stress, but surviving the fire will make me stronger and a better teacher in the long run. More importantly, stress or no, I will always have the personal reward of the joy of having worked with and taught the kids that I did. Ten years from now that is what I will remember and cherish the most.

Appendix A1

Selected Developed Course Materials

Name: Greg Case
Grade: 12
Subject: Statistics
Strand: Data Analysis, Statistics, and Probability
Learning Standards: 12.D.4

Date: 11 / 17
No. in group: 2
Time: 1:09 PM
School: CCHS
Text: Triola, Elementary Statistics, 8th ed.

Topic: Binomial Probability Distributions

Materials: 10 pennies per group; TI-8X calculators; textbooks; handout

Objectives: To get the students to understand, internalize, and work with the concept of binomial probability distributions and their underlying abstract function.

Comments:

The methodology here is one of mental experimentation explored through lots of questions. Very few 'answers' should be provided by the instructor. As often as possible, encourage the students to discuss possible answers or furthur questions with their partners, provided that such discussion does not disrupt the class ~ aka great for when a question has been posed but not okay when someone is addressing the class.

I scrapped the idea of doing a physical experiment and went to the mental experiment for a few reasons. First, I felt I could connect with many of the students in the class with the sports example much better than with pennies. Second, the goal was to get the class to develop the formula on their own. While pennies or dice would have been a good experiement, they would not have assisted in the goal of understanding why or how the formula described binomial probabilities. With the mental exercise, students are given a better chance of understanding and remembering the formula than if they were simply told what it was and that it described some data, even if they had generated the data themselves.

INTRODUCTION

Newly aquired Red Sox pitcher Curt Shilling records approximately 40% of his outs by strikeout. What are the chances that, in any given inning, he will strike out the side? Get no strikeouts? 1strikeout? 2 strikeouts?

DEVELOPMENT

Questions to ask:

What is the problem really asking? Do we care if there are hits or walks mixed in with outs?

Any ideas on methods we could use to solve this? [Independent, order doesn't matter, so tricky. Small enough to use prob. tree]

Are there any events we can identify? [First out is by strikeout, etc.]

How many trials are there? [Out = trial, constant 3 per inning]

Are the trials dependant or independent?

How many outcomes for each trial?

Does the probability change? [Maybe for real life, but not for us]

What would a good random variable choice be? [Remember what a random variable is: single numerical value determined by chance for a given procedure. Best r.v. would be number of strikeouts in the inning]

Repeat problem with words (SSO), checking previous results.

What's the probability of getting three strikeouts in an inning?

SSS -> (.4)(.4)(.4) = .064

What's the probability of getting two strikeouts?

SSO -> (.4)(.4)(.6)

SOS -> (.4)(.6)(.4)

OSS -> (.6)(.4)(.4)

Also equivalent to:

P(SSO) * number of ways to mix two S's and one O

= (.4)^2 * (.6) * 3!/2!

But 3!/2!1! is the same as 3 C 2

[Continue solving the problem, asking for each step along the way from the class, getting answers from different students whenever possible and waiting a bit before taking an answer and moving on.

Since students are in groups of 2, encourage interaction when answering the questions before someone is called on.]

[Make a table of what's been found: x, p(x), and the way p(x) was found]

Can we establish a formula for p(x) using x as the number of strikeouts?

P(x) = (.4)^x(.6)^3-x * 3!/(x!3-x!) = (.4)^x(.6)^3-x * 3 C x

Lets go to an alternate universe where there are 4 outs per inning. What would the probabilities look like?

[Repeat above process for n=4, again discussing with class and encouraging partners to work together as above]

Without doing calculations, what would the probabilities look like if there were 6 outs per inning?

What about if there were n outs per inning?

Lets say Shilling records 30% of his outs by strikeout. What would change?

What if Shilling records a strikeout with probability p?

[Wrap-up]

So now if there are n outs per inning and an out is a strikeout with probability p, the chance of getting x strikeouts is:

P(x) = [(p)^x] * [(1-p)^n-x] * [n C x]

This is a the binomial distribution formula! What makes it 'binomial'?

[Go back to the four criterion and stress each point. Ask the class why each one is important. What if the events were not independent? Would the forumula work? Why not?]

Assign homework and explain what is necessary to do it.

EVALUATION

Possible follow-up homework problems:

Pp. 201-204, Sec. 4-3

#1 – 8 [Conceptual check]

#9 [Exploration]

#17,19 odd [Practice]

#29

#32

6-3 Estimating a population mean for small sample size

Feb 24

- Hand out worksheet while checking homework

- Go over homework (#3 due today – p. 311, #20 – 24, 25, 27)

- Assign hw #4 (p. , #)

Now we're pretty good at finding confidence intervals as estimates of population means when our sample size is >30.

However, it is not always possible to get large samples. A greater sample size might lead to a much greater cost, time, or other spent resource. For example, lets say you're working in quality control for a widget company. You want to find the mean of a particular measure for your widgets, but it costs the company thousands of dollars to make each widget. Is there an economical way to estimate the population mean?

There is, but lets make sure we know what we're assuming:

- n <= 30
- Sample is a simple random sample
- The sample is from a roughly normally distributed population.

What does "roughly normally distributed" mean? It means that the population needs to be close to a normal distribution but doesn't need to be exact. It cannot be multi-modal (what does this mean?) and it must be symmetric.

What is our best point estimate for the pop. mean?

The sample mean is still the best point estimate. Why? It's an *unbiased estimator*

However, we're no more sure about how good our estimate is than we were with larger sample size – if anything, our estimate is probably not as good as it would have been for larger sample size. However, we have a tool to help us now – *confidence intervals*.

We need to divide the methods for n < 30 into two cases.

Case 1: Pop. standard dev. is known: We can use the same method as for large sample size: E = Z(a/2)*sigma/root n, confidence interval limits x bar +/- E.

However, this case is somewhat unrealistic ~ there are few cases where the pop. mean will be unknown while the pop. standard deviation will be known. However, in these rare cases we can use this method as long as our assumptions are met.

Case 2: Pop. standard dev. is unknown:

William Gosset was a Guinness Brewery employee working in quality control somewhere in the early 1900's. Since it was expensive to use large samples, he needed a distribution that could be used with small sample sizes. To this end he developed a new distribution called the *Student t distribution*.

Why Student t distribution? Guinness did not allow publication of results by employees, so Gosset used the psuedonym Student to publish his work.

Important things to note when using the Student t distribution:

- Our assumptions must hold – n <= 30, simple random sample, population roughly normally distributed. (Why n <= 30? If n > 30, then we can use the normal distribution method)

- The Student t distribution is roughly normal, but has greater variability. (Why? More on this tomorrow)

- The Student t distribution changes with different n. We express this as *degrees of freedom*; when using the Student t distribution, degrees of freedom = n - 1

So if our sample size is 20, we have 19 degrees of freedom – more on this tomorrow.

- Our basic methodology is the same. Since the distribution is symmetric, we'll still be finding our confidence interval as x bar +/- E. However, finding the margin of error E is a bit different.

Instead of finding our critical values using the normal distribution (Z a/2 – z scores) we'll find them using the Student t distribution (t a/2 – t scores). To do this we use table A-3

Match up alpha (1 - degree of confidence) on top row with degrees of freedom (remember: for this n-1) to get the critical value. For now always use 2-tails.

For example, for alpha = .02 (how confident? 98% confidence) and 15 degrees of freedom (what's our sample size? 15 + 1 = 16), our critical t-score is: 2.602

We'll calculate E using the same basic idea:

Margin of error = critical score * sample std. dev / square root of sample size
E = t a/2 * s / sqrt(n)

Now we have everything we need to construct a confidence interval! Tada!

So our confidence interval will be:

(x bar – E, x bar + E)

Feb 27

- Warm up problems: (Check HW)

Find the following critical t-scores:

10 degrees of freedom, 98% confidence
17 degrees of freedom, 99% confidence
29 degrees of freedom, 50% confidence (We will almost never construct such an interval)
1 degree of freedom, 95% confidence
22 degrees of freedom, 90% confidence

Why are the critical values for 1 degree of freedom so high?

Our sample size is only 2, so we do not expect our point estimate to be very good at all. Consider how critical values affect the margin of error, E. With a smaller sample size, would we expect a larger or smaller margin of error? In our equation, we now have both the square root of n and t-scores that change based on sample size.

- Assign hw #5

- Quiz on 6-3, 6-2 on Tuesday. We will be quizzing about once per section in this chapter.

- Go over homework (#4 due today -)

Let's take a closer look at the Student t distribution. Open to the diagram on p. 316.

(Draw standard normal distribution followed by 2 student t distributions for varied degrees of freedom)

Which of these do we expect to have the greatest variance / standard deviation? Why? Smaller sample sizes should have more variance; this is reflected in the Student t distribution.

The standard normal distribution is taller and has smaller tails, while the Student t distribution gets shorter and has thicker tails as degrees of freedom gets smaller. Why is this?

Shorter and thicker tails shows the greater variance in the Student t distribution. What happens when we try to find critical values for the same confidence but different degrees of freedom? Thicker tails means that more area fits there, so our critical t-values will be greater (show)

Decision tree:

Is n > 30?

- Yes : Use critical Z scores
- No: Is the population roughly normal?
  - Yes: Do we know sigma?
    - Yes: Use critical Z-scores
    - No: Use Student t distribution
  - No: We don't have the tools to solve this

- Work on worksheet, answer individual questions

6-4 Determining sample size required to estimate mu

Mar 1

- Warm up :

We want to estimate the mean body temperature of humans to within .10 degrees with 99% confidence. Assume that the population standard deviation is 0.62 degrees F.

What's our value of E?

What's our appropriate critical value Za/2?

Using E = Za/2 * sigma / sqrt(n), how large must our sample be?

- Review homework

(At least 2. #14, #15, #19, #20)

Going back to our example, what do we need to know in order to determine minimum sample size? Critical value/confidence level, pop. std. dev., desired margin of error.

Note: When determining minimum sample size, always round up! You want the smallest integer that is larger than or equal to the decimal result; rounding down will result in an insufficient n. Err on the side of caution will be the M.O. of the day.

(Modus Operandi is a way of doing things)

We will talk about what to do if we don't know the standard deviation in a few minutes.

Is our required sample size dependant on the size of the population? No, but there are cases when it will matter, specifically when we're drawing from a small, finite population without replacement. For now we'll assume that our populations are sufficiently large, so our population size does not affect sample size.

Find the minimum sample size if we change our desired margin of error to .05 degrees, half of .10 degrees. How does it compare to our minimum sample size for .10 degrees?

Find the minimum sample size if we change our desired margin of error to .20 degrees, twice as large as .10 degrees. How does it compare to our minimum sample size for .10 degrees?

Can we make a general statement about modifying our desired margin of error and what it does to minimum sample size?

Halving E quadruples n. Doubling E quarters n. In general, multiplying E by *k* will muliply n by $1/k^2$.

However, as noted before, what we've got right now isn't terribly practical. We do not expect there to be many cases where we know the pop. standard deviation but not the pop. mean. What can we do to estimate the std. dev. if it is not known?
We have three options / tactics to use:

1. Range rule of thumb.

   What does the range rule of thumb estimate sigma to be?

   sigma = range / 4

2. Pilot study of at least 31 values to establish s, then refining results along the way

3. Estimate the standard deviation using the results of a previous study

A few comments on the third option: Often we can be creative with using previous results as long as we are careful to err on the side of caution. For example, lets say we want to estimate the mean IQ of math teachers. We know that for the population of all IQ scores, sigma is 15. Noting that math teachers are a more homogeneous population than the population of all humans, which of these do we expect to be true?

Sigma > 15, < 15, = 15?

We don't know for sure, but a reasonable assumption would be to guess that sigma <= 15. However, if we guess too low for sigma, we'll end up with an insufficient sample size, so we'll err on the side of caution and estimate sigma to be 15 – the highest reasonable value.

