



STUDENT PREPAREDNESS IN COLLEGE MATHEMATICS

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JANUARY 1, 2015
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Abstract

Educators everywhere are always wondering – is the work we are doing here preparing our students for the next step? Is what we teach in high school relevant to what they need for college? With all of the educational reforms that have been taking place in the past twenty years, it is important to examine how students' preparedness for college math has changed.

This report explores student preparedness for college in math through many different mediums. From faculty perceptions over the course of their careers, to students thoughts on how well they felt high school prepared them for what they are going through now, and what the numbers have to say about it all. WPI has a fast-paced curriculum, so students who are ill prepared often fall behind. What causes those things to happen and how can we prevent it in the future?

This paper shows that students are just as prepared as they were five years ago, but that still is not enough. Through some analysis of the pre-calculus and calculus background students have before attending college, it is hard to say that recent improvements are impacting students enough to show significant growth over the last five years. Basic skills are essential to advancement in education, but are often overlooked to accommodate time for standardized testing and AP preparation.

Chapter 1: Background

Mathematics education in Massachusetts has been changing over the course of the last twenty years. From the Education Reform Act of 1993 to the rising numbers of students taking Advanced Placement (AP) courses and changes in curriculum frameworks, a great deal has been done to improve the standards of mathematics education which we have come to know as acceptable. These reforms have led to a larger focus on standardized testing, which has had an impact on the ways students learn and the ways educators teach.

Massachusetts Education Reform Act of 1993

The Education Reform Act was developed in 1993 as a seven-year plan to improve education in Massachusetts. The act has increased financial assistance on the state level dramatically with the goal to have equal state and local shares. Other reforms include implementing school councils, providing opportunities for educators to continue their education, adding authority for principals, defining school committee roles better, and creating measurable and consistent standards for students and schools throughout the state.

The progress made by the Education Reform Act can be seen in all facets of its efforts. A foundation budget was developed bring all schools to the same level of spending and grants are being distributed to enhance technology in school districts. Teaching requirements have changed to include passing a test in their subject area and another in communication and literacy skills. Previously, the only statewide educational requirements were for history and physical education. Now the state has standards for all core subjects. The Massachusetts Comprehensive Assessment System, or MCAS, was developed as a standardized statewide examination to determine which

students and schools need the most aid in education. Passing this examination has also been made a requirement for high school graduation.

An article in the New York Times commends Massachusetts on its efforts, stating “behind Massachusetts’ raw numbers are two decades of sustained efforts to lift science and mathematics education. Educators and officials chose a course and held to it, even when the early results were deeply disappointing.” It has taken some time to see the lasting effects of the reform act, but studies are coming to light now. However, there is still debate over what led to its success. Was it the additional money, the testing holding administrators accountable or the clearer standards? Are all of these aspects leading to positive growth?

Advanced Placement

Advanced Placement courses were implemented into high school curriculums so that advanced students could take college level courses while still in high school. Students would be taught higher level material and could receive college credit by taking a standard final examination. When this program was first developed, only the highest-ranked, most qualified students took part in the program. Now that College Board is pushing for higher participation numbers, the quality of the program is at stake.

Since College Board began the Advanced Placement (AP) program in 1953, significant changes have taken place. Earlier studies show that students who take AP courses in high school are better prepared for college courses. Those receiving college credit (with an AP score of 3, 4, or 5) and starting in the second course of the corresponding topic in college are shown to do as well, if not better, than students who had taken the first course in that topic. This shows that at the time the research was done (in the 80s and early 90s) AP courses and exams were equivalent to introductory college courses. Looking at more recent research, another story can be told. The

College Board has been increasing participation in the AP Program in American high schools, to the point that it has increased more than two-fold every decade. Such participation rates have brought critics to question whether high standards of these tests and their qualifications for college credit are still valid.

In 2000, William Lichten from Yale University reviewed the AP Program to see if College Board claims about qualification were accurate. He began by evaluating the scoring rubric and how it correlates to placement for college courses. The scoring rubric was developed to determine whether or not colleges would allow the student to start in an advanced course. The following table shows the AP scores, their interpretations, and approximate college grade equivalents.

| AP Score | Interpretation | Approx. Grade Equivalent |
|----------|--------------------------|--------------------------|
| 5 | Extremely Well Qualified | A |
| 4 | Well Qualified | B |
| 3 | Qualified | C |
| 2 | Possibly Qualified | - |
| 1 | No Recommendation | - |

Along with this grading rubric, in 1999, the College Board made the claim that,

“Almost two-thirds of the students achieved grades of 3 or above on AP’s 5-point scale – sufficiently high to qualify for credit and/or enrollment in advanced courses at virtually all four-year colleges and universities, including the most selective.”

This claim is consistent with previous research, such as the studies that were previously mentioned, but Lichten’s more recent research states

“It is an open secret...that both this claim and scale [as in the above table] disagree with college standards. This disparity is a sign of remarkably poor communication between the colleges and the College Board.”

As times have changed, the College Board has not changed their grading scale or the rubrics associated with it. Colleges and universities have changed their grading systems, so the claim can be made that the College Board should make adjustments as well. In the 1950s, when the AP Program began, a B or a C in a college course was considered satisfactory. Today, a C is unsatisfactory and a B is satisfactory. Since colleges are changing what they view to be acceptable amounts of knowledge for courses, AP scores should change along with them. Colleges only provide course credit for AP scores of 3, 4, or 5, with most only approving 4s and 5s. This means that only these scores really match college level of knowledge.

Lichten’s study shows that only 22% of students with an AP score of 3 took an advanced course in that subject area even though 61% placed out of the remedial course. For students who received a score of 3 or below, this means that AP can be seen as more of a placement exam, rather than an *advanced* placement exam.

The Education Trust distributed a survey to students from Kindergarten through college in 1999 that showed the gap between high school and college. Of the three quarters of U.S. high school graduates who attend college, about half take a remedial course, one third do not make it into sophomore year, and less than half actually graduate from college. The Trust states, it “doesn’t make any sense” that college level, or AP courses, are the growing the quickest in high schools, while remedial courses have the largest growth amongst college courses.

With this knowledge, it is hard to justify giving incoming college students credit for their AP scores when they may not have achieved sufficient knowledge in the subject area to skip an introductory course or excel through a program. The lack of communication with colleges and

universities diminishes the credibility for AP exams and the results obtained by taking AP courses in high school. So much focus has been placed on the AP Program over the last 50 years, yet it appears that more basic level skills are truly the place where growth needs to occur for students to be successful in college.

Impacts of High-Stakes Testing

In recent years, many states have begun to implement high-stakes testing in their school systems. The scores on these tests largely impact schools and students in a variety of ways including serving as a requirement for graduation, determining school standings and teacher effectiveness, and establishing budgets for individual schools. These tests are often promoted as diagnostic tools to identify students' strengths and weaknesses in various subjects, however they are hardly ever used for that. Scores are often reported in the summer, after students have already finished school. There are not enough questions on each individual topic to determine whether or not a student has achieved competency in that area. For these reasons, along with others, high-stakes testing scores are usually just used for rewards and sanctions. Twenty-two states give incentives for high or improved test scores and twenty states give financial awards to successful school. Even worse, forty-five states punish schools or districts based on poor test results. One of the major issues with the method of sanctions and rewards can be described by the Uncertainty Principle, described below.

“The more important that any quantitative social indicator becomes in social decision-making, the more likely it will be to distort and corrupt the social process it is intended to monitor.”

This principle implies that using scores from high-stakes testing to evaluate schools and educators is not practical or effective. The more these examinations are developed and enforced, the less valid they become. The corruption is brought to light by the news, research studies, and

stories told by teachers throughout the country. Administrators and teachers cheat by excluding low-performing students, changing students' answers, giving extra time, and more. Students are more tempted to cheat because of high stakes – they fear the failure that could come if they do not do well. Students are being encouraged to skip school the day of the testing, enter GED programs, or drop out of school all together so that their scores are not counted against schools or districts. This leads to misrepresented high school completion rates, dropout rates, and achievement rates. Minorities and students with low socioeconomic status are impacted negatively by these tests. They are not receiving the proper education that is needed for them to succeed in school or in society.

