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# Holy Name High School: Mathematics Curriculum

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**Abstract**

We worked with Holy Name High School to improve mathematics scores on the SAT. A diagnostic test with SAT and ACT questions was administered to approximately 160 students. A statistical analysis was then performed on the results in order to determine significant deficiencies. Problem-based learning was implemented in experimental lessons. The effectiveness of these lessons was examined via pre-class and exit surveys. Using these results we formulated several recommendations for the teachers to improve the students' math scores.

## Authorship

All members of the group contributed equally to:

- Project Proposal
- Introduction
- Background
- Project Overview and Direction
- Sample Lectures Comparisons and Conclusions
- Conclusions
- Diagnostic Tests and Lecture Materials
- Final Document Editing

Dante Amoroso contributed primarily to:

- Diagnostic Test Analysis
- Educational Theory
- Diagnostic Test Miscellany
- Final Document Compilation (LaTeX)

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- Initial Project Direction
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- Diagnostic Test Conclusions
- Recommendations
- Educational Theory
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- Diagnostic Test Motivation and Creation
- Sample Lectures
- Meetings with Holy Name High School

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# 1 Introduction

Holy Name High School, a local Catholic school in Worcester, MA, recently reported having historically low SAT scores in mathematics, but did not have a clear idea as to why. The administration was concerned that the students were not getting the best education and that parents might be driven away by low math SAT scores, reducing both student population and income from tuition. Having had positive previous experience with a WPI IQP group, they contacted WPI about setting up another IQP to investigate the causes. Initial meetings with the headmaster suggested that uneven preparation and background between students due to their matriculation from different middle schools could be responsible. Statistical analysis of a diagnostic test would later reveal that students instead have several key areas of weakness, mostly in logical thinking skills and correctly approaching more freeform problems.

## 1.1 Goals

Our primary goal was to help Holy Name with its math department to ensure that students' mathematical intuition and thinking ability was further developed by the new curriculum. In doing this, we aimed to have an understanding of the student body's strengths and limitations, and investigated new teaching methods which we hoped would work well with Holy Name students.

## 1.2 Project Scope

To gain an understanding of the student body, we gave a diagnostic test built from SAT and ACT questions. We had junior and senior math students take the test, and we analyzed the results for particular weaknesses, both in topical knowledge and problem solving abilities. Our motivation here was to answer the question of where students were going wrong: Do they understand the material? Do they simply not know it? Have they learned to think mathematically?

Having addressed this question, we moved on and prepared some sample lectures for a few classes at Holy Name. These lectures were designed to address the shortcomings noticed in the diagnostic test, as well as observe the students in a classroom environment. They focused on teaching logical thought rather than new material, highlighting the general question of "Does this make sense?". We also gave a short survey to each with both mathematical questions and opinions, which we used to learn more about the student body's perception of mathematics.

### 1.3 Results Summary

The results of the diagnostic test revealed several key points about the abilities of the Holy Name students. First and most importantly, the students performed significantly below national averages in every difficulty level and field of mathematics. The largest weakness by far, however, was trigonometry, with only 14.17% of all questions answered correctly. The hard questions saw a low percentage of correct answers as well, with only 21.81% answered correctly. Due to the nature of the hard questions, which generally apply math concepts in very nonstandard and deliberately confusing manners, we were able to deduce that the students had serious issues applying the equations and basic math concepts that they appeared to know fairly well.

In forming our lectures, we tried to create math problems that we knew were based on material the students we were teaching already knew, with the entire purpose being to see if the students could use this old material in new ways. From the surveys we administered, we learned that the students did not have much previous experience with these types of less formulaic problems. Within the lectures, the lack of experience with nonstandard math problems also became apparent. Generally speaking, when left to think on their own, the students had trouble thinking through our math problems from beginning to end. When we gave them some hints as to how to approach the problem, they usually were able to complete them correctly. This suggests that the students are indeed intelligent enough to succeed in mathematics, but they currently lack the confidence and experience to flourish.