Examples

In Pizzaville, the cheapest large cheese pizza can be purchased for $6.0 while the most expensive large cheeze pizza can be purchased for $14.0. Use the range rule of thumb to estimate the pop. standard deviation, then find the minimum sample size required to estimate the mean cost within $0.10 with 98% confidence.

Find the minimum sample size required to estimate the mean IQ of all math teachers to within 2 IQ points with 95% confidence, noting that IQ scores for everyone are normally distributed with a mean of 100 and a standard deviation of 15.

- Return 6-2 quiz at end of class, go over if time.

Statistics
Warm Up
Section 6-6

Name: _______________
Date: _______________

A simple random sample of 25 women's weights from around the world gave a sample mean of 145.6 pounds with a sample standard deviation of 35.0 pounds. Construct 95% confidence intervals for the population mean and the population standard deviation.

Consider instead a simple random sample of 100 women with the same sample statistics. Construct 95% confidence intervals for the population mean and the population standard deviation.

Using the above study as a pilot study, how large should a sample be to estimate the population mean with 95% confidence within 2.5 pounds?

How large should a sample be to estimate the population standard deviation with 95% confidence to within 5% of the actual value?

Statistics
Warm Up 2
Section 6-6

Name: _______________
Date: _______________

First, for the curious - statistics in the news:
http://www.cnn.com/2004/WEATHER/03/18/average.winter.ap/index.html

Now, the warm-up…

Quality Control testers at a candy company want to make sure their new process for making a particular type of candy bar is consistent. Each candy bar is supposed to be 50.700g and they don't want the population standard deviation of weight to be any more than 1.000g. They have already established that the population is normally distributed. How many candy bars should they include in their study if they want to estimate the population standard deviation to within 20% of the actual value with 95% confidence?

a) Careful! Some of the information in the question isn't relevant (yet). Which numbers / facts are relevant?

b) Which numbers / facts aren't relevant?

c) How large should their sample be?

The testers take a sample of minimum size (use what you got for part c) and find a sample standard deviation of .930g. Construct a 95% confidence interval for the population standard deviation.

d) What's the lower limit of the confidence interval?

e) What's the upper limit of the confidence interval?

f) If you were in charge of Quality Control at the company, would you be happy with the results? Why or why not?

Statistics
Section 7-2
Practice

Name: ________________
Date: ________________

For each claim, write the appropriate conclusion based on if the test statistic is in the critical region or not.

Claim: The Dandy Candy Company claims that the mean weight of their premier candy bars is 50.7g.
Result: The test statistic was not in the critical region.
Conclusion:

Claim: A scientist claims the mean nitrogen dioxide level in West London is greater than 28 parts per billion.
Result: The test statistic was in the critical region.
Conclusion:

Claim: A weight loss program claims that program participants have a mean weight loss of at least 10 pounds after one month.
Result: The test statistic was in the critical region.
Conclusion:

Claim: Acme Light Bulbs claims that the mean life of a certain type of bulb is at least 750 hours.
Result: The test statistic was not in the critical region.
Conclusion:

Claim: An SAT prep course claims that the mean score of all of its students is 1350.
Result: The test statistic was in the critical region.
Conclusion:

Claim: A consumer watchdog group claims that the mean weight of a particular kind of cereal is less than the posted 12.4 oz.
Result: The test statistic was not in the critical region.
Conclusion:

For each claim, determine the null hypothesis, alternative hypothesis, if the test is one- or two-tailed, the range of the critical region, and the value of the test statistic.

Claim: The Dandy Candy Company makes fudge-filled chocolate eggs during Easter season. They claim that the mean weight of the eggs is 8.20 oz.
Data: A random sample of 50 eggs found a sample mean of 8.12 oz and a sample standard deviation of .10 oz. Testers want a level of significance of .05.

Claim: In 1994, the national mean cost to community hopsitals per patient per day was $931. Researchers claim that the mean cost of patient care in Massachusetts is higher than the national average.
Data: A random sample of 35 daily costs of in Massachusetts hospitals yielded a mean of $1131 with a sample standard deviation of $333. Significance level is 5%.

Claim: The manufacturer of a new car, the Alpha, claims that a typical Alpha gets at least 27.0 mpg.
Data: A random sample of 40 new Alphas gave a sample mpg of 26.8 with a standard deviation of 1.4 mpg. Use a significance level of .02.

Claim: *Highway Statistics* claimed that in 1990 the average passenger vehicle was driven more than 10300 miles.
Data: A random sample of 500 passenger vehicles had a mean of 10400 miles driven in 1994. Assume that the population standard deviation is 6000 miles. Use a significance level of 1%.

Statistics
Section 7-4
Practice

Name: _______________
Date: _______________

For each claim, find: a) The test statistic b) The range of the critical region
c) Whether or not the claim can be supported or rejected (word the final conclusion)

Claim: The Dandy Candy Company claims that the mean amount of fat in their premier candy bars is 6.3g. Assume the population is normally distributed.
Data: Sample size 16, sample mean 6.8g, sample standard deviation 0.2g
Significance: 0.05
Conclusion:

Claim: A car company claims that the mean repair cost of a particular model of car in a particular crash test is no more than $6,300. Assume the population is normally distributed.
Data: Sample size 10, sample mean $6,423, sample standard deviation $596
Significance: 0.10
Conclusion:

Claim: A weight loss program claims that program participants have a mean weight loss of more than 10.0 pounds after one month. Assume the population is normally distributed.
Data: Sample size 25, sample mean 11.2 lbs., sample std. dev. 4.5 lbs.
Significance: 0.01
Conclusion:

For the next problem, use the P-value method to determine if the answer can be supported or rejected.

Claim: The mean number of M&Ms in a bag is greater than 56.0 M&Ms.

Data: (Based on the data we collected in class)

57	56	56	57	56	56	57	59	57
57	58	58	58	55	54	55	57	58

Significance: 0.05

Statistics
In-Class Project
40 Points

Name: ________________
Partner: ________________
Group Num: ____________
Date: ________________

For centuries (okay, a couple of decades) philosophers (okay, inquisitive children) have been pondering questions about the color distribution of a bag of M&Ms. Finally, we have the tools to come up with a good, solid estimate for the proportion of a particular color of M&Ms. Today we will use our new-found skills with statistics to estimate the population proportion of red M&Ms.

I will be collecting and grading this project, so be sure to show all supporting work.

1) Just for amusement, come up with an initial guess as to the proportion of red M&Ms:

1. ______________

2) Before we can begin sampling, we need to know how large our samples need to be. For 90% confidence, what minimum sample size is required to estimate a population proportion with a margin of error of no more than 5%?

2. ______________

3) The result of part 2 is too high, so lets try broadening our allowed margin of error. For 90% confidence, find the minimum sample size required to estimate a population proportion with a margin of error of no more than 10%.

3. ______________

4) Get some sample data and begin! By the way, what do we need to assume about how M&Ms are packaged for our work to be valid?

4a) Record your results on your data sheet. Break down each bag individually first, then combine them to get a larger n.

5) What should be considered a "success"? ______________________

What should be considered a "failure?" ______________________

6) Find x, n, and $\hat{p}$ for your sample data.

x : __________

n : __________

$\hat{p}$: __________

7) When you've found them, send someone from your group up to write them on the board.

8) What kind of distribution are we working with where we count the number of successes in a certain number of trials? 8. __________

9) Before we can construct a confidence interval, we need to verify that that the _________ (fill in) approximation to our distribution is appropriate. Justify that we are indeed able to construct a confidence interval (or show why we cannot).

10) Using your sample data, construct a 90% confidence interval for the true population proportion of red M&Ms.

10. ______________

11) Write your results on the board next to your earlier results.

12) Name three ways to reduce the margin of error when constructing similar confidence intervals.

12a. ________________________________

12b. ________________________________

12c. ________________________________

13) According to Mars, Incorporated, the proportion of red M&Ms is %20. Does your confidence interval contain the actual proportion? If it did not, would you necessarily have made a mistake somewhere along the way? Defend your answer using what you know about confidence intervals.

14) Record the results of the other groups.

15) How many of the confidence intervals generated contain the actual population proportion?

15. ______________

16) If you were to combine all of the sample data into one sample, what would happen to the margin of error for constructing a 90% confidence interval?

16. ______________

17) If instead of 90% confidence a 99% confidence interval was generated, what would happen to the margin of error?

17. ______________

18) Using all of the sample data as one simple random sample, construct a 99% confidence interval for the proportion of red M&Ms.

18. ______________________

x = ________
n = ________
$\hat{p}$ = ________

19) Compare the two confidence intervals you generated, one with 90% confidence and smaller n and one with 99% and larger n. Which gives better results? In what situations would you prefer the interval which your group did? In what situations would you prefer the interval constructed with data from the entire class?

Extra Credit! (Very difficult! Show all work!) Name : ________________

Consider the 90% confidence intervals that the class was generating. Using the binomial distribution:

A) Find the probability that all of the confidence intervals generated would actually contain the mean.

B) Find the probability that at most 5 of the confidence intervals generated would actually contain the mean.

C) Find the minimum number of confidence intervals that would need to contain the mean for the results to be considered 'usual'.

D) Can a binomial distribution always be used to calculate the probability of a certain number of confidence intervals containing the mean out of n similar intervals with the same level of confidence? Why or why not?

Data Sheet

Name: ________________
Partner: ________________
Group Num: ____________

Sample Results, Bag 1:

Color								Total
Amount								

Sample Results, Bag 2:

Color								Total
Amount								

Sample Results, Total:

Color								Total
Amount								

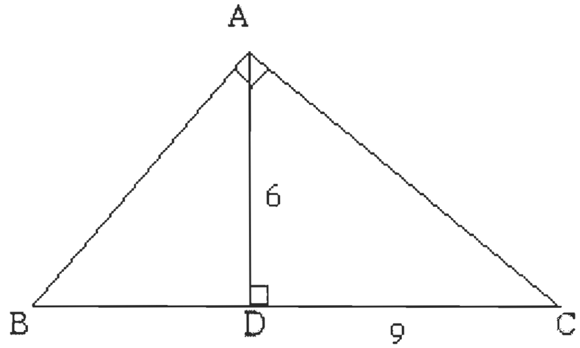
Does the CI contain the actual proportion?

Class results:

Group #	Bag 1	Bag 2	n	x	p	E	Confidence Interval	(y/n)

CP2 Geometry
Altitude-Hypotenuse
Review Sheet

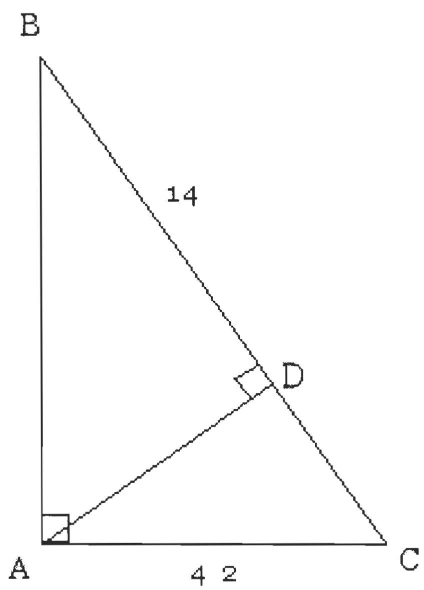
Name: ____________________
Date: ____________________



1) BD = ________

AB = ________

AC = ________



2) DC = ________

AB = ________

AD = ________

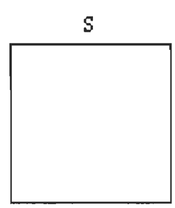
CP2 Geometry
Area!