With so much focus on rewards and repercussions from test results, some teachers are beginning to teach to the test, rather than teach all of the necessary material for the course in order to remain in good standing in their school systems. By focusing significant amounts of class time on how to test, instead of the curriculum, the results of the tests cannot truly be seen as valid. Since the tests are seen as such high importance, the curriculum has become narrower to accommodate. This means important information is not being taught, but test-taking skills are. Since the curriculum is narrowing and so much time is focused on getting high test scores, teacher morale is dropping. Not all teachers support the tests or the results they create. Teachers lost their sense of freedom to teach what they like in fear of their students receiving low scores. Too much pressure is coming from the students' performance evaluation and the states' responses.

Mass Math + Science Initiative (MMSI) was introduced in Massachusetts to gain more interest in the AP program. It was developed to help develop “a talented workforce of science, technology, engineering, and math professionals.” The program helps pay for students to take AP

exams as well as provides professional development and Saturday student study sessions to better prepare students and teachers for the test. Incentives are given to students who achieve passing scores and their teachers. This program is increasing AP performance, but at the same time it is adding more pressure and monetary value to test-taking. The program encourages teachers to teach to the test to obtain results that benefit themselves, their school, and their students.

It is hard to tell if what these tests are reporting is accurate. Who decides what “proficient” means? Is it the same across the board? Do schools with high state test scores score as well on federally administered tests? Not necessarily – those schools might just have better test preparation or exclude more students. Score reporting errors are occurring across the country, completely wiping out the hope that these tests are giving useful information in those areas.

With such uncertainty relating to the validity of high-stakes testing, it is hard to support the idea of it ruling school systems throughout the country. The benefits do not seem to outweigh the consequences. Research has even shown that states that have implemented high-stakes testing have done worse on other measures of academic achievements than states with no/low-stakes testing programs. The move towards high-stakes testing has weakened our education system and we are beginning to see the repercussions.

Chapter 2: The Background of WPI Math Students

WPI Mathematics Courses and Requirements

At WPI, students are expected to have a wide knowledge of science, math, and even humanities. For all majors, at least some calculus is required. Students start out in the calculus sequence which consists of four courses – MA1021 (Calculus I), MA1022 (Calculus II), MA1023 (Calculus III), and MA1024 (Calculus IV). Students who need to start at a slower pace may begin in MA1020, a semester-long calculus I course. Incoming students may start in various stages of the calculus sequence, depending on the amount of knowledge they have coming from high school. Typically, if a student has taken AP Calculus AB, and achieved a qualifying score, they will start in MA1023. Students who have a qualifying score in AP Calculus BC usually start in MA1024. If a student has no previous calculus background they will begin in MA1021 or MA1020. Many students decide to start a level behind for a refresher and introduction to college math courses. A math placement test was developed to help students determine which course they should begin with, but that will be addressed later in this paper.

Changes in First Mathematics Course Enrollment

Over the course of the last five years, there has been a shift in enrollment for the various levels of the calculus sequence. Less students are beginning in MA1021 and the numbers for upper level calculus courses are growing. The table below gives the number of students enrolled in MA1021 and MA1023 in the first term from 2010 to 2014.

| Year | Students in MA1021 | Students in MA1023 | Total New Students |
|------|--------------------|--------------------|--------------------|
| 2010 | 305 | 34% | 380 |
| 2011 | 317 | 32% | 412 |
| 2012 | 283 | 30% | 381 |
| 2013 | 284 | 26% | 492 |
| 2014 | 305 | 29% | 464 |

The shift of incoming students to higher levels of calculus courses could occur for a few reasons. The first, and most ideal, being that students are coming in to college better prepared in math than before. Each year students are developing more and learning more, which sets them apart from the students who came before them. Another reason for the shift could be changes in admission requirements. If admissions is growing stricter based on the number of applicants, then the students accepted to WPI would be held to a higher standard. Applicants would need to impress admissions with their previous math experience, so once accepted, that population would be starting in higher level courses. The last reason I will discuss is the growing numbers of students taking Advanced Placement courses. The College Board has been growing its numbers in schools throughout the country. With more students taking these exams, there are more incoming college students who have those prerequisites under their belts. This does not mean, however, that these students are better prepared than those who came before them. It may only mean that they think they are more prepared and therefore sign up for higher level courses.

Student Cohorts

It could be said that there is a divide between WPI students who begin in MA1021 or 1022 and those that begin in MA1023 and 1024. With different levels of background coursework, these students enter WPI with varying math backgrounds, study skills, and motivation levels, to name a few. The two cohorts have can be defined by their first course at WPI, but also by their progression from that. We will explore whether this divide is growing over time or shrinking, the affects it has on students as they continue to grow their math background, and how much of a role Advanced Placement has in this issue.

Chapter 3: Faculty Perceptions

Faculty members have a special insight into their students and how they have changed over recent years. Seeing different classes and levels of talent, they are able to make assumptions about student preparedness and see trends that might not be apparent to all. The math faculty at WPI was surveyed to gain feedback on how students have changed in levels of preparedness in math over the span of their years teaching as well as their opinions on the divide between Calculus I/II and Calculus III/IV students.

The faculty members surveyed ranged from zero to 20+ years of teaching experience at WPI. They were asked to compare today's students with those that they taught ten years ago by ranking a series of statements about student preparedness. The statements included levels of preparedness, as well as prerequisite knowledge of trigonometry, algebra, geometry, and logarithms. The students were broken down into two cohorts by their first calculus course at WPI. Cohort 1 is made up of the students who began in MA1020, MA1021, or MA1022. Cohort 2 is the students who first took MA1023 or MA1024. The faculty members ranked the cohorts separately so that comparisons could be made.

One bold professor claimed that students have improved preparedness in every category in the past ten years. He believes that there is not a divide between the students who start in different levels of calculus. He seems to be an outlier from the rest of the data and has been removed for further analysis, though the point he has made is still relevant.

Changes Over Ten Years

The faculty was asked to rank the statements on a scale from 1 to 5, with 1 meaning students less prepared, 3 meaning they are the same, and 5 meaning they are more prepared than students from ten years ago. Overall, the faculty believes that students are less prepared than ten

years ago, but not by much. When the faculty ranked the students on overall preparedness, their score was a 2.32 – only slightly under a 3.0 rating. Trigonometry and geometry seem to have declined the most, with scores below 2.0, while algebra and logarithms sit closer to 2.5. The table below shows the numbers explicitly.

| Trigonometry | Algebra | Geometry | Logarithms | Overall |
|--------------|---------|----------|------------|---------|
| 1.96 | 2.46 | 1.96 | 2.32 | 2.32 |

The variation in these numbers is not large enough to determine whether or not students are more prepared for college math than they were ten years ago, but some evidence is there.

Initial Math Course Cohort Analysis

The faculty rated preparedness levels in the second cohort (those students starting in MA1023 or 1024) slightly higher than the first cohort (students starting in MA1020, 1021, or 1022). Across the board, each category has an average score of about 0.5 higher for cohort 2 than for cohort 1. Remember, though, that this survey is not ranking the students’ math ability – it is ranking how the current students in this category compare to those in the same category ten years ago. This means that the faculty see the more advanced cohort keeping up with the students from ten years ago better than the lower-level cohort. The first cohort seems to be slipping behind further and faster than the second. As time has gone by, what has changed in preparation for cohort 1 that has not affected cohort 2 nearly as much? Will this divide continue to grow and will students fall further and further behind those that came before them?