### 1.4 Recommendations

We feel that there are several ways to begin solving these problems. In the short term, one of the most obviously neglected areas is clearly trigonometry. We feel that this facet of a standard pre-calculus/trigonometry course needs to be emphasized more. Another relatively simple addition to courses is the introduction of more leading questions. In our lectures there were many instances of us needing only to gently prod a group of students to get them to either make the final conceptual leap needed, or even just to speak up and report a correct answer they had already found. There are also a number of quick things, such as using non-standard letters for variables in problems (to avoid the conditioned reflex of always solving for $x$) or emphasizing that the order in which subtraction and division are performed in solving a linear equation doesn't actually matter, one way is just usually easier than the other.

We also feel there are some general and longer term changes that the department might want to consider. One of these is the introduction of some occasional 'problem-based learning' sessions[1]. We do caution moderation in this regard, however, as an entirely problem-based curriculum runs the risk of ignoring many fundamentals of a standard mathematics education [10]. We also feel it would be useful to have a general shift towards using non-explicit coefficients in the course of a problem, for instance asking students to solve $ax^2 + b = c$ for $x$.

[1]Problem-based learning is discussed in Sections 2.2 and A.5.

# 2 Background

In this section we discuss some basic principles of educational theory and give an overview of the SAT and ACT standardized tests. We also offer background information on Holy Name High School and its math department.

## 2.1 Educational Theory

Here we present a brief summary of the education theory background used in this report. A more thorough overview is given in Appendix A.

### 2.1.1 Learning Styles

Learning Styles is a label that gets applied to several distinct, albeit similar, educational theories. For clarity we acknowledge Felder and Solomon [3] for the particular version we are referencing here.

The general idea behind learning styles is that different individuals have different approaches to learning material. It is possible to categorize these approaches by placing students at a location within each of four categories (it need not be specifically four, that's just the breakdown from [3]). Based on their position on each of these 'axes', individuals can be determined as more or less responsive to various types of teaching. These four axes are referred to by their two extremes, despite most people being somewhere in between, and are enumerated as follows:

- Active/Reflective Learners
- Sensing/Intuitive Learners
- Visual/Verbal Learners
- Sequential/Global Learners

For a detailed description of what these categories represent, see Section A.1. An important distinction to make is that these categories do not sort students into being better at a given subject in school, but rather indicate how students would best learn in any subject, a good example being a verbal learner and Mathematics. The fact that a specific person is categorized as a verbal learner doesn't mean they are only fit for a field like English, rather that they need to be taught differently from a highly visual learner if they wish to understand the same material. Thus, while intuition might lead you to label such a person as less fit for Mathematics, they need only be taught in a manner which accommodates their learning style to succeed in it.

### 2.1.2 Structure of Intellect

The Structure of Intellect (SI) Theory [23], is basically a means of dissecting the concept of intelligence into many subdivisions. It starts by defining everything a person can do as an operation on contents resulting in products. It then specifies five operations, five contents and six products, yielding 150 archetypal types of thought. Types of operations, contents and products defined by the theory, as well as the meaning of each, are discussed in detail in Section A.2. However, it isn't worth discussing too deeply here as the sheer number of possible combinations means any particular one would apply to a very small number of Holy Name students, thus making it impractical for application.

### 2.1.3 Multiple Intelligence

Multiple Intelligence (MI) theory is very similar to SI Theory, but has the virtue of being a great deal simpler. Both share the fundamental idea that intelligence cannot be measured by a single, stand-alone metric, but rather has multiple independent factors. MI Theory, however, begins by deciding on a criterion by which a way of thinking could qualify as one of these factors [5]. These criterion are discussed in detail in Section A.3. The current ([22] [11]) list of intelligences that have passed the criterion is:

- Spatial Intelligence
- Linguistic Intelligence
- Logical-Mathematical Intelligence
- Bodily-Kinesthetic Intelligence
- Musical Intelligence
- Interpersonal Intelligence
- Intrapersonal Intelligence
- Naturalist Intelligence

These are again discussed in more detail in Section A.3. It is important to note that the theory specifically cautions against expanding it into a device aimed to prescribe behavioral norms to people, and not to trivialize it through over-simplification [5]. An example of this would be teaching math to musically intelligent students by having them set it to song or listen to enjoyable music while studying, which would be both condescending and ineffective.

### 2.1.4 Aptitude Treatment Interaction

Aptitude Treatment Interaction (ATI) most generally refers to the idea of looking at the performance of a group of students as a function of the various teaching options available [21]. ATI is based on the cognitive styles possessed by the student or students who make up the group. What is meant by cognitive styles is essentially the same thing as Learning Styles from Section 2.1.1. ATI then seeks to find a teaching option that best enhances the performance of the students as a group based on something such as the MI or SI breakdown that each student falls into combined with their learning style.