Name: ________________

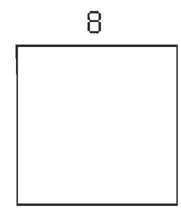
Date: ________________

See how many formulas for area your group can come up with!

Square

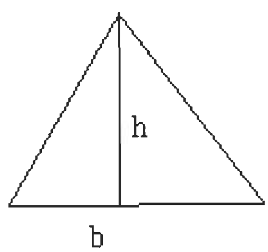


Formula for area:

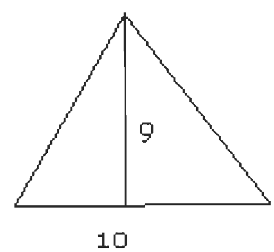


Area:

Triangle

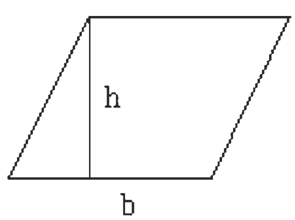


Formula for area:

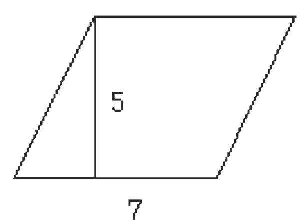


Area:

Parallelogram / Rectangle

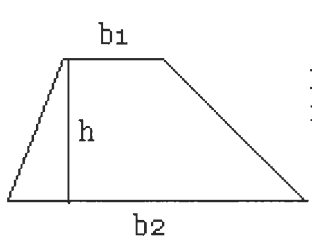


Formula for area:

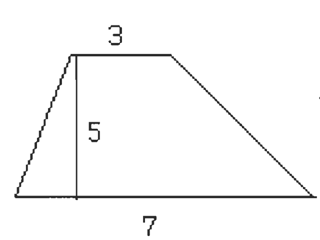


Area:

Trapezoid



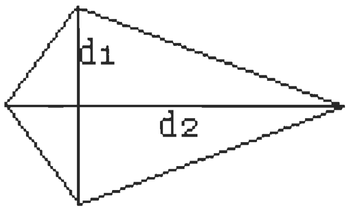
Formula for area:



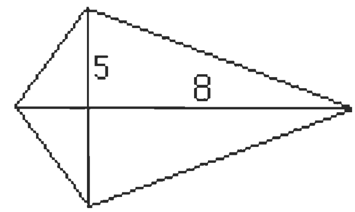
Area:

P.2

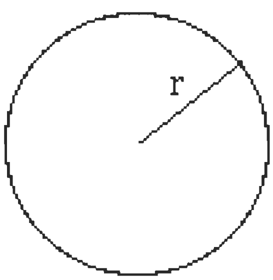
Kite



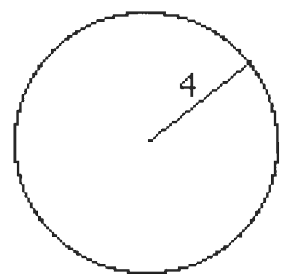
Formula for area:



Circle

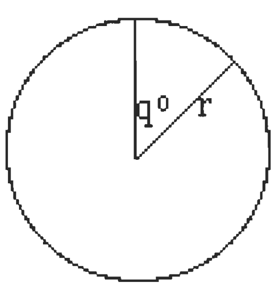


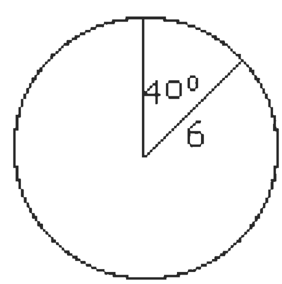
Formula for area:



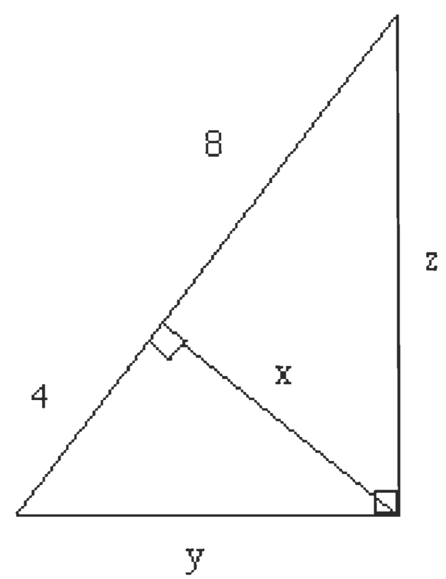
Sector

Formula for area:

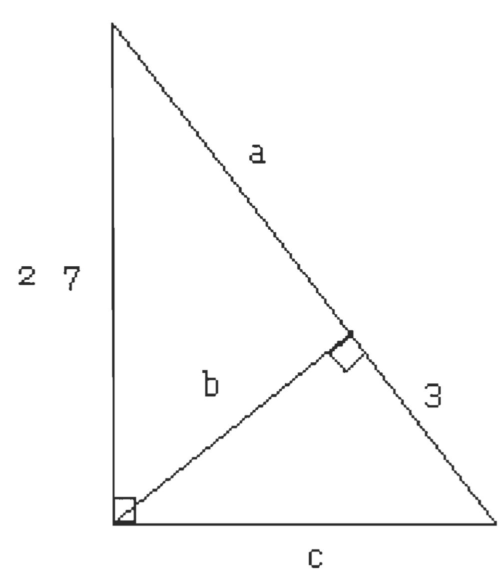




#1 Find lengths x, y, and z! Leave your answers as reduced radicals.

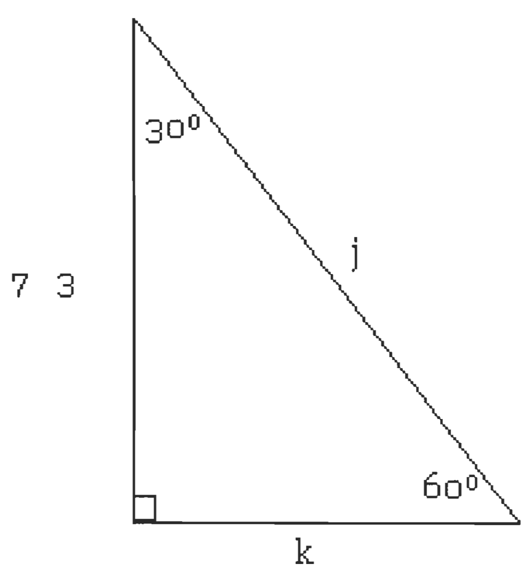


#4 Find lengths a, b, and c! Leave your answers as reduced radicals.

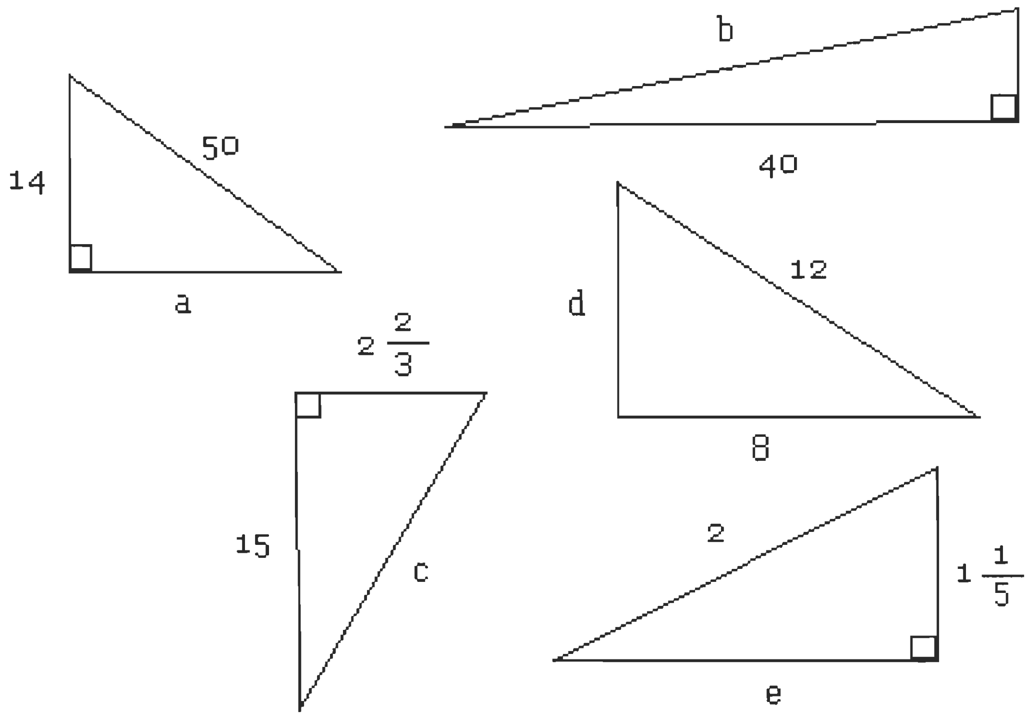


#2 A square has diagonals of length 10. What's the perimeter of the square? (Hint: Can you find a 45-45-90 triangle?)

#5 Find lengths j and k.



#3 Find the missing sides in each triangle (careful ~ one's not a pythagorean triple!)



#6 Mr. Case travels 4 miles south, 4 miles west, 3 miles south, then stops. Mr. Leone travels 4 miles east, 2 miles north, and 4 more miles east then stops. How far apart are Mr. Case and Mr. Leone? (Hint: Look for a right triangle so that the hypotenuse is between Mr. Case and Mr. Leone)

CP2 Geometry
Problem Set #2
Trigonometry Project

Name: ________________
Team Members: ________________
____
Date: ________________

Step 1: Construct a measuring device. You'll need:

1 protractor
5-6" string
1 weight
1 straw

Tie the weight to one end of the string. Loop the other end of the string through the protractor and tie off. Tape the straw along the edge of the protractor so that looking through the straw will give you a straight line parallel to the base of the protractor. Víola! You now have your measuring device.

Step 2: Start from the base of the crayon and walk 40' straight away in any direction. Mark the spot!

Now, have a team member stand in the spot 40' away from the crayon with the measuring device. Measure the angle of elevation to the top of the crayon and record it on your results sheet.

Step 3: Use your collected data to calculate the height of the crayon. Draw a diagram first, and make sure to show all work and write your final results **neatly**.

Step 4: Choose 2 other objects in the courtyard to measure. You can choose how far away from the base of each object to be, but remember to write it down on your results sheet. Make sure everyone in the group gets a chance to use the measuring device!

Record all of your results on your results sheet and work together to calculate the height of the objects measured. Again, write your final results **neatly**!

How to use the measuring device

1) Stand in the marked spot facing the object you're measuring

2) Aim the device towards the top of the object being measured. The base of the protractor should be parallel to your line of sight.

3) Let the string hang until it stops swinging. Grasp the string where it touches the protractor, then you can bring the device down.

4) See where the string is touching the protractor. Record the angle showing at that point.

5) The angle of elevation is the angle complementary to the angle showing on the protractor.

CP2 Geometry
Problem Set #2
Trigonometry Project

Name: ________________
Team Members: ________________
____
Date: ________________

Results Sheet

1) First object measured: Purple Crayon

Distance from base: 40 feet

Measured angle of elevation: ________

Work:

Calculated height: ________

Diagram

2) Second object measured: ______________

Distance from base: _________

Measured angle of elevation: ________

Work:

Calculated height: ________

Diagram

3) Third object measured: ________________

Distance from base: __________

Measured angle of elevation: _________

Work:

Calculated height: _________

Diagram

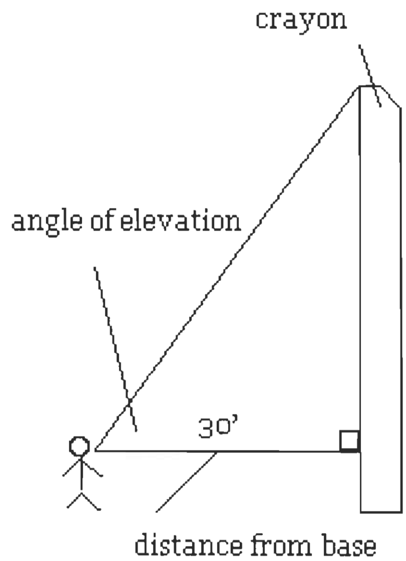
4) Write-up

Explain your steps. What was your process?