The Advanced Placement students were given just about the same preparedness ratings as the second cohort. This is probably because most of the students overlap in these categories. However, the faculty has rated AP students’ preparedness in trigonometry and geometry slightly higher than cohort 2. AP may have been able to keep these students on track with their

trigonometry and geometry skills. However, this brings back the point of the decline of the AP program. The AP students should be improving as the program develops, not falling behind. The College Board needs to look into their preparation and how it has changed over the last ten years, since these faculty members are seeing some decline.

Faculty Advice

The faculty who completed the survey had an opportunity to give their feedback on what creates the divide between the types of students and come up with solutions to minimize the gap in the future. Some focused on issues with high school education, while others wrote of the transition to college.

Many faculty members focused on the idea that more **theory** needs to be taught in schools. Students are so used to following examples and just replicating what they have seen before, rather than developing their own solutions to problems. Their basic skills are lacking, and such focus on standardized testing is not helping. Students rely on calculators to solve simple problems, rather than learning trigonometry or other topics on their own. Teachers sometimes push students ahead to get through material, rather than slowing down and giving students time to achieve mastery in each area. One faculty member suggested a basic skills review for new students to get to the level they need to be at before taking college level calculus.

According to the faculty, basic skills, such as algebra and trigonometry, are the areas that need to improve the most. The focus on standardized testing has forced teachers to brush over important skills to get to the end goal. College professors are having to re-teach students topics that they should have learned in high school in order to be able to complete the lessons and get through the material that they have planned. The fast-paced nature of WPI does not allow time

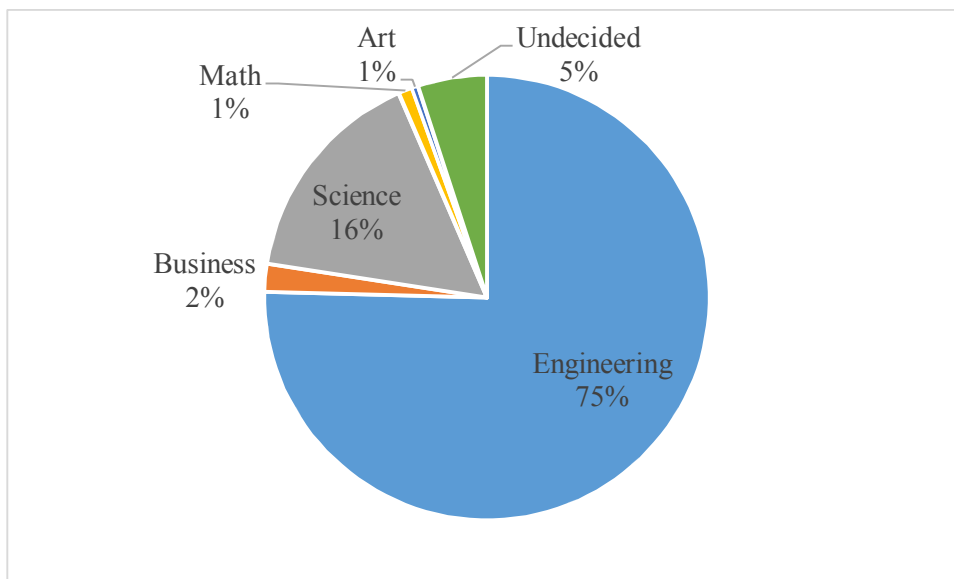
for review of basic skills, so faculty members are relying on high schools to have already taught the students that material.

Chapter 4: Student Perceptions

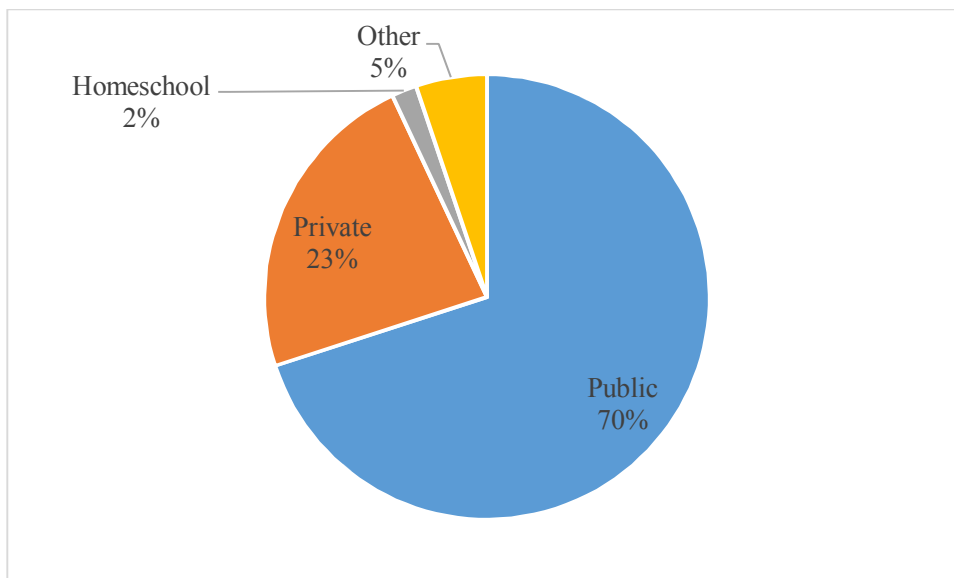
Students have their own perceptions of their preparedness for college math courses. They are able to reflect on their experiences in college courses and figure out how high school could have prepared them better. Students were surveyed in their math classes during B term to figure out how prepared they felt in various topics related to calculus from their high school education. The participants were from MA1022 and MA1024 courses that ran that term. This ensured that the two cohorts could be examined for their own feelings of preparedness.

Student Background

The surveyed students came from a variety of backgrounds even though they were surveyed in just two of the math courses offered at WPI. Most students were from the class of 2018, but surprisingly about 5% of the survey respondents were upperclassmen taking the basic calculus courses. The students had varying majors, with engineering being most popular at 75%. The chart below gives the complete breakdown.



The students came from public schools, private schools, home schools, and even other types, as shown in the chart below.



Most students come into WPI having taken AP Calculus AB as their highest math course, followed by honors calculus, then AP Calculus BC, and pre-calculus. 70% of students entering MA1023 had AP Calculus AB background. It is a little surprising that most of the students entering MA1021 (40%) had AP Calculus AB background since the same material is covered in MA1021 and MA1022 as in that course. Questions about the math placement test give some insight into this situation.

The students were asked if they have taken the math placement test that was offered before coming to WPI. The test is a pretty good judge of previous math skills and accurately placed 78% of the surveyed students who took it. The students who did not take the test came up with a variety of excuses, from thinking the test was too much work to not feeling prepared enough to take it or even forgetting that it existed, responses were all over the board. The most meaningful, though, were the most popular answers. 43% of the students stated that they already knew what they wanted to take, so the test was not necessary. The students that started in MA1021 either had never taken calculus before, did not feel strong enough in their abilities, or wanted a review of some of the calculus they learned in high school before jumping into college

calculus. This explains why so many AP Calculus AB students started in MA1021. The second most popular answer (from 21% of respondents) was that they had AP credit, so they knew where to start. These would be the AP Calculus students who got qualifying scores on their exams. Their scores allowed them to skip the introductory courses and go into MA1023. The AP Calculus BC students had a combination of these reasons for entering MA1023. The students taking the survey had mixed feelings about their preparedness from high school.

Preparedness from High School

The students were asked to rank the following topics on how well they think they were prepared for them from high school: trigonometry, algebra, analytical geometry, and logarithms. The scale is a 1 to 5 rating, with 5 being the most prepared. Overall, the students seemed fairly confident in their abilities from high school. Logarithms received the lowest average score (3.49), followed by analytical geometry (3.83), trigonometry (3.90), and algebra with the highest score of 4.59.