There are some unfortunate complications with the ATI approach, however, namely that learning styles vary not only from person to person, but also within a person as the subject material is being varied [21]. This makes it difficult to maximize whatever index is being used to indicate performance. Further information is presented in Section A.4.

We used the ideas in this section throughout the report to help guide us in analyzing Holy Name students' abilities in mathematics so that we had a clearer idea of what weaknesses were present. Moreover, we hoped to be able to differentiate between a problem in knowledge and a problem in learning.

## 2.2 Problem-Based Learning

A problem-based mathematics curriculum entails teaching mathematics in the context of problem solving. In this approach students work together in small groups to try and solve problems with minimal input from the instructor. These problems are also heavily based on real-world examples rather than purely mathematics. The power of problem-based teaching is that it directly addresses a common concern among students regarding how useful their math education truly is. Furthermore, problem-based learning has been shown to improve student flexibility in their approach to problem solving, and to improve a student's intuition of mathematical concepts [8]. For instance, rather than solving the equation of a line with slope $b$ passing through the point $(c, d)$, a student may be asked to find a model for a store's total earnings if it makes $b$ dollars a day and had $d$ dollars on the $c^{\text{th}}$ day of the year. This isn't intended to be a simple word problem; students collaborate amongst themselves without expressly being told that the model is linear. Gains in SAT scores seem to be, at best, modest [12], and some recent evidence [10] has cast doubt on the effectiveness of problem-based learning in certain situations, such as remedial math classes. However, this approach to curriculum reform seems well-suited to a math program that fails to engage the students. A more complete description of problem-based learning is given in section A.5.

## 2.3 The SAT and the ACT

The Scholastic Aptitude Test (SAT) and the ACT[2] are the two most common and widely accepted standardized tests for evaluating high school students in the context of admission to United States colleges and universities [2]. In this section, we provide an overview of the format of and topics covered on these tests.

The College Board [18] gives a general outline of the material students are expected to know for the mathematics portion of the SAT:

Number and Operations

- Arithmetic word problems (including percent, ratio, and proportion)
- Properties of integers (even, odd, prime numbers, divisibility, etc.)
- Rational numbers
- Logical reasoning
- Sets (unions, intersections, elements)
- Counting techniques
- Sequences and series (including exponential growth)
- Elementary number theory

Algebra and Functions

[2]Which, incidentally, doesn't stand for anything anymore.

- Substitution and simplifying rational expressions
- Properties of exponents
- Algebraic word problems
- Solutions of linear equations and inequalities
- Systems of equations
- Quadratic equations
- Rational and radical equations
- Equations of lines
- Absolute value
- Direct and inverse variation
- Concepts of algebraic functions
- Newly defined symbols based on commonly used operations

Geometry and Measurement

- Area and perimeter of a polygon
- Area and circumference of a circle
- Volume of a box, cube, and cylinder
- Pythagorean theorem and properties of isosceles, equilateral, and right triangles
- Properties of parallel and perpendicular lines
- Coordinate geometry
- Geometric visualization
- Slope
- Similarity
- Transformations

Data Analysis, Statistics, and Probability

- Data interpretation
- Statistics (mean, median, and mode)

- Basic probability

As for the format, most of the questions are multiple choice, however some require the student to write out the full answer without predefined choices. The SAT mathematics section is scored out of 800. Massachusetts, on the whole, averages 524, compared with the 518 national average [9]. Improving SAT scores is often done by "coaching", which is an intensive review tailored specifically for the purpose of raising SAT scores. This is often done by taking many practice tests, drilling formulas, tutoring, and test-taking strategies. However, it seems that the SAT cannot be easily coached. Powers and Rock, who analyzed SAT score differences between coached and uncoached students, found that there was only a weak correlation between coaching and score improvement, with math scores rising, on average between 10 and 17 points per group tested [17]. This is roughly in agreement with previous studies on the same subject. Furthermore, instances where coached students made marked gains in SAT scores were matched by uncoached students [16], which suggests that a good portion of SAT improvement comes about simply from taking the test again. Although Powers and Rock focused primarily on commercial SAT coaching (e.g., the Princeton Review, Kaplan), the results would also extend to other SAT preparatory programs. Another reason why SAT coaching tends to be somewhat ineffective is, as Powers points out, "SAT questions are selected partly on the likelihood that they will not be susceptible to short-term coaching." This means that the SAT tends to pick questions that require more thought than mere drilling can prepare for. Therefore, it seems unwise to focus solely on SAT preparation as a means of improving SAT scores, and instead focus on more effective methods which entail reforming curriculum at a baser level.