CP2 Geometry
Trigonometry Project

Name: ________________
Team Members: ________________
____
Date: ________________

Results Sheet



1) First object measured: Purple Crayon

Distance from base: 30 feet

Measured angle of elevation: ________

Work:

Calculated height: ________

2) Second object measured: ________________

Distance from base: __________

Measured angle of elevation: ________

Work:

Calculated height: ________

CP2 Geometry
Working Lunch
Learning Guide
Circles

Name: ________________
Date: __________________

Stop!

Before anyone takes any food, we should figure out how large one whole pizza is. Lets have two people estimate the diameter of a pizza (two people so we can double-check each other).

Diameter of a pizza: ___________________

Since we know the diameter, we can calculate the radius: ______________

Okay! Now lunch can begin, but keep working on this learning guide ~ I'll be collecting it at the end of class!

1) Circumference and Area

Find the circumference of a whole pizza: ________________
(If you don't remember the formula, look it up!)

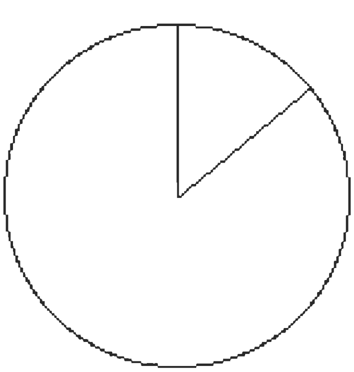
Find the area of a whole pizza: ________________

2) Sectors and Arcs

Assuming that all pieces of pizza are made equal, what fraction of a pizza is one piece? __________________

Since the area of the whole pizza is ______ and one piece represents ______ of a whole pizza, what's the surface area of one piece of pizza?____________

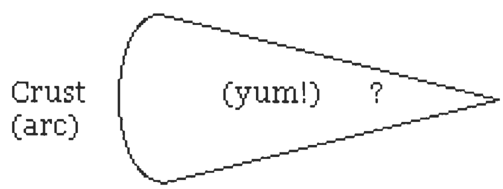
This is called a ***sector*** of a circle!



Sectors and Arcs, continued

How many degrees are there total in a circle? ___________

Since the number of degrees total is ______ and one piece is ______ of the whole, the number of degrees in the angle at the point of one piece is: _________



Since the circumference of the pizza is ______ and one piece is ______ of the whole, the amount of crust on one piece of pizza is __________

This is called an ***arc*** of a circle!

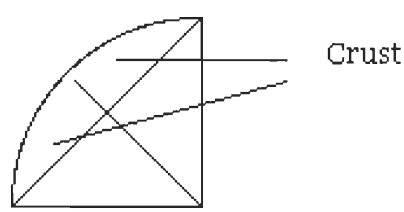
An arc has both an angle measure (called the *measure* of the arc) and a length (called the *length* of the arc).

The measure of an arc is the same measure as the angle forming the arc from the center of the circle ~ what's the measure of the arc for one piece of pizza? __________

3) Segment

By now some people will have finished eating the cheesy part of the pizza ~ a lot of people tend to not eat the left-over crust, though. How much surface area is there in the crust?

Lets do it for not one but instead two pieces of crust, and assume it looks something like this:

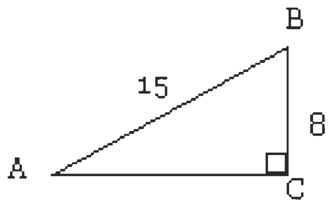


Any ideas on how to find the area of this ***segment***?

Hint: You know the area of the *sector* (now the area of **two** pieces). How much of the pizza have you already eaten? (It's a triangle!)

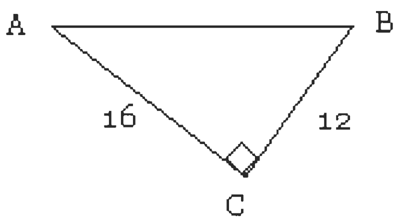
CP2 Geometry
SOHCAHTOA
Group Activity

Name: ____________________
Date: ____________________



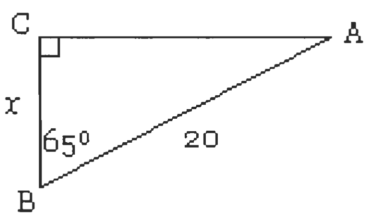
What's the *sine* of angle A? __________

What's the measure of angle A? __________
(Use your calc!)



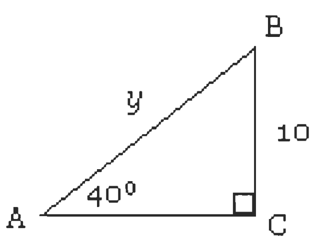
What's the *tangent* of angle B? __________

What's the measure of angle B? __________



What's the *cosine* of angle B? __________
(Use your calc!)

What's the measure of CB? __________

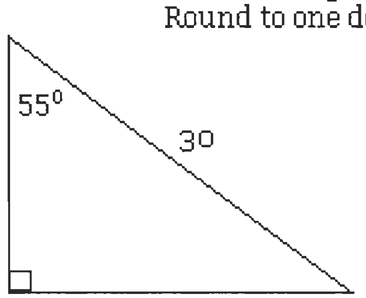


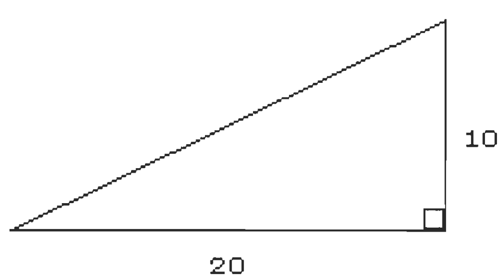
What's the *sine* of angle A? __________

What's the measure of the hypotenuse AB?

____

In the triangles below, fill in all the missing angles and lengths. Round to one decimal place.

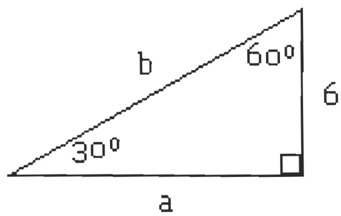




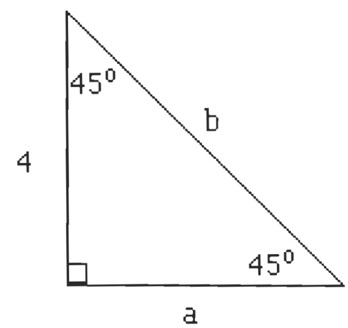
CP2 Geometry
Warm Up
Special Right Triangles

Name: ____________________
Date: ____________________

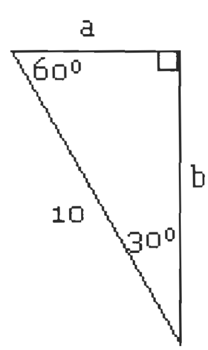
#1)



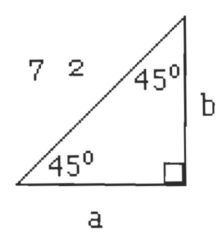
#2)



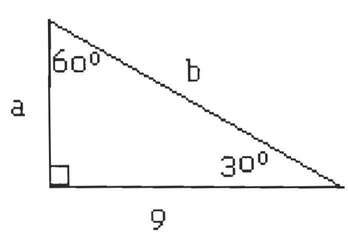
#3)



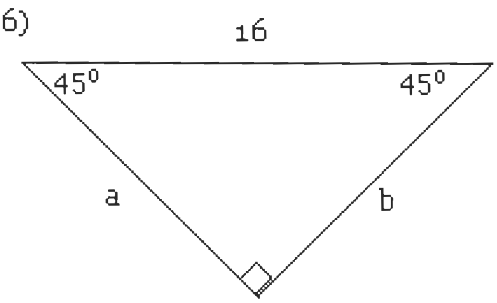
#4)



#5)



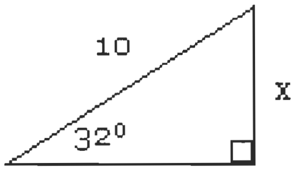
#6)



CP2 Geometry
Trig Practice

Name: ____________________
Date: ____________________

Answer each to the nearest tenth (one decimal place)

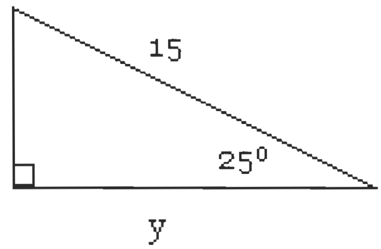


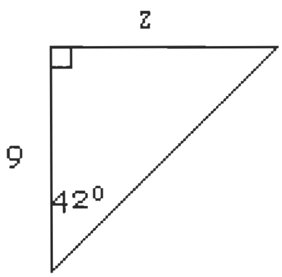
1. Find x

____

2. Find y

____



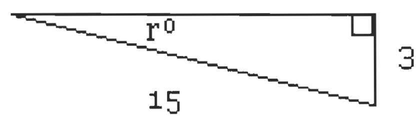


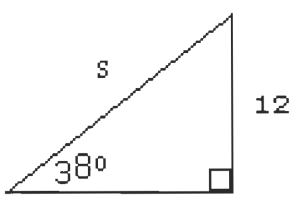
3. Find z

____

4. Find r

____



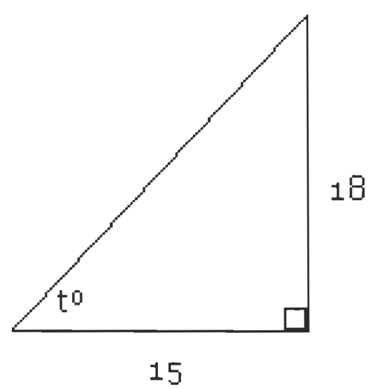


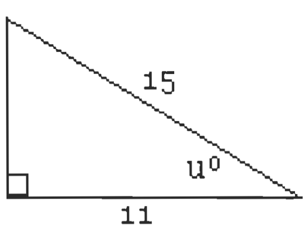
5. Find s

____

6. Find t

____



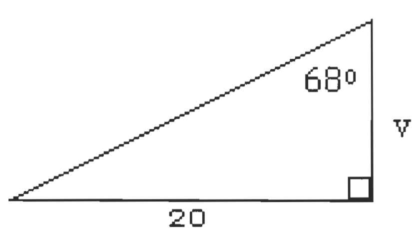


7. Find u

____

8. Find v

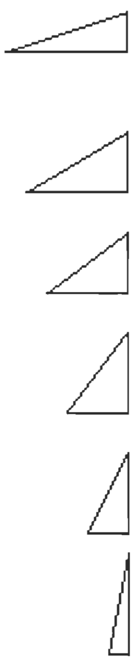
____



CP2 Geometry
SOHCAHTOA
Exploration

Name: ________________
Date: ________________

Use your calculator (or the table from the book) to fill out the table. Go out to at least 3 decimal places.



angle A	sin A	cos A
10°	______	______
15°	______	______
20°	______	______
30°	______	______
45°	______	______
60°	______	______
70°	______	______
75°	______	______
80°	______	______
85°	______	______
90°	______	______

What's happening to the sine as the angle increases?

____

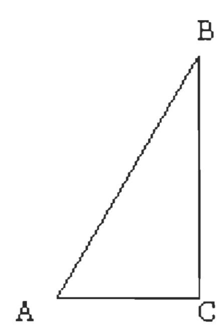
What's happening to the cosine as the angle increases?