When divided into subgroups, comparisons can be made. Public and private school students had very similar scores. The average difference between the two was only 0.04. The students could also be divided by the highest level of math they took in high school. The pre-calculus students had the lowest scores in every category, though they tied with AP Calculus BC students in geometry. The AP Calculus AB students had the highest scores in algebra and geometry, while the BC students felt more prepared in trigonometry and logarithms than them. The table below shows the preparedness scores of the three groups in each of the topics.

| | Trigonometry | Algebra | Geometry | Logarithms |
|---------------------|--------------|---------|----------|------------|
| Pre-Calculus | 3.65 | 4.46 | 3.73 | 3.23 |
| AP Calc AB | 3.91 | 4.63 | 3.86 | 3.46 |
| AP Calc BC | 4.03 | 4.47 | 3.73 | 3.8 |

It makes sense that the AP students felt more prepared in trigonometry and algebra since they used those skills in more diverse ways throughout high school. The AP Calculus BC students might feel less prepared in geometry because they have not taken courses that use it in a while. The differences between these groups are not very large, but they are larger than those of the public versus private high school students.

Besides ranking their preparedness for the above topics, the students were asked what areas from high school would have helped them to be more successful in calculus or Differential Equations at WPI if they had been more prepared in them. The students gave a variety of answers, including some from the chart above. Fifty students claimed that a better background in trigonometry would have been beneficial and forty-three students said it was knowledge of logarithms. This data is consistent with the scores achieved from the previous question. Many students also pointed to basic calculus skills such as differentiation and integration. Only twenty students blamed geometry, though many students said they were not very prepared in it in the previous question. Other areas for improvement included algebra skills, graphing shapes and functions, solving problems without calculators, and learning more theory or having harder problems to solve.

Cohort Analysis

The data could also be analyzed as the two cohorts that have been mentioned previously based on first calculus courses at WPI. The table below shows the average scores that students gave to the four topics of interest.

| | Trigonometry | Algebra | Geometry | Logarithms |
|----------|--------------|---------|----------|------------|
| Cohort 1 | 3.70 | 4.55 | 3.77 | 3.27 |
| Cohort 2 | 4.10 | 4.64 | 3.88 | 3.70 |

Across the board, the second cohort of students felt more prepared than the first cohort from high school. Geometry and algebra skills are not too different between the cohorts, but the largest gaps are in logarithms and trigonometry – the two problem areas that keep coming up. The first cohort has had less practice with the applications of trigonometry and logarithms, so seeing them in their introductory calculus class might be a bit of a shock. The second cohort feels more prepared, but a score of 3.70 in logarithms is not that strong. Both groups need to improve their knowledge of logarithms, which means that high schools need to be helping their students to be more prepared in that area before they go to college. That material will not go away – they will need it for calculus, Differential Equations, and even higher math and engineering courses.

Chapter 5: Math Placement Test Data

Background

The math placement test was developed about ten years ago by Bill Farr as a tool to create recommendations for the first math course a student should take at WPI. It is a comprehensive forty-three question test that students complete through the WeBWorK testing system. There are four algebra problems, four trigonometry problems, four geometry problems, and four function problems that make up a sixteen question pre-calculus section. Eight problems cover differential calculus (MA1021 material) and nine cover integral calculus (MA1022 material). Ten MA1023 questions were added in 2014 so that students could be placed in MA1024 as their first course. Students receive one point for each correct answer and their total scores in each category determine which course they will be recommended to take. The placement test has been able to improve NR rates throughout all calculus courses at WPI.

Change Over Five Years

Based on faculty perceptions, students are either less prepared in math than before or just about the same. The placement data can add to this analysis so that it is not just relying on subjective perceptions. The table below gives the average scores for each subcategory over the past five years.

| Year | Algebra | Trigonometry | Geometry | Functions | MA1021 | MA1022 | MA1023 |
|--------------|---------|--------------|----------|-----------|--------|--------|--------|
| 2009 | 3.6 | 3.0 | 2.9 | 3.4 | 5.6 | 5.8 | N/A |
| 2010 | 3.7 | 3.0 | 3.1 | 3.5 | 5.7 | 5.9 | N/A |
| 2011 | 3.6 | 3.1 | 3.1 | 3.5 | 5.8 | 6.1 | N/A |
| 2012 | 3.6 | 2.9 | 3.1 | 3.5 | 5.9 | 6.3 | N/A |
| 2013 | 3.7 | 3.1 | 3.2 | 3.5 | 6.2 | 6.6 | N/A |
| 2014 | 3.7 | 3.1 | 3.2 | 3.5 | 6.2 | 6.6 | 2.512 |
| Total | 4.0 | 4.0 | 4.0 | 4.0 | 8.0 | 9.0 | 10.0 |

There are no clear trends in the pre-calculus topics of the placement test. The numbers are too similar to determine whether or not students have increased knowledge of algebra, trigonometry, geometry, and functions from high school. The calculus topics show steady increases, but still the differentials are too small to make any solid arguments about preparedness.

The math placement test has been placing less students into MA1020 and more into MA1023 over the years. The table below shows the percentages of students who took the test placed into each course.

| Year | MA1020 | MA1021 | MA1022 | MA1023 | MA1024 |
|-------------|--------|--------|--------|--------|--------|
| 2009 | 23% | 9% | 6% | 63% | N/A |
| 2010 | 21% | 6% | 6% | 66% | N/A |
| 2011 | 19% | 8% | 6% | 67% | N/A |
| 2012 | 18% | 6% | 6% | 70% | N/A |
| 2013 | 16% | 5% | 6% | 73% | N/A |
| 2014 | 15% | 5% | 5% | 64% | 11% |

This shift implies that students are becoming increasingly well prepared, or at least are receiving more information about higher math and calculus topics in high school. Note that the percentage of students placed into MA1023 drops in 2014, but that is only because 11% of the students were placed into MA1024, a higher level course.

According to a report by Professor Bill Farr in 2012, NR (or “No Record”) rates have been decreasing ever since the math placement test was implemented. Students who take the test – and follow its recommendations – are more likely to do well in the following math courses. Unfortunately, the participation rates for this test have been decreasing since 2009. More

students are taking their course placement into their own hands, which may be hurting them in the long run.

Cohort Analysis

The data can be divided into our two cohorts to see where their prior math skills differ.

The following table shows the average scores each cohort received in all of the math placement categories.

| | Algebra | Trig | Geometry | Functions | MA1021 | MA1022 |
|-----------------|---------|------|----------|-----------|--------|--------|
| Cohort 1 | 3.2 | 2.4 | 2.4 | 2.9 | 3.2 | 2.2 |
| Cohort 2 | 3.8 | 3.3 | 3.4 | 3.7 | 7.1 | 8.0 |

The second cohort consistently beats the first in prerequisite knowledge. It can be expected that those students entering MA1023 and MA1024 would have significantly higher knowledge of MA1021 and MA1022 material. However, the large gaps in knowledge of pre-calculus skills is a bit surprising. The second cohort appears to have better prior knowledge of basic skills before entering their coursework at WPI.

Now the next question is have the cohorts changed over the past five years? The tables below give the scores for each of the math placement categories since 2009.