There is recent evidence that both curriculum reform and SAT preparation can lead to marked increases in SAT scores. In Georgia [25], schools that had 50+ point SAT score improvements were interviewed, and their collective strategies for SAT improvement were documented. The schools extensively used SAT prep programs, including the software "One on one with the SAT," [25] and weekend SAT prep classes (Kaplan) offered four times per semester. However, the schools did not focus solely on SAT improvement; efforts were also made to improve students' abilities in problem solving, adjust focus on math in middle schools, implement independent research projects, and place more emphasis on taking advanced math courses. In particular, they used the PSAT as diagnostic measurement, which helped schools address shortcomings in their math curriculum. Based on these cases, we draw the conclusion that SAT prep programs may be useful for improving SAT scores, but a school needs to have a proper foundation to do so, which comes from curriculum reform.

Another popular standardized test is the ACT. Broadly, the ACT and SAT are very similar, and generally scores between the two tests correlate well [2]; a student who does well on one test is highly likely to do well on the other. Good scores in math, reading, and language with the SAT correspond to good scores on the ACT, and vice versa. There are, however, some interesting differences on what the two tests cover [1]. The ACT includes a "science reasoning" section which emphasizes understanding data, charts, graphs, etc. Also, the ACT mathematics includes trigonometry, unlike the SAT. It is generally thought [2] [1] that, on the whole, the SAT focuses on problem-solving while the ACT focuses on knowledge. This is, of course, not a dichotomoy, but instead a weak trend. Still, some students [1] may find that they do better on one test than another.

Although we do not wish to delve into the debate of which test makes for a better indicator of academic potential, a short overview will be given. Recent news [20] indicates that ACT math and science scores have declined over recent years, consistent with many conclusions coming out of educational research literature that high school math and science aptitude has been slipping. However, in 2003, the SAT reported a 35-year high in math scores, indicating a significant disconnect between the two tests. Theories explaining this vary widely, with little consensus. Some feel that the discrepancy is due to an effort by the two tests to distinguish themselves, as standardized testing is a competitive business. The CollegeBoard, however, credits the discrepancy to more students taking the SAT being enrolled in rigorous college preparatory mathematics courses.

Interestingly, whether a student is more likely to take the SAT or ACT is regional. [13], and many others, show a strong preference of the ACT in non-costal regions, while the SAT dominates on the coasts. For instance, in 2005, 12% of Massachusetts high school graduates took the ACT, while 86% of graduates took the SAT. For comparison, 69% of Oklahoman high school graduates took the ACT, while 6% took the SAT. However, the vast majority of universities accept SAT and ACT scores with equal weight, and (in principle) a student may take any test he or she wishes. Unfortunately, in practice a high school often teaches to the SAT or ACT [13], which gives one test an inherent advantage over another.

## 2.4 Holy Name High School

Near the end of D term 2007, the IQP group visited Holy Name for the first time. We discovered from the headmaster at the time that the students were having serious difficulties understanding the mathematical concepts necessary to succeed on the SAT and in a higher education environment. According to the headmaster, SAT scores in mathematics were poor, and qualitative assessments of student performance from Holy Name's teachers were, generally, that students were weak in their mathematical abilities. However, the school did not have SAT scores on record, nor did the headmaster clarify what 'weak' really meant. In order to obtain a better understanding, we obtained copies of every math course curriculum at the school. We found that there was no particular centralized curriculum throughout the department, and that the school's curriculum records were, at best, incomplete, and sometimes not even consistent between teachers.