____

Looking at the triangles to the left of the table, does this make sense? (Think about what's happening to the opposite and adjacent sides)

Is it possible for angle A to be 90° in the triangle to the right? (Try to draw such a triangle)

What will the sine of 0° be? Cosine of 0°?



Appendix B

Selected Developed Assessments

Statistics H-Block Name ________________
Test – Chapter 5 Date ________________

Calculators happily active. Use the table or the calculator where indicated, otherwise ou may use whichever method you are most comfortable with. If you use the calculator, make sure to write down what you typed. Be sure to show all work and pictures where appropriate for full credit!

1. Use the probability density function graphed below to answer the questions.

a. What kind of distribution is being modeled?

1a. _____________

b. Find the probability that the random variable has a value above 6.0

1b. _____________

c. Find the probability that the random variable has a value between 9.8 and 14.6

1c. _____________

2. If Z is a standard normal variable, find each of the following:

a. The probability that Z is less than –0.44 (Table)

2a. ______________

b. $P(Z > 1.13)$ (Calc)

2b. ______________

c. P_{34} (Table)

2c. ______________

d. D_6 (Calc)

2d. ______________

3. The random variable X has a normal distribution with mean 75.0 and standard deviation 5.0.

a. Find $P(X<67.0)$ (Table)

3a. ______________

b. Find $P(71.3 < X < 82.7)$ (Calc)

3b. ____________

4. A bank's loan officer rates applicants for credit worthiness. The ratings are normally distributed with a mean of 200 and a standard deviation of 50.

   a. If an applicant is randomly selected, find the probability that the applicant has a rating between 170 and 220. (Table)

   4a. ____________

   b. If 16 applicants are randomly selected, find the probability that their average credit rating will be greater than 215. (Table)

   4b. ____________

5. Assume that the price of regular unleaded gas around the Concord area is normally distributed with a mean of \$1.649 per gallon with a standard deviation of \$0.049.

   a. Find the probability that a random gas station will be charging between \$1.699 and \$1.749 per gallon. (Calc)

   5a. ____________

b. Find the prices that separate the bottom 5% from the top 5%. (Calc)

5b. _____________

6. Acme Bulbs produces standard light bulbs with life times that are normally distributed with a mean of 800 hours and a standard deviation of 100 hours.

a. Acme wants to label their bulbs with a rated life such that more than 75% of the bulbs last longer than the advertised time. What should Acme put on the label of their bulbs? (Table)

6a. _____________

b. If 50 bulbs are randomly selected, what is the probability that their average lifetime will be greater than 820 hours? (Calc)

6b. _____________

c. An Acme quality tester experimented with 50 randomly selected bulbs and found they had a mean lifetime of 820 hours. Is this result unusual? Defend your answer using probability.

7. 7% of Americans have type O^- blood. A blood drive draws 150 random donors.

a. Would it be appropriate to approximate this experiment using the normal approximation? Defend your answer with what you know.

b. Find or estimate the probability that of the 150 donors at least 10 will be type O^-. Use the normal approximation if it is suitable. Otherwise, use binomial probabilities. In either case, use your calculator.

c. Find or estimate the probability that of the 150 donors exactly 10 will be type O^-. Use the normal approximation if it is suitable. Otherwise, use binomial probabilities. In either case, use your calculator.

Statistics Name: ____________________

Quest – Sections 6.2 – 6.4 Date: ____________________

Calculators allowed. Show <u>all</u> supporting work for full credit. Do each problem with the method specified. If you use your calculator, write down what you entered. Be mindful of rounding. Lastly, if you cannot solve the problem with the given information state so.

1. In Pizzaville, concerned health officials want to estimate the average amount of pizza consumed by citizens of the town on a weekly basis. A simple random pilot study of 35 citizens has been conducted with sample mean 4.30 kg and sample standard deviation 1.53 kg. The officials want to estimate the population mean within 0.50 kg with 98% confidence. How many people should be included in the final study? (Table)

   1. ________________

2. A bank would like information concerning the incomes of its local customers. It wants to estimate the average income within $2000 of the actual mean with 92% confidence. A recent report for the town reported that the standard deviation of incomes is $12,031. How many customers should the bank survey? (Calc)

   2. ________________

3. In major-league Baseball this year, the league minimum salary is $300,000 while the largest salary is $21,000,000. Find the minimum sample size needed to estimate the average salary of MLB players within $1,000,000 with 90% confidence using the range rule of thumb to estimate σ. (Table – note that using range rule of thumb here does not give a very good estimate for σ)

   3. ________________

4. *Consumer Reports* gave the following data about calories in a 30 gram serving of chocolate chip cookies. Both fresh-baked such as Duncan

Hines and Pillsbury and packaged cookies such as Pepperidge Farm and Nabisco were included.

153	152	146	138	130	146	149	138	168
147	140	156	155	163	153	155	160	145
138	150	135	155	156	150	146	129	127
171	148	155	132	155	127	150	110	

Sum = 5128

Assume this is a simple random sample and find the 99% confidence interval for the mean number of calories in all chocolate chip cookies. (Calc)

4. ___________________

5. How big are professional football players? Consider the following positions: defensive end, defensive tackle, offensive guard, offensive tackle, and center. A random sample of professional players in these positions was taken, and the following list gives body weight in pounds.

292	290	295	306	310	281	270	278	303	293
275	305	306	291	275	269	290	275	309	292
270	265	260	315	295	285	290	305	260	280
320	280	298	291	266	273	281	274	300	283

Sum = 11496

Find the 98% confidence interval for the mean weights of all professional Football players at these positions. (Table)

5. ___________________

6. A computer programmer wants to see if a particular random number generator generates the expected mean. The random numbers are believed to be uniformly distributed.

a. If she takes a sample of 100 numbers and gets a sample mean of .499 and a sample standard deviation of .301, what is her 97% confidence interval for the mean? (Calc)

6a. ______________

b. If she instead takes a sample of 20 numbers and gets a sample mean of .440 and a sample standard deviation of .298, what is her 98% confidence interval for the mean? (Table)

6b. ______________

c. If she instead takes a sample of 20 numbers and gets a sample mean of .532 but knows that the population standard deviation is .280, what is her 99% confidence interval for the mean? (Table)

6c. ______________

d-f. Defend why you chose each method for parts a-c, above.

g. If she wants to estimate the actual mean to within .002 with 90% confidence and knows the standard deviation is .280, how large should her sample size be?

6g. ______________

7. It always pays to shop around before puchasing a used car. Nine customers purchased the same model car with the same mileage and options from nine different used car dealers and paid an average of $7200. The standard deviation was $812. Construct a 99% confidence interval for the average used-car price for this particular car with particular

mileage and options. Assume the prices are normally distributed. (Table)

7. ____________________

8. The kidneys of a human being

Statistics
Quiz – Section 6.5

Name: ____________________
Date: _____________________

Calculators allowed. Show <u>all</u> supporting work for full credit. Be mindful of rounding.

1. The Gallup Organization, a notable polling organization, took a poll of 1005 american adults and found that 492 of them said they approved of President Bush. They want to construct a 95% confidence interval for President Bush's overall approval rating.

   a-c) For this poll, what are the values of x, n, and p?

   1a. x = ______________

   1b. n = ______________

   1c. p = ______________

   d) Find the margin of error, E.

   1d. E = ______________

   e) Construct the 95% confidence interval for President Bush's overall approval rating based on Gallup's poll results.

   1e. ________________

   f) In this poll, does q represent the proportion of people who said they disapproved of the President? Defend your answer.

2. The *New York Times* wants to do a poll to estimate the proportion of people who have already decided who they are going to vote for in the

upcoming election. An initial study gave a sample proportion of 76.0%. They want to estimate the actual proportion within 2% with 98% confidence. How many people should the *Times* include in its poll?

2. ________________

3. Emerson Hospital would like to know how many people in Concord caught the flu this past season. They took a simple random survey and found that of the 425 people surveyed, 67 had contracted the flu. Construct a 90% confidence interval for the overall population proportion of people who had the flu in Concord last season.

3. ________________

4. The *Boston Globe* wants to do a poll of voters in Massachucetts to see who would support a Constitutional Ammendment banning same-sex marriage in the state. They want to estimate the proportion within 4% with 99% confidence. How many people should the *Globe* include in its poll?

4. ________________

5. On one evening during the week of March 1, NBC Nightly News drew 521 of the 4010 homes involved in Neilson's TV Ratings Survey. Construct a confidence interval to estimate overall viewership with 99% confidence.

5. ________________

Statistics
Quiz – Section 6.6

Name: ____________________
Date: ____________________

Calculators allowed. Show all supporting work for full credit. Be mindful of rounding and what the question is asking.

1. Find the critical values X^2_R and X^2_L corresponding to the given sample size and confidence level.

   a) Sample size 12, Confidence level 95%

   1a. ________________

   b) Sample size 24, confidence level 90%

   1b. ________________

   c) Sample size 59, confidence level 99%

   1c. ________________

2. Use the given degree of confidence and sample statistics to find a confidence interval for the population standard deviation. Assume the population has a normal distribution.

   College students' annual earnings: $n = 13$, $\bar{x} = \$4168$, $s = \$832$

   98% confidence

   2. ________________

3. A home appliance company wants to estimate the population standard deviation of replacement time for a particular model of toaster oven. They conduct a simple random survey of 31 people who bought the oven and find a sample mean of 12.3 years with a standard deviation of 1.4 years. Construct a 99% confidence interval for the variance of replacement time for all toaster ovens sold of that model.

   3. ________________

4. Find the appropriate minimum sample size for the following:

a) You want to be 99% confident that the sample variance is within 20% of the population variance.

4a. ___________________

b) You want to be 95% confident that the sample standard deviation is within 20% of the population standard deviation.

4b. ___________________

c) You want to be 99% confident that the sample standard deviation is within 5% of the population standard deviation.

4c. ___________________

5. The amount in ounces of juice in eight randomly selected juice bottles are as follows:

15.1   15.8   16.2   15.6   15.4   15.6   15.2   15.3

Find a 95% confidence interval for the population standard deviation.

5. ___________________

Statistics Name: ____________________
Test – Chapter 6 Date: ____________________

Calculators allowed. You know the drill - show <u>all</u> supporting work for full credit. Do each problem with the method specified. If you use your calculator, write down what you entered. Be mindful of rounding. Lastly, if you cannot solve the problem with the given information state so.

1. Researchers from a national health organization are conducting a series of studies about the health of Americans. A simple random sample of 20 american men gave the following sample statistics for height: sample mean 70.2 in., sample standard deviation 3.0 in. Assume that men's heights are normally distributed.

   a) Construct a 90% confidence interval for the mean height of all american men. (Table)

   1a. ____________________

   b) Construct a 90% confidence interval for the population standard deviation.

   1b. ____________________

   c) The researchers have decided that their results are not precise enough. Using their initial results as a pilot study, find the minimum sample size required to estimate the population mean to within 0.4 in. with 95% confidence.

   1c. ____________________

   d) Find the minimum sample size to estimate the population standard deviation to within 10% of the actual value with 95% confidence.