Cohort 1

| Year | Algebra | Trigonometry | Geometry | Functions | MA1021 | MA1022 |
|-------------|---------|--------------|----------|-----------|--------|--------|
| 2009 | 3.2 | 2.4 | 2.3 | 2.8 | 2.9 | 2.0 |
| 2010 | 3.3 | 2.4 | 2.3 | 2.8 | 3.0 | 1.8 |
| 2011 | 3.2 | 2.5 | 2.5 | 2.9 | 3.1 | 2.2 |
| 2012 | 3.2 | 2.3 | 2.4 | 2.9 | 3.2 | 2.3 |
| 2013 | 3.2 | 2.4 | 2.6 | 2.9 | 3.5 | 2.4 |
| 2014 | 3.3 | 2.4 | 2.5 | 2.8 | 3.4 | 2.5 |

Cohort 2

| Year | Algebra | Trigonometry | Geometry | Functions | MA1021 | MA1022 |
|------|---------|--------------|----------|-----------|--------|--------|
| 2009 | 3.8 | 3.4 | 3.3 | 3.8 | 7.2 | 8.1 |
| 2010 | 3.8 | 3.3 | 3.4 | 3.8 | 7.0 | 8.0 |
| 2011 | 3.8 | 3.3 | 3.4 | 3.7 | 7.1 | 8.0 |
| 2012 | 3.7 | 3.2 | 3.4 | 3.7 | 7.0 | 8.0 |
| 2013 | 3.8 | 3.3 | 3.5 | 3.8 | 7.3 | 8.1 |
| 2014 | 3.8 | 3.3 | 3.4 | 3.7 | 7.2 | 8.0 |

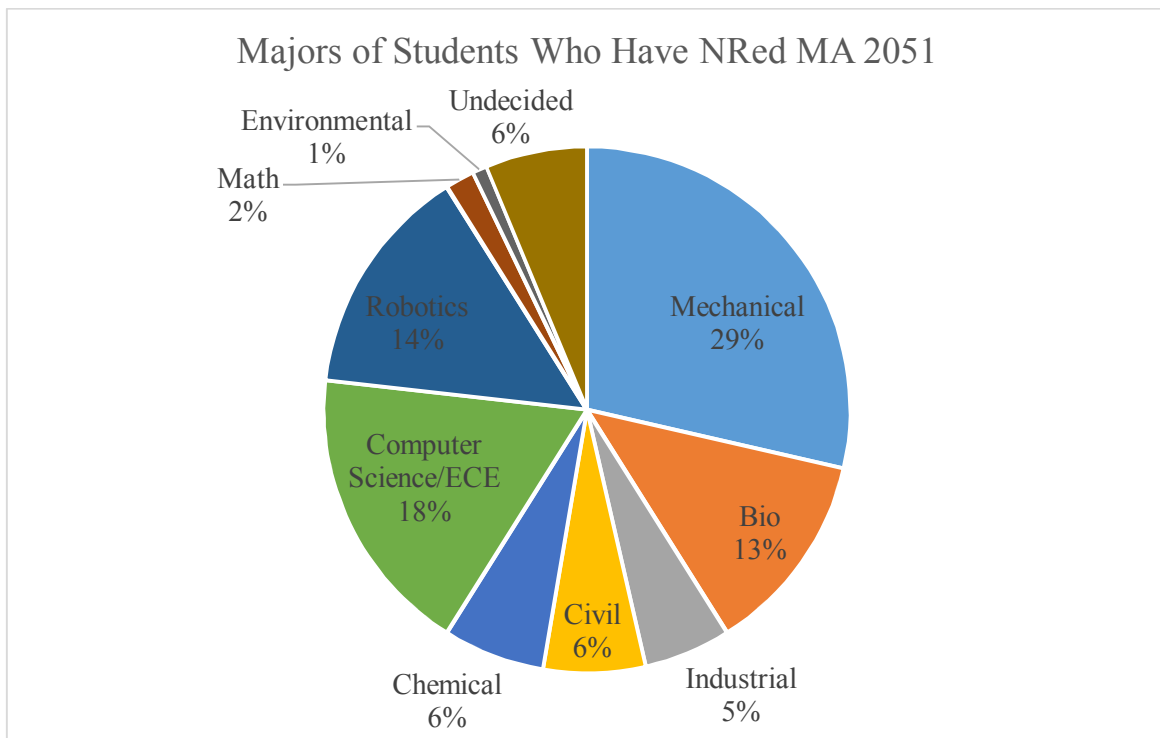
As with the complete set of data, it does not seem that the changes between the years of either cohort are significant enough to make any assumptions. However, one thing that is valuable is that the difference between the cohorts is pretty consistent through the years. The cohorts did not drift away or come closer together in the last five years.

Chapter 6: Differential Equations

Differential Equations, or MA2051, is typically the first math course a WPI student takes after completing the calculus sequence. It is a requirement for many majors, especially engineering. Unfortunately, many students fail this course year after year. In 2014 alone, 112 students NRed the course. The goal of this analysis is to identify which students are failing Differential Equations and figure out what led to their failure.

Student Backgrounds

The students taking Differential Equations come from a variety of backgrounds, therefore those that NR the course are equally as diverse. Their majors range from Mechanical Engineering to Computer Science, Environmental Studies, and Math. The pie chart below shows the breakdown of majors of students who have NRed a Differential Equations course at WPI.



The majority of students who have NRed a Differential Equations course are Mechanical Engineering, Computer Science/ECE, Robotics Engineering, or Biology majors. These percentages line up with the overall percentages of students who attend WPI in each major, so not too much can be said about the breakdown by majors. However, Differential Equations is especially important for Mechanical Engineering and Physics majors to grasp, so it is a little disheartening that these students are making up the majority of the population that is NRed this course.

The grades of all students who NRed MA2051 in the past year were collected to see what might have led to the failure. 37% of the students had previously NRed a calculus course at WPI and 26% NRed more than one! The next question might be, which courses these students have the most trouble with. The course-by-course GPAs were calculated to determine if there is one course that the majority of these students struggle with. The table below gives these GPAs.

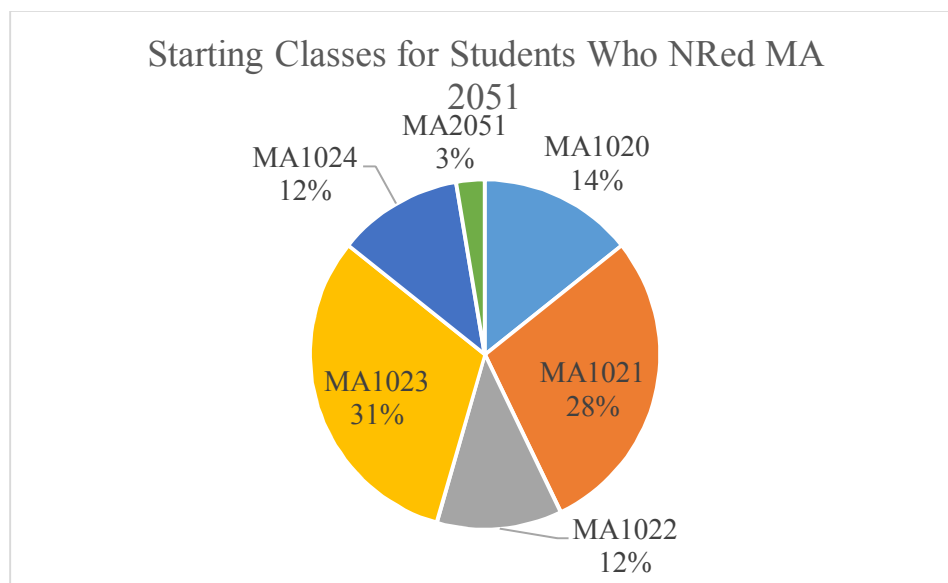
| Course | GPA |
|---------|------|
| MA1020 | 3.13 |
| MA1021 | 2.47 |
| MA1022 | 2.03 |
| MA1023 | 1.91 |
| MA1024 | 2.10 |
| Average | 2.24 |

The students clearly struggled with MA1023 the most, though none of the GPAs are very high. The students in MA1020 seem to have done the best, which could mean that having the extra term to learn the material was beneficial for them. As the students progressed, their grades

got worse, but a little improvement came in MA1024. However, that upward climb was not enough to help them succeed in MA2051.

The course that most students NRed before NRed MA2051 was MA1022. Some students even NRed the course up to three times! This makes sense because many of the topics in MA1022 overlap with those in MA2051.

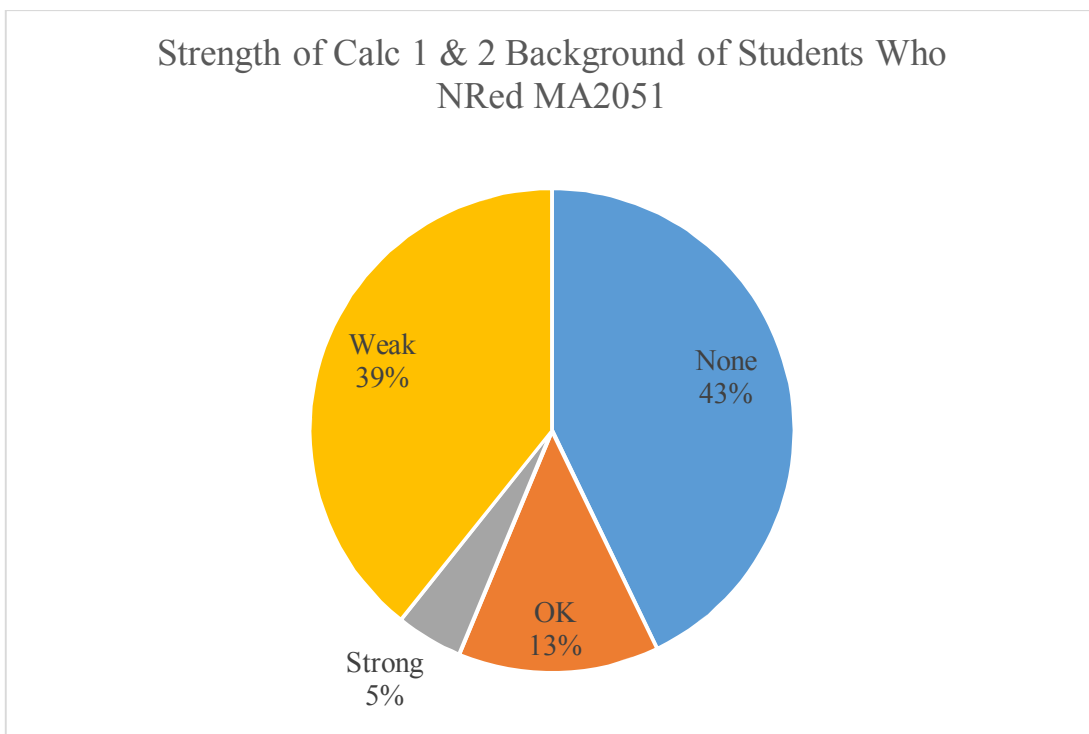
The students who NRed Differential Equations started their math coursework at WPI in many different places. The chart below shows this distribution.



| Course | # of Students |
|--------------|---------------|
| MA1020 | 16 |
| MA1021 | 32 |
| MA1022 | 13 |
| MA1023 | 35 |
| MA1024 | 13 |
| MA2051 | 3 |
| Total | 112 |

The majority of students who NRed MA2051 began in MA1023, followed closely by MA1021. These two courses are the ones that most students start with at WPI. The interesting part, though, is that the percentage of students who took MA1023 first is higher. Yes! These students are typically seen as the more advanced students with better background knowledge. However, the fact that these students make up the majority of those who NRed Differential Equations tells a different story.

Another way to look at this conjecture is that students who started in more advanced calculus courses do not have as strong of a background in basic calculus skills. It has been a while since they brushed up on the skills or they did not get a strong enough background on the topics while in high school. The chart below shows a breakdown of students by the strength of their background in MA1021 and MA1022 topics.

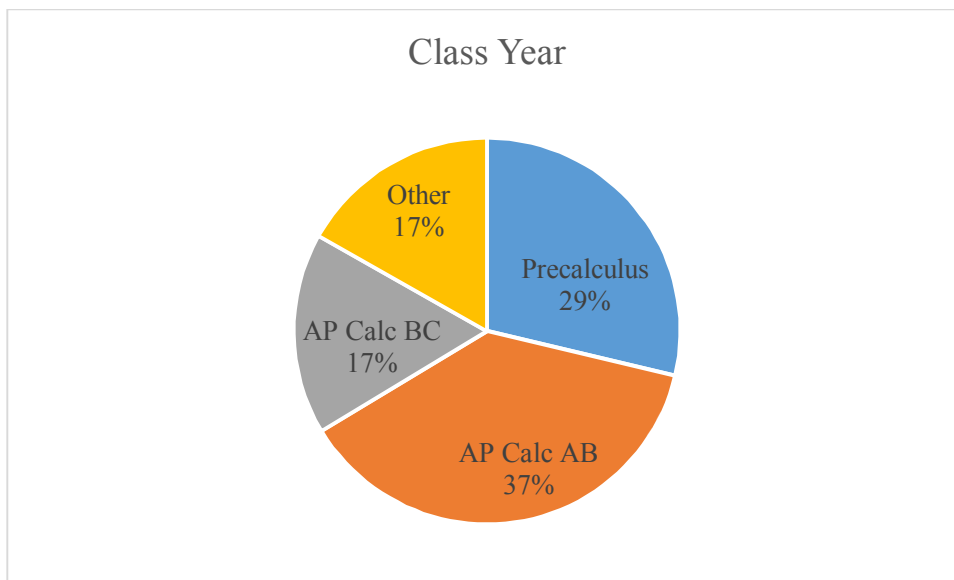


Students are considered to have no background if they did not take MA1020, MA1021, or MA1022 at WPI. A “weak” background means that they may have taken those courses, but they received only Bs or Cs in them. An “OK” background consists of a B average in previous courses, and a “strong” background means they got mostly As.

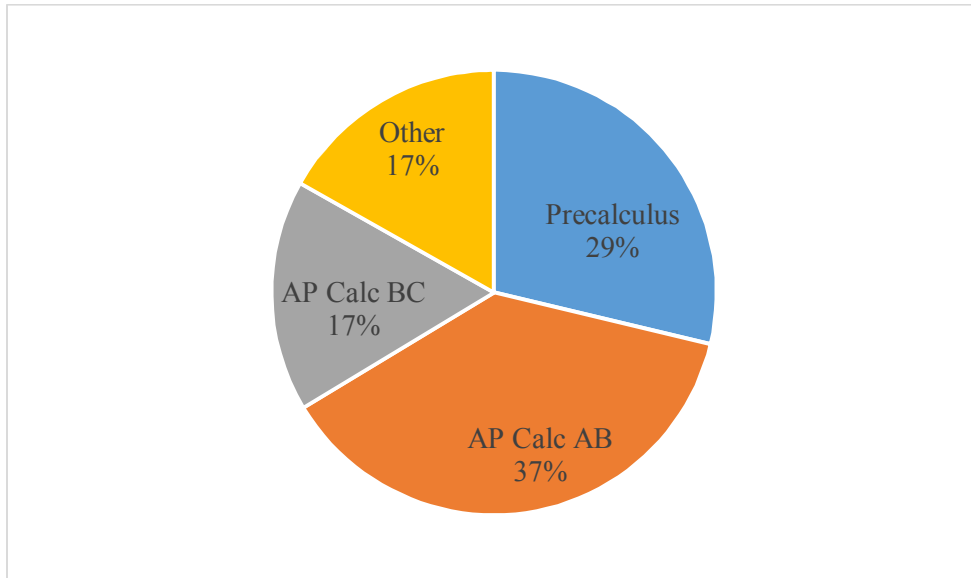
It does not come as much of a surprise that the students with weak or no backgrounds in the first few WPI calculus courses are most likely to NR Differential Equations. Without the proper skill set, or a current memory on those skills, it is difficult for students to excel in the course. The outliers here are the students with strong backgrounds in calculus. In order to figure out what went wrong with these students, and the others, a survey was sent out to any student who NRed MA2051 in the past year to see why they thought they did not succeed.