   1d. ____________________

2. Find the critical value ____ that corresponds to a degree of confidence of 93% (Table)

2. ____________________

3. A random sample of 145 full grown lobsters had a mean weight of 17.01 oz. and a standard deviation of 4.02 oz. Construct a 90% confidence interval for the mean weight of all full grown lobsters. (Calc)

3. ____________________

4. The data below consists of the pulse rates in beats per minute of a sample of 36 students. Construct a 98% confidence interval for the population mean of all similar students. (Either method)

80	51	60	57	74	60	67	76	61	
66	71	71	93	87	79	92	69	72	Sum = 2642
89	73	74	77	75	72	80	84	66	
68	64	96	70	74	73	69	78	74	

4. ____________________

5. Given the following sample statistics taken from a simple random sample, construct a 99% confidence interval for the population mean. (Table)

$n = 100$, $\bar{x} = 63.39$, $s = 2.44$

5. ____________________

6. State the three conditions that must be met to use the student-t distribution to estimate a population mean.

6a. ____

6b. ____
6c. ____

7. Find the critical value ___ corresponding to a sample size of 12 and a degree of confidence of 98%.

7. ____________________

8. A football coach randomly selected 10 players from the team and timed how long each took to perform a particular drill. The times in seconds were:

5.10	11.04	11.97	12.29	9.22
12.14	6.68	13.30	8.31	9.59

Construct a 99% confidence interval for the mean time of all players on the team, assuming that times are normally distributed. (Either method)

9. ____________________

9. Given the sample statistics, determine which method for estimating the population mean is appropriate or that neither applies. You do not have to find critical values.

a. From a sample of 40 observations, $\bar{x} = 62.4$, s = 16.2. The population appears to be normally distributed.

9a. ____________________

b. From a sample of 28 observations, $\bar{x} = 18.9$. The population appears to be extremely skewed with ___ = 5.3.

9b. ____________________

c. From a sample of 40 observations, x = 6.8, s = 3.1. The population appears to be normally distributed.

9c. ____________________

d. From a sample of 115 observations, $\bar{x} = 45$, $s = 12$. The population appears to be extremely skewed.

9d. ______________

10. Of 124 adults randomly selected from one town, 19 of them smoke. Construct a 95% confidence interval for the true percentage of all adults in the town that smoke. (Table)

10. ______________

11. An advertising firm wishes to confirm Nielson ratings independantly for a particular show. Nielson estimates that 17% of homes were tuned into the show. How many homes should the firm include in their survey if they want a margin of error of no more than 5 percentage points with 99% confidence?

11. ______________

12. The *Concord Journal* wants to estimate how much support a particular upcoming ballot question has in the town. How many people should they survey if they want to estimate the population proportion to within 5 points with 90% confidence? No current estimate exists.

12. ______________

Statistics
Quiz – Sections 7-2, 7-3

Name: ______________________
Date: ______________________

Calculators allowed, but use the method indicated and show everything you type in when using the calc. Show all supporting work for full credit. Draw pictures where apppropriate to illustrate your methods. Be mindful of rounding and what the question is asking.

1. Find the test statistic and determine if it falls in the critical region or not for the given hypothesis and sample data. (Table)

   a) H_0: $\mu \geq 74.3$; $n = 45$, $\bar{x} = 74.0$, $s = 2.1$; Significance = 0.05

   1a. ___________________

   b) H_1: $\mu \neq 18.64$; $n = 60$, $\bar{x} = 18.70$, $s = 0.22$; Signficance = 0.10

   1b. ___________________

2. Find the P-value for the given hypothesis and sample data. (Table)

   a) H_0: $\mu = 1540$; $n = 100$, $\bar{x} = 1505$, $s = 185$

   2a. ___________________

   b) H_0: $\mu \geq 80.00$; $n = 85$, $\bar{x} = 79.00$, $s = 2.00$

   2b. ___________________

3. A Cruise Line claims that it scores on average greater than 7.8 on a particular customer satisfaction scale. To test this claim, an independent

organization performs a simple random survey of 50 customers of the cruise line and finds a sample mean of 8.1 with standard deviation 1.2. They want a level of significance of 0.05.

a) Determine the null hypothesis and alternative hypothesis.

3a. ____________________

b) Find the test statistic.

3b. ____________________

c) Does the test statistic fall in the critical region? (Show work!)

3c. ____________________

d) Write the final conclusion of the independent study.

4. *Highway Statistics* claimed that in 1995 the average passenger vehicle was driven at least 10700 miles. A random sample of 350 passenger vehicles had a mean of 10500 miles driven in 1995. Assume that the population standard deviation is 6000 miles and use a significance level of 0.02.

a) Determine the null hypothesis and alternative hypothesis.

3a. ____________________

b) Find the test statistic.

3b. ____________________

e) Find the P-value

3c. ____________________

f) Write the final conclusion of the test.

5. Health officials in Pizzaville have determined that the safe limit of pizza consumption is 4.3 pizzas per week. The officials want to determine if residents of Pizzaville consume on average no more than the safe limit.

A random sample of 45 citizens yielded a sample mean of 5.2 pizzas with a standard deviation of 2.1 pizzas. At a significance level of 0.01, what must the officials conclude based on their evidence? (Use a method of your choice, but show all work neatly – if I cannot find it, I cannot give credit for it)

6. A Brokerage Firm claims that its clients have an average percent yield of greater than 6.0%. A blind random sample of 50 clients of the firm showed a sample mean of 6.2% with a sample standard deviation of 0.6%. Using a level of significance of 0.05, what can be concluded about the firm's claim? (Use a method of your choice, but show all work neatly – if I cannot find it, I cannot give credit for it)

Statistics Name: ______________________
Quiz – Section 7-4 Date: ______________________

Calculators allowed, but use the method indicated and show everything you type in when using the calc. Show <u>all</u> supporting work for full credit. Draw pictures where appppropriate to illustrate your methods. Be mindful of rounding and what the question is asking.

1. Find the critical values for the given hypothesis, sample size, and significance level. (Table)

   c) H_0: $\mu \geq 84.2$
   n = 22
   Significance = 0.05

   1a. ____________________

   d) H_1: $\mu \neq 18.64$
   n = 12
   Signficance = 0.01

   1b. ____________________

2. You wish to test the claim that $\mu > 3.42$ at the 0.02 significance level. For a simple random sample of n = 18, the sample mean is 3.14 and the sample standard deviation is 0.84. Compute the value of the test statistic.

   2. ____________________

3. Compute the P-value for the given sample data and hypothesis.

   $\bar{x} = 15.7$
   s = 3.2
   n = 27
   H_0: $\mu \geq 16.8$

   3. ____________________

4. A test of sobriety involves measuring the subject's motor skills. Twenty-three randomly selected sober drivers take the test and produce a mean

score of 45.0 with a standard deviation of 4.1. At the 0.01 significance level, test the claim that the mean score for all sober subjects is equal to 38.5. (Traditional method with table)

a) Determine the null hypothesis and alternative hypothesis.

3a. ___________________

b) Find the test statistic.

3b. ___________________

g) Find the critical value(s).

3c. ___________________

h) Conclusion

5. A large software company give job applicants a test of programming ability. In the past the mean score has been 150. Thirty five randomly selected applicants from a particular large university are tested and produce a mean score of 169 with a standard deviation of 10. Use a significance level of 0.05 to test the claim that the sample comes from a population with a mean greater than 150 (Traditional method with table)

a) Determine the null hypothesis and alternative hypothesis.

3a. ___________________

b) Find the test statistic.

3b. ___________________

e) Find the critical value(s).

3c. ___________________

f) Conclusion:

6. A curious fasion reporter wants to test the claim that the average number of ties that men in a particular city own is greater than 3.0. 15 randomly selected men give a sample mean of 4.2 ties with a standard deviation of

1.1. Use the 0.02 significance level to test the claim. (P-value method - calc)

a) Determine the null hypothesis and alternative hypothesis.

3a. ____________________

b) Find the test statistic.

3b. ____________________

c) Find the P-value.

3c. ____________________

d) Conclusion:

7. A researcher wants to check the claim that convicted burglars spend an average of 18.7 months in jail. He takes a random sample of 7 such cases from court files and finds a sample mean of 21.6 months with a sample standard deviation of 5.8 months. Test the hypothesis at the 0.05 significance level. (P-value method – calc)

a) Determine the null hypothesis and alternative hypothesis.

3a. ____________________

b) Find the test statistic.

3b. ____________________

c) Find the P-value.

3c. ____________________

d) Conclusion:

Statistics
Quiz – Section 7.5

Name: ____________________
Date: ____________________

Calculators allowed. Show <u>all</u> supporting work for full credit. If I can't find what you did, I can't give credit for it!

1. In 1997, the national graduation rate (the percent of college students who complete a degree within 5 years) was 52.8%. In simple random sample of 120 student records from students who matriculated in 1998, 59 graduated within 5 years. At the .05 significance level, test the claim that the national graduation rate is less than it was in 1997. (Traditional method with table)

2. A local TV station claimed that "At least 70% of the town saw our show last night." In a simple random sample of 75 people, 43 reported watching at least part of the show. Test the TV station's claim at the .01 significance level. (Traditional method with table)

3. A recent Gallup poll reported that 33% of Americans frequently experience stress in their daily lives. Interested about the impact of having children on stress, a researcher polled 100 adults who did not have any children

under the age of 18 and found that 28 of them reported experiencing stress frequently. Is there enough evidence to suggest that not having any children reduces stress compared with the national average at a significance level of .02? (P-value method with the table)

4. A researcher working on college rankings wants to know what proportion of students at a particular University smoke. The University reports that no more than 5% of their students smoke, but the researcher wants to test this claim. In a simple random sample of 500 students, 31 said they smoked. Can the researcher reject the University's claim at the .05 significance level? (P-value method with the calc)

Statistics
Quiz – Section 7.6

Name: ______________________
Date: ______________________

Calculators allowed. Show all supporting work for full credit. If I can't find what you did, I can't give credit for it!

1. The Dandy Candy Company wants to see if a new chocolate bunny making process makes more consistent chocolate bunnies than their older machines. The older machines produced bunny ears with a standard deviation of 0.30". A simple random sample of 24 bunnies created with the process gave a sample standard deviation of 0.26". At the .05 significance level, test the claim that the new process produces bunnies with less standard deviation than the old process.

   H_0: ______________________ H_1: ______________________

   Test Statistic: ______________ Critical Value(s): ____________

   Conclusion: ____
   ____
   ____
   ____

2. A large appliance company estimates that the variance of the life of its appliances is 3.0 months. A consumer advocacy group wants to test this claim. The group took a simple random sample of 27 appliances and found a variance of 2.8. At the .01 significance level, what should the advocacy group conclude?

   H_0: ______________________ H_1: ______________________

   Test Statistic: ______________ Critical Value(s): ____________

   Conclusion: ____
   ____
   ____
   ____

3. A hospital claims that the standard deviation of wait time experienced by patients at its minor emergency depeartment is no more than 0.5 minutes. If a random sample of 25 wait times has a standard deviation of 0.7 minutes, what can be concluded about the hospital's claim if a significance level of .10 is used?

H_0: ______________________ H_1:______________________

Test Statistic:______________ Critical Value(s): ____________

Conclusion: ____

____

____

____

4. The amount in ounces of juice in 8 randomly selected bottles is as follows. With the sample data, test the claim that the bottles of juice come from a population with a variance of 0.35 at the .05 significance level.

15.5 15.8 16.1 15.6 15.4 15.5 15.2 15.3

H_0: ______________________ H_1:______________________

Test Statistic:______________ Critical Value(s): ____________

Conclusion: ____

____

____

____

Statistics
Test – Chapter 6

Name: ______________________
Date: ______________________

Calculators allowed. Show <u>all</u> supporting work for full credit. Be mindful of rounding. All samples are simple random samples drawn from normally distributed populations.