Survey Data

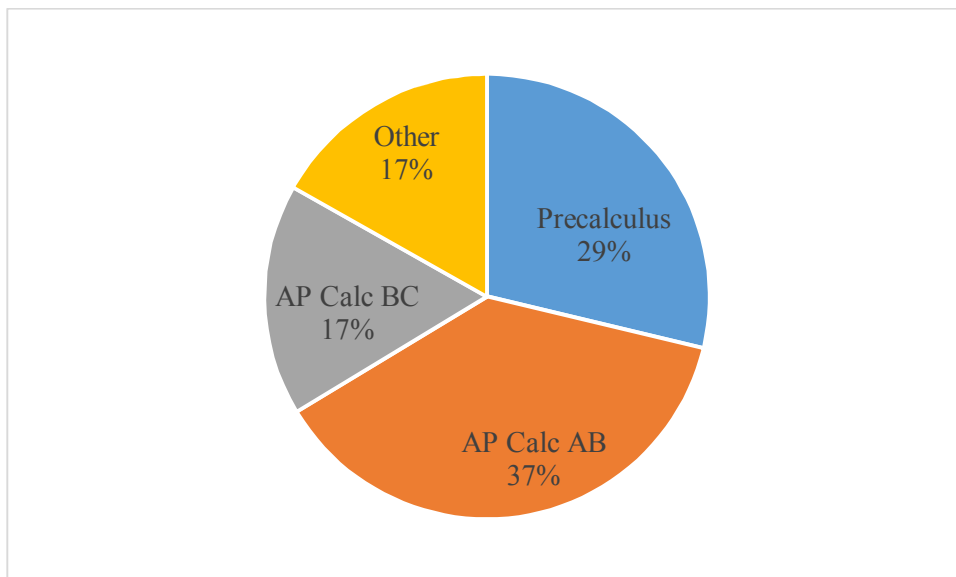
About 20% of the students who NRed Differential Equations responded to the preparedness survey. They came from a variety of classes, with half of them in the junior class.



More than half of the respondents went to a public high school, while others went to private, charter, vocational, or home schools.



Most took AP Calculus AB in high school and half started in MA1020 or MA1021, as shown below.



All of the students were required to take Differential Equations for their major.

The survey asked the students to rate how much they felt prepared in trigonometry, algebra, analytical geometry, and logarithms from high school. They used the same ranking system (a scale of 1 to 5) as the survey used in Chapter 4. They ranked logarithms the lowest with an average score of 3.63, trigonometry next with 3.92, then geometry (4.00), and the highest score was given to algebra (4.38). When compared with the original sample set, these students claimed to be more confident in logarithms, geometry, and trigonometry. Algebra is the topic that set them apart, which makes sense because Differential Equations requires a lot of algebra in order to get the correct answers.

| Trigonometry | Algebra | Geometry | Logarithms |
|--------------|---------|----------|------------|
| 3.90 | 4.59 | 3.83 | 3.49 |

When asked what they could have been better prepared in from high school, the students who NRed Differential Equations almost unanimously said calculus skills or an introduction to Differential Equations. Only a few blamed the basic skills, which explains their confidence in the four topics.

Next, the students were asked what they thought led to them NRing MA2051. They were given a variety of options such as lack of calculus skills/background, lack of organizational and study skills, lack of effort, instructor’s teaching style, and personal problems. The option that most students ranked highest was instructor’s teaching style. These students blame the professor for their struggles in Differential Equations, which may be completely valid. However, another issue to think about is the fact that humans do not like to be wrong. They especially do not like to admit that they are wrong. These students may be blaming the professors because all of the other

options mean that they could have done something better to improve. The rest of the responses were all across the board, so it is hard to tell if there is another cause besides professors.

When asked about what campus resources the students used to help them in MA2051, the responses were scarce. Those who said that they went to office hours, MASH, or the Math Tutoring Center did so infrequently. They did not take the initiative to improve their own grades and pass the course. Many of the students retaking the course, though, seem to be improving. Seeing the material a second time has helped them to progress. Maybe if they had taken the initiative to get extra help, the larger exposure might have helped in the first place.

There was another discovery that should not go unnoticed. When sending the survey to students, it was discovered that 20% of the students who had NRed Differential Equations within the past year are no longer enrolled at WPI. The students could have NRed other courses or been discouraged by their failure in this one. With such demanding needs to pass this course, they decided (or were recommended) not to continue on.

Placement Test Recommendations

The placement test data could easily be connected to the students who had NRed MA2051. The table below shows the scores of these students in each of the categories.

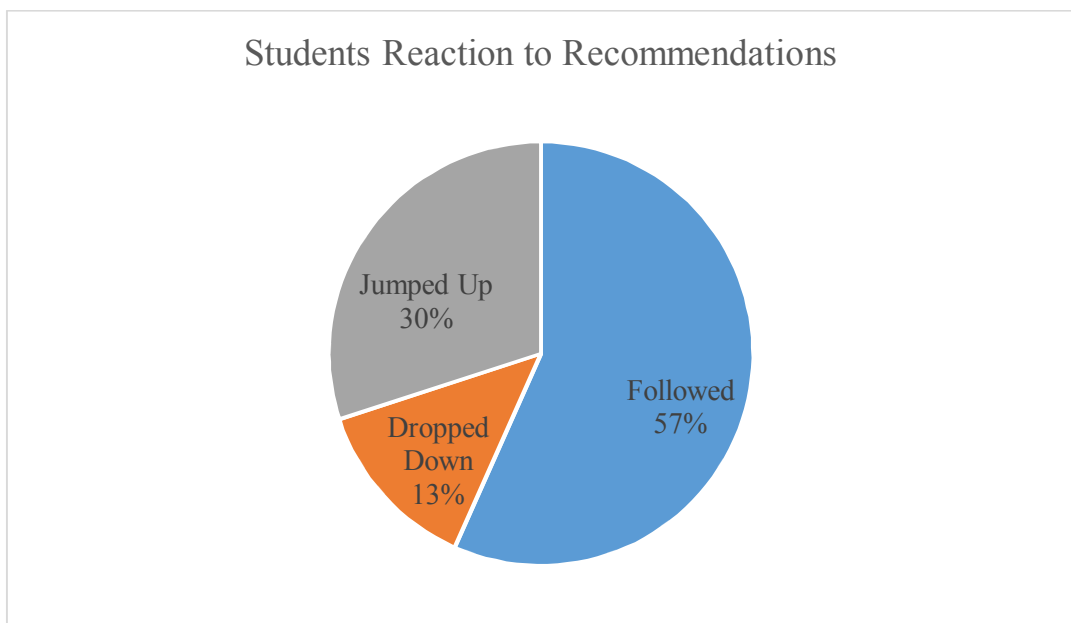
| | Algebra | Trig | Geometry | Functions | MA1021 | MA1022 |
|------------|---------|------|----------|-----------|--------|--------|
| MA2051 NRs | 3.4 | 2.8 | 2.8 | 3.3 | 4.9 | 5.2 |

These placement test scores that these students received fall directly in the middle of the two cohorts. That could be because the students in this category fall into both cohorts and the average places them in the middle. In order to fix this issue, the data can be separated into which calculus course the students started in, as in the table below.

| | Algebra | Trig | Geometry | Functions | MA1021 | MA1022 |
|-----------|---------|------|----------|-----------|--------|--------|
| MA1020 | 2.4 | 1.7 | 1.7 | 2.2 | 2.2 | 1.5 |
| MA1021/22 | 3.4 | 2.7 | 2.8 | 3.3 | 4.6 | 4.5 |
| MA1023/24 | 3.9 | 3.7 | 3.4 | 3.9 | 7.3 | 8.1 |
| MA2051 | 4.0 | 3.5 | 3.5 | 4.0 | 6.3 | 7.5 |

This table shows the students who NRed differential equation in an interesting perspective. The scores in each category are all over the place. That means that initial pre-calculus and calculus skills before coming to WPI are not really an indicator of success or failure in Differential Equations.

The placement test is not just used for fining out prerequisite knowledge – it is also used for placement. So, did the students who NRed MA2051 follow their recommended placement? The chart below shows the percentage of students who followed their recommendation, jumped up a level, or dropped down to a level below what they were recommended.