1. Find the critical values for each of the following.

   a. H_0: μ = 30.2
   n = 35
   Significance = 0.05

   1a. ___________________

   b. H_1: μ ≠143
   n = 16
   Significance = 0.01

   1b. ___________________

   c. H_0: SIGMA = 5.6
   n = 25
   Significance = 0.10

   1c. ___________________

   d. H_0: p ≤0.440
   n = 100
   Significance = 0.02

   1d. ___________________

2. Find the test statistic for each of the following.

   a. H_0: μ ≥16.0
   n = 10
   s = 3.2; $\bar{x}$ = 17.2

   2a. ___________________

   b. H_0: $SIGMA^2$ ≤4.4
   n = 71
   s^2 = 4.5

   2b. ___________________

   c. H_0: p = 0.55
   n = 50
   x = 29

   2c. ___________________

   d. H_0: μ = 1.60
   n = 45
   s = 0.35; $\bar{x}$ = 2.00

   2d. ___________________

3. The Dandy Candy Company wants to make a foray into the cola business. A possible producer claims that the mean caffeine content per 12 ounce bottle of their cola is 40 milligrams. A researcher from the DCC samples 35 bottles and finds a mean caffeine content of 39.2 millligrams with a standard deviation of 7.5 milligrams. At a level of significance of 0.01, what should the researcher conclude about the producer's claim? (Use the traditional method)

H_0: ____________________ H_1: ____________________

Test Statistic: ____________ Critical Value(s): ___________

Conclusion: ____
____
____
____

4. An automotive battery manufacturer gaurantees that the mean reserve capacity time of a certain battery is greater than 1.5 hours. A consumer watchdog group samples 50 batteries and finds a sample mean of 1.55 hours with a standard deviation of 0.32 hours. At a level of significance of 0.10, what can the watchdog group conclude? (Use the P Value method)

H_0: ____________________ H_1: ____________________

Test Statistic: ____________ P Value: ________________

Conclusion: ____
____
____
____

5. A computer repairer believes that the mean repair cost for damaged computers is more than $95. To test this claim, a researcher samples 7 computers and finds a mean repair cost of $100 with a standard deviation of $42.50. At a level of significance of 0.01, what can the researcher conclude? (Use the traditional method)

H_0: ____________________ H_1:_____________________

Test Statistic:_____________ Critical Value(s): ___________

Conclusion: ____
____
____
____

6. The dean of a university estimates that the mean number of classroom hours per week for full-time faculty is 11.0. The student council wants to test this claim, so they sample 8 faculty members and find the following data (in hours per week):

11.8 8.6 12.6 7.9 6.4 10.4 13.6 9.1

At a level of significance of 0.05, what can the student council conclude about the dean's claim? (Use the P Value method)

H_0: ____________________ H_1:_____________________

Test Statistic:_____________ P Value: _________________

Conclusion: ____
____
____
____

7. The Pew Research Center claims that no more than 38% of American adults regularly watch a cable news broadcast. In a random sample of 75 Americans, 33 responded that they did regularly watch a cable news broadcast. At a level of significance of 0.05, what can be concluded about the Pew Center's claim? (Use the traditional method)

H_0: ____________________ H_1: ____________________

Test Statistic: ____________ Critical Value(s): ____________

Conclusion: ____

____

____

____

8. A state school administrator claims that the standard deviation of test scores for eighth-grade students who took a life-science assesment test is less than 30. A subordinate of the administrator is assigned the task of testing this claim. 10 scores are randomly selected and the standard deviation is found to be 28.8. At a level of significance of 0.01, what should the subordinate conclude about the administrator's claim?

H_0: ____________________ H_1: ____________________

Test Statistic: ____________ Critical Value(s): ____________

Conclusion: ____

____

____

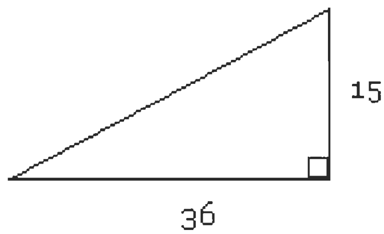
____

CP2 Geometry
Pythagorean Triples
Special Right Triangles
Quiz

Score: ________

Name: ________________
Date: ________________

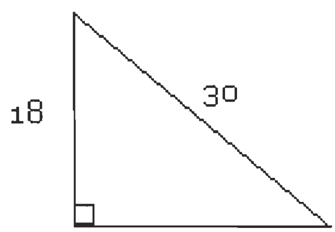
1.



a) Which family of Pythagorean triples does this triangle belong to?

b) What's the length of the missing side?

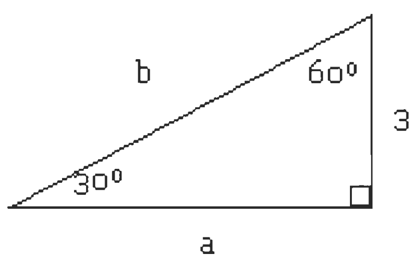
2.



a) Which family of Pythagorean triples does this triangle belong to?

b) What's the length of the missing side?

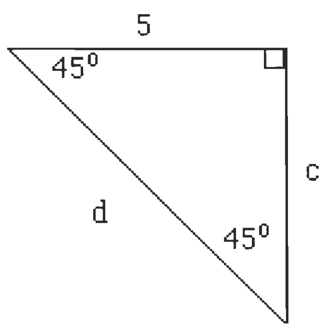
3.



a = ________

b = ________

4.



c = ________

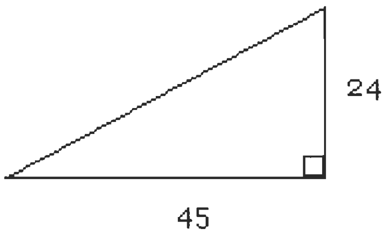
d = ________

CP2 Geometry
Pythagorean Triples
Special Right Triangles
Quiz

Score: ________

Name: ________________
Date: ________________

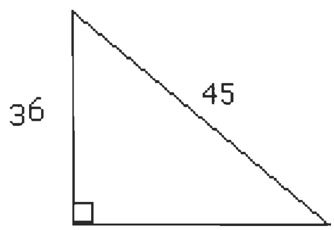
1.



a) Which family of Pythagorean triples does this triangle belong to?

b) What's the length of the missing side?

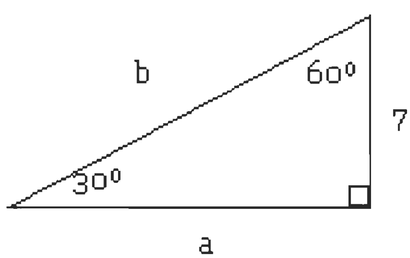
2.



a) Which family of Pythagorean triples does this triangle belong to?

b) What's the length of the missing side?

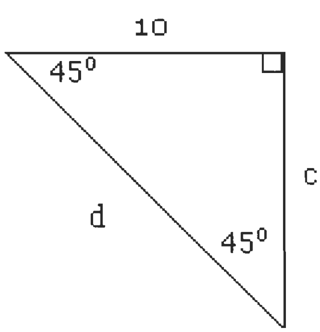
3.



a = ________

b = ________

4.



c = ________

d = ________

CP2 Geometry
Chapter 8 Test

Score: ______

Name: ________________
Date: ________________

1. Simplify the ratios

a) $\frac{18x^2y}{12xy^2}$

b) $\frac{18xy}{10xy}$

2. Evaluate the proportions for the unknown variable

a) $\frac{7}{12} = \frac{9}{x}$

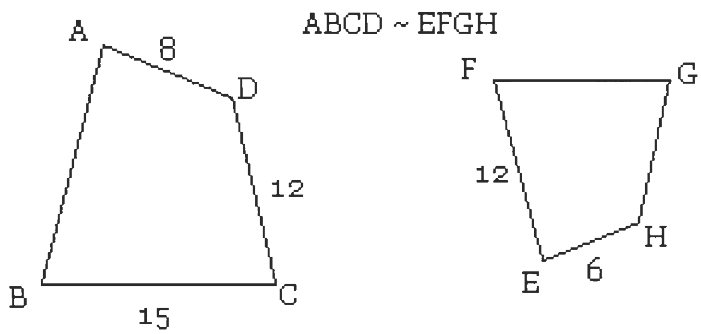
b) $\frac{x}{12} = \frac{10}{8}$

c) $\frac{(x+4)}{(x-6)} = \frac{10}{5}$

d) $\frac{(x+6)}{10} = \frac{(x+4)}{6}$

3. Find the unknown sides

ABCD ~ EFGH



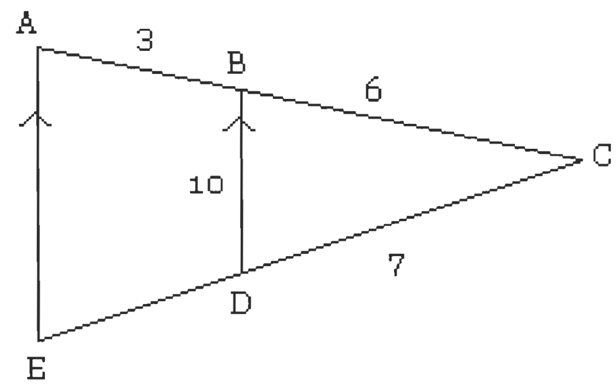
a) AB = ______
b) FG = ______
c) GH = ______

d) What is the dilation ratio of ABCD to EFGH? ______

4. Find the geometric means of 6 and 24

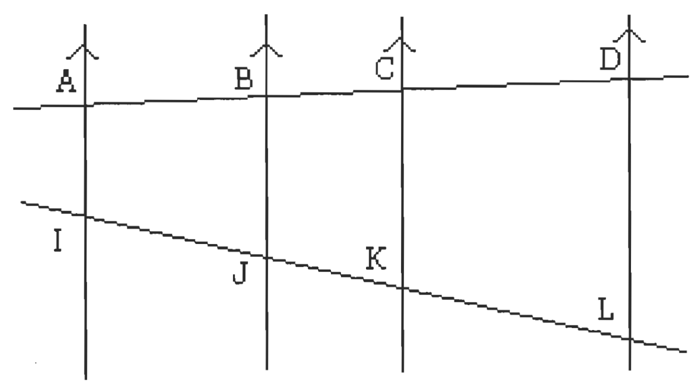
CP2 Geometry
Chapter 8 Test

5.



Find AE and ED AE = _______ ED = _______

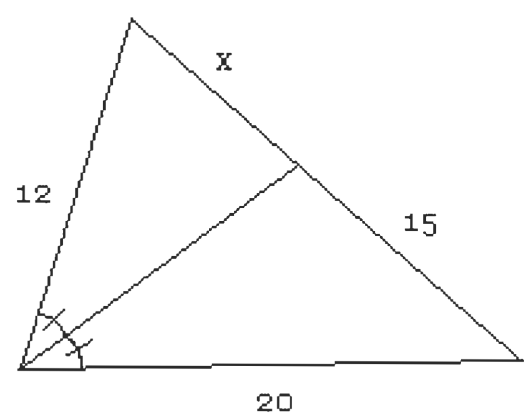
6.



AB = 5
BC = 3
CD = 7
IL = 20

IJ = _______
JK = _______
KL = _______

7.



Find x

x = _______

CP2 Geometry
Chapter 8 Test

7. Find the ratio of x to y

a) $10x = 12y$

d) $\frac{(x+6)}{9} = \frac{(y+4)}{6}$

8. A person is standing next to a 12' tall lamppost. The person is standing 4' away and casting a 3' shadow away from the lamppost. How tall is the person?

9. a) Mr. Case has a model train set. The scale of the train is 1:60. If the actual train is 35 feet long, how long is Mr. Case's model?

b) A separate model car in the same set (with the same scale) is 6" long. How long is the actual train car?

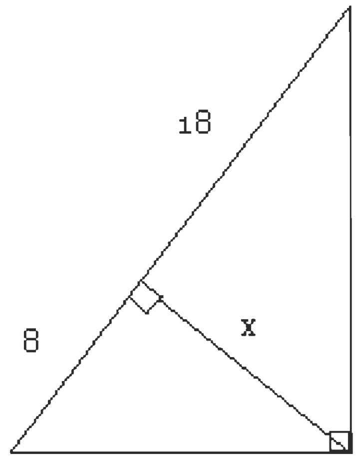
10. What's the difference between a *ratio* and a *proportion*?