A surprising 30% jumped up to the next course after getting their recommendation from the placement test. It is not really a surprise that they did not have all of the basic skills necessary

to be successful in Differential Equations. Of the students who were recommended to take MA1020 and skipped it, half got As in MA1021 and the other half got Bs or Cs. They started to NR courses in MA1023, but all of their grades got worse over time. This shows that their weak pre-calculus background could only get them so far. The students who skipped MA1021 NRed almost every class after it. All of the students who skipped MA1023 received Bs or Cs in MA1024. This shows that without the basic calculus knowledge, it is difficult for any student to be successful in more advanced math courses.

Chapter 7: Conclusion

Math education has been changing over the past twenty years. Standardized testing and AP exams are forcing students to learn a lot of material quickly, rather than the essentials thoroughly. Changes in curriculum are causing students to miss out on basic skills and suffer for it later in their educational careers.

According to math faculty at WPI, students' preparedness for college math has not changed much over the past five years. This might mean that a study like this should be done a little further down the road to see what the educational reforms truly are doing to our young students.

The students themselves would like some help in the basic areas. They do not have strong knowledge of trigonometry or logarithms – some important topics for higher mathematics. The students would like teachers to slow down and teach them the fundamental knowledge they will continually need. They want more challenges in high school like harder word problems, more theory, and less focus on calculators. The students are asking for it, so why should the educators keep it from them?


The placement data agrees with faculty claims that students have not changed much in the past five years. They seem just as prepared in basic topics and calculus as before. One point to think about, though, is that the placement test is just that – another test. Students are used to taking tests, so this one should not be too hard for them. What if the students were asked to take some other form of assessment that measured their math background? Would they succeed? Or would they fail to show progress in a way other than the tests they have grown so accustomed to?

A majority of the students who did not succeed in Differential Equations did so because they would not ask for help. They relied on their own advice and knowledge to get them through a complicated course. They ignored recommendations from the placement test and suffered for it. They did not seek out tutoring from professors or MASH tutors. This might mean that professors need to do more of the reaching. If students are not going to seek out help on their own, then teachers must become more available or encourage an atmosphere where help is justified and normal. Students need to be comfortable asking for help because once they get into the work force, that is all they will be doing.

Math students may not be falling far behind those who came before them, but that does not mean that they are headed in a better direction. Students should be becoming more prepared for college without the opportunity of being less prepared. High schools and colleges should have more communication so that they are on the same page. Students cannot be expected to make a seamless transition between the two levels of education if there is no one to bridge that gap.

Appendix A: Surveys

Faculty Survey

WPI

Default Question Block

How long have you taught at WPI?

- 0-5 years
- 5-10 years
- 10-15 years
- 15-20 years
- 20+ years

Students at WPI can sometimes be viewed as two cohorts:

1. Students who started in Calculus 1 or 2 (MA 1020, 1021 or 1022)
2. Students who started in Calculus 3, 4 or higher (MA 1023, 1024, etc.)

Answer the following questions for each cohort individually and how you think they have changed over the last 10 years, or if you have not been teaching at WPI for 10 years, how they have changed since you started.

Please rate the following statements about **cohort 1** (Calc 1/2), with 1 being least true and 5 being most true.

| | 1 | 2 | 3 | 4 | 5 |
|--|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| Students are more prepared from high school than before. | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| Students have more prerequisite knowledge of trigonometry from high school than before. | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| Students have more prerequisite knowledge of algebra from high school than before. | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| Students have more prerequisite knowledge of (analytical) geometry from high school than before. | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| Students have more prerequisite knowledge of logarithms from high school than before. | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |

Please rate the following statements about **cohort 2 (Calc 3/4)**, with 1 being least true and 5 being most true.

| | 1 | 2 | 3 | 4 | 5 |
|--|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| Students are more prepared from high school than before. | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| Students have more prerequisite knowledge of trigonometry from high school than before. | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| Students have more prerequisite knowledge of algebra from high school than before. | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| Students have more prerequisite knowledge of (analytical) geometry from high school than before. | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| Students have more prerequisite knowledge of logarithms from high school than before. | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |

Please rate the following statements about **students who have taken AP courses in high school**, with 1 being least true and 5 being most true.

| | 1 | 2 | 3 | 4 | 5 |
|--|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| Students are more prepared from high school than before. | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| Students have more prerequisite knowledge of trigonometry from high school than before. | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| Students have more prerequisite knowledge of algebra from high school than before. | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| Students have more prerequisite knowledge of (analytical) geometry from high school than before. | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| Students have more prerequisite knowledge of logarithms from high school than before. | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |

What do you think causes the divide? Do you think there is any way to bring the students back to an even playing field?

If you have taught Differential Equations (MA 2051), which of the following do you think attribute to student failure in the course?

- Study Skills
- Algebra
- Differential and Integral Calculus
- Other

Do you have any other comments on this subject?

Student Survey



WPI

What is your major?

- Engineering (ME, BME, CHE, CE, etc.)
- Business (Management, Industrial Engineering, etc.)
- Science (Biology, CS, Chemistry, etc.)
- Math (including Actuarial Science)
- Art (IMGD, Humanities, etc.)
- Undecided

What is your class year?

- 2018
- 2017
- 2016
- 2015

What type of high school did you attend?

- Public
- Private
- Other

What was the highest level of math you took in high school?

- Precalculus
- AP Calculus AB
- AP Calculus BC
- Other

>>



WPI

Did you take the WPI Calculus Placement Test?

- Yes
- No

If you **did not** take it, why not?

If you **did** take it, do you think it gave you an accurate placement?

>>



What was the first math course you took at WPI?

- Calculus 1 (MA 1020 or 1021)
- Calculus 2 (MA 1022)
- Calculus 3 (MA 1023)
- Calculus 4 (MA 1024)
- Other

Do you have to take Differential Equations (MA 2051) as a requirement for your major?

- Yes
- No
- Don't Know

On a scale from 1 to 5, which 5 being the most prepared, please rate how prepared you were for the following topics after your high school education.


| | 1 | 2 | 3 | 4 | 5 |
|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| Trigonometry | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| Algebra | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| Geometry (analytical) | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| Logarithms | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |

What areas from high school do you believe you could have been better prepared in to be more successful in Calculus and/or Differential Equations at WPI?

Have you NRred a Calculus or Differential Equations course at WPI?

- Yes
- No

Differential Equations Student Survey



WPI

Did you pass MA 2051 (Differential Equations) the first time you took it?

Yes

No

>>



What is your class year?

- 2018
- 2017
- 2016
- 2015

What type of high school did you attend?

- Public
- Private
- Other

What was the highest level of math you took in high school?

- Precalculus
- AP Calculus AB
- AP Calculus BC
- Other

On a scale from 1 to 5, which 5 being the most prepared, please rate how prepared you were for the following topics after your high school education.

| | 1 | 2 | 3 | 4 | 5 |
|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| Trigonometry | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| Algebra | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| Geometry (analytical) | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| Logarithms | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |

>>



What was the first math course you took at WPI?

- Calculus 1 (MA 1020 or 1021)
- Calculus 2 (MA 1022)
- Calculus 3 (MA 1023)
- Calculus 4 (MA 1024)
- Other

Did you have to take Differential Equations (MA 2051) as a requirement for your major?

- Yes
- No
- Don't Know

Who was your professor for MA 2051?

What do you think led to you NRing MA 2051? Please rank the following with 1 being the most prominent reason and 6 being the least. If the statement does not apply to you, leave it blank.

- Lack of calculus skills/background
- Lack of organizational and study skills
- Lack of effort
- Instructor's teaching style
- Personal problems
- Other

To what degree did you make use of resources related to MA 2051?

| | Never | Rarely | Sometimes | Often | All of the Time |
|----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| Office Hours | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| MASH | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| Math Tutoring Center | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |

>>



WPI

What areas from **high school** do you believe you could have been better prepared in to be more successful in Differential Equations at WPI?

What areas from **college** do you believe could be improved to increase your chances of passing MA 2051?

Did you retake MA 2051?

- Yes
- No

>>



WPI

What difference, if any, did you see in taking MA 2051 a second time?

Any additional comments?



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