CP2 Geometry
Chapter 9 Test

Score: ________

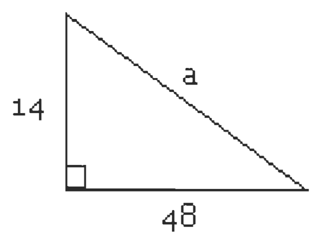
Name: ________________
Date: ________________

1. Find x



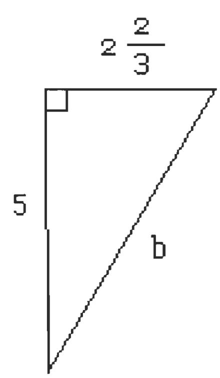
1. ________

2. Find a



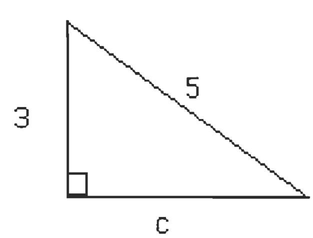
2. ________

3. Find b



3. ________

4. Find c



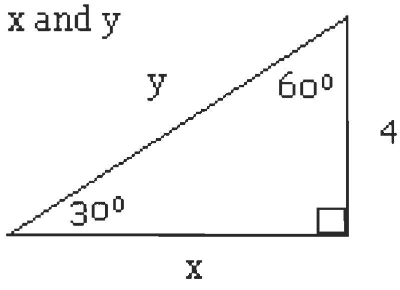
4. ________

5. List as many families of Pythagorean triples as you can

6. Mr. Case starts at point A. He drives 15 miles east to point B, then 18 miles north to point C, then 12 miles east to point D. How far is he from his starting location? Draw a diagram.

6. ________

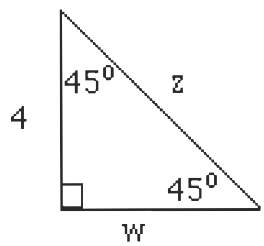
7. Find x and y



7. x = ________
y = ________

8. In a 30-60-90 triangle, what is the ratio of the sides? (In terms of x - you can draw a picture if you want)

9. Find w and z

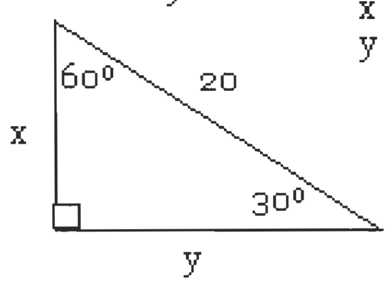


9. w = __________

z = __________

10. In a 45-45-90 triangle, what is the ratio of the sides? (In terms of x - you can draw a picture if you want)

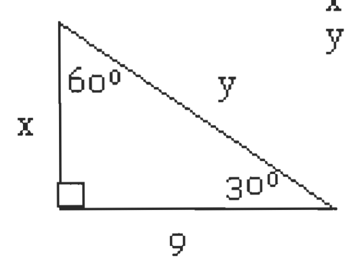
11. Find x and y



x = _______

y = _______

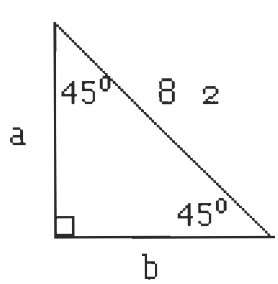
12. Find x and y



x = _______

y = _______

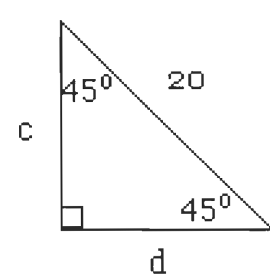
13. Find a and b



a = _______

b = _______

14. Find c and d



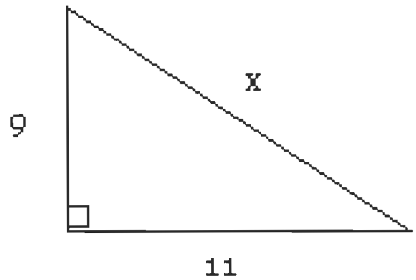
c = _______

d = _______

15. To the nearest thousandth or in reduced radical form...

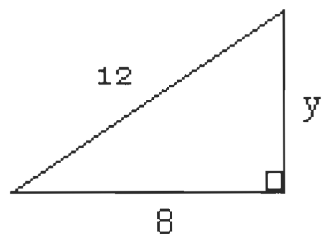
a) Find x

15a. ________



b) Find y

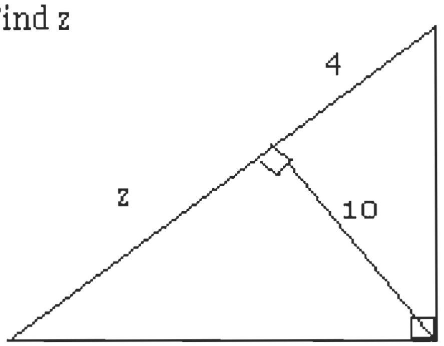
15b. ________



16. What theorem did you use to solve #15?

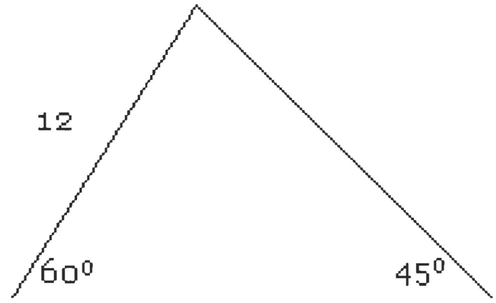
17. Find z

17. ________



18. Find the perimeter

18. ________

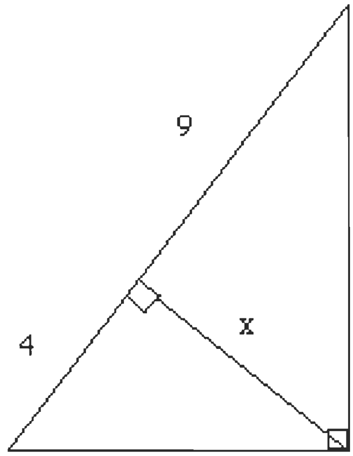


CP2 Geometry
Chapter 9 Test

Score: ________

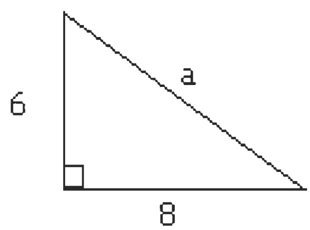
Name: ________________
Date: ________________

1. Find x



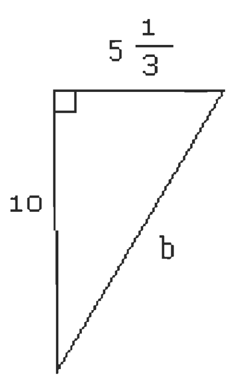
1. ________

2. Find a



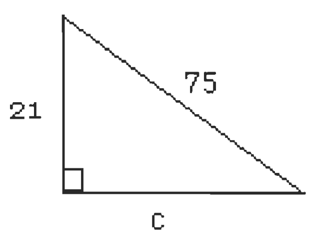
2. ________

3. Find b



3. ________

4. Find c



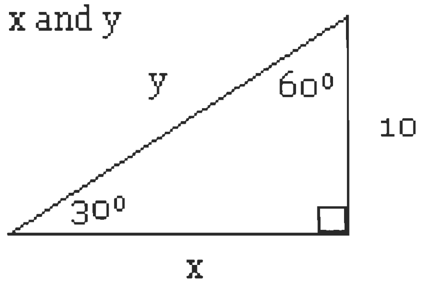
4. ________

5. List as many families of Pythagorean triples as you can

6. Mr. Case starts at point A. He drives 10 miles east to point B, then 19 miles north to point C, then 15 miles east to point D. How far is he from his starting location? Draw a diagram.

6. ________

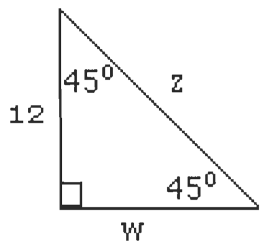
7. Find x and y



7. x = ________
y = ________

8. In a 30-60-90 triangle, what is the ratio of the sides? (In terms of x - you can draw a picture if you want)

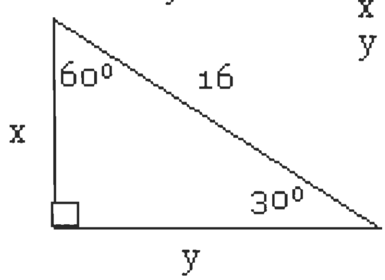
9. Find w and z



9. w = ________
z = ________

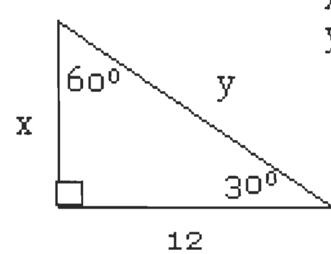
10. In a 45-45-90 triangle, what is the ratio of the sides? (In terms of x - you can draw a picture if you want)

11. Find x and y



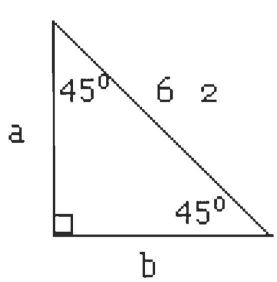
x = ______
y = ______

12. Find x and y



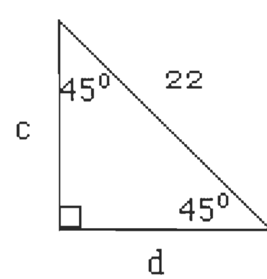
x = ______
y = ______

13. Find a and b



a = ______
b = ______

14. Find c and d

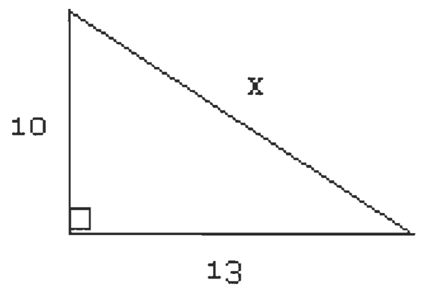


c = ______
d = ______

15. To the nearest thousandth or in reduced radical form...

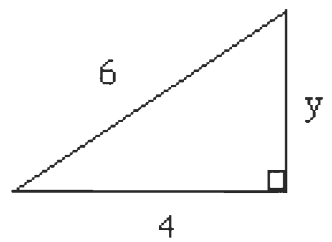
a) Find x

15a. ________



b) Find y

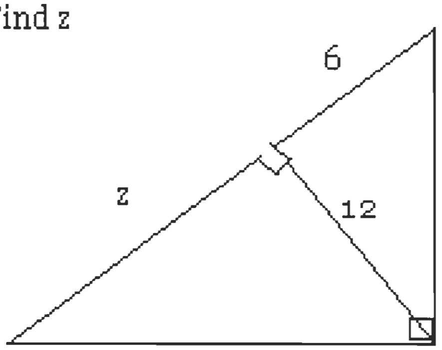
15b. ________



16. What theorem did you use to solve #15?

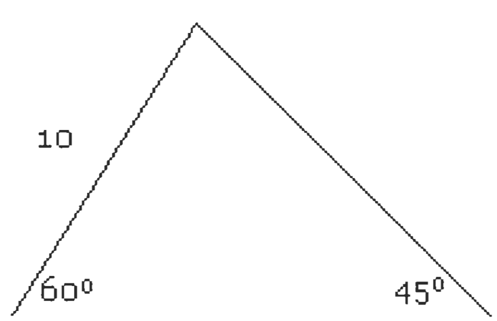
17. Find z

17. ________



18. Find the perimeter

18. ________



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[4] Concord School District, *Student Profile – Concord-Carlisle High School*, http://www.colonial.net/district/stu_profile.php (2004)

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