Holy Name offers both middle and high school educations, with math classes ranging from Pre-algebra to AP Calculus. In middle school, all students take both Pre-algebra and Algebra. Once high school begins, the students branch off into several other classes. Ninth graders are divided into Algebra, Geometry, or Integrated Math I. The Holy Name course description describes the Integrated Math course as covering a broad variety of material which "strengthens their math skills and prepares them to take more advanced courses." Tenth graders take Geometry, Algebra I/trigonometry, or Integrated Math II. Eleventh graders take Introduction to Analysis, which is a Precalculus course that combines concepts from Algebra and Trigonometry, Algebra II/Trigonometry, or Integrated Math III. Twelfth graders can take AP Calculus, Calculus, or Advanced Math Topics (a project-based course that combines concepts from previous classes).

We immediately noticed that every math class curriculum had one major thing in common. Almost every class focused on the SAT. Some classes had SAT performance as a primary goal, while others had SAT preparatory work as part of the curriculum. The extent to which the SAT was emphasized seemed curious, as one of the supposed main problems with the school was low SAT scores. The group hypothesized that, based on the studies of Powers and Rock [17], SAT coaching may ironically be one of the main causes of the poor performance. Their results indicate that excessive focus on SAT score coaching deemphasizes an understanding of the material. We used this to conjecture that it may be of benefit to scale back SAT material and focus instead on mathematical foundations.

## 2.5 Holy Name Math Department

As mentioned earlier, we were given a folder containing the curricula for all of Holy Name's math courses. These consisted of syllabuses and course outlines for the courses, as well as teachers' notes, sample homework assignments, and other relevant documents. We read through these in order to assess potential gaps in the curriculum as compared to SAT standards (Section 2.3). A comprehensive summary is offered in Appendix 2.1; we present the more notable findings here.

Holy Name offers, generally, two different routes for math education. One is a typical Algebra I - Geometry - Algebra II - Trigonometry sequence, and the other is an Integrated Math sequence. The Integrated Math sequence is designed for students with less experience or some weaknesses in mathematics. It is more broad in scope than the traditional math courses, and emphasizes projects, word problems, and group work. It is also heavily focused on test taking skills, particularly SAT performance. Based on the syllabuses, the Integrated Math sequence seems to cover all the subject areas on the SAT. Unfortunately, the syllabuses for Integrated Math II and III were incomplete, so we could not get a full grasp of what was covered.

Courses in the traditional sequence had more complete syllabuses and seemed to cover most of the standards, but there were some glaring and curious gaps. There was nothing in the courses involving sequences, counting techniques/combinatorics, and logic. While these topics are not critically important for the SAT, they are part of the covered topics, and there are often several questions related to them. One curious omission was the lack of study of perpendicular lines and coordinate geometry, neither of which could be found in Algebra I, II, or Geometry syllabuses. Not only are these very important concepts for understanding the Cartesian plane, they are often addresed on the SAT, and their absence is somewhat troubling.

Perhaps the most notable gap in the covered material of the traditional sequence is the lack of data analysis, statistics, and probability in any of the courses. These topics are expected knowledge for the SAT. Integrated Math did cover these concepts, but their absence in the traditional sequence is certainly a notable flaw that should be investigated.

All this being said, we did not expect that the omissions in the covered material were what most strongly affect SAT scores. The vast majority of the frameworks did seem to be covered in the class, particularly with the integrated math sequence. In fact, the integrated math sequence seemed to cover a wider range of topics than the traditional sequence. We also suspected that some of the gaps we did notice were due to incomplete syllabuses. Based on the documentation we received, the math department seemed to be somewhat disorganized, so it was entirely possible that the syllabus the school kept as a record was not the actual syllabus used in the course. Based on this, we hypothesized that the low SAT scores were not due to weakness in coverage, but rather weakness in ability and understanding.

The math department appointed Mr. Gregory Marcotte as the new department head in 2007. Mr. Marcotte was our primary contact at Holy Name, and he expressed personal interest in both our project and general departmental reform.

# 3 Project Overview and Direction

Holy Name High School has lower math SAT scores than desired. These scores are hurting the students' chances of getting into good colleges and universities, as well as reducing parents' incentives to pay Holy Name for their children's educations. In addition to the internal reforms being made by Holy Name, we were tasked with independently assessing the problem. In particular, they wanted us to discover the specific weaknesses present in the student population's mathematical abilities.

To do this, we wanted to begin investigating new methods of teaching that might help better engender understanding. We first sought to learn more about mathematics educational theory to expand our background in the subject. We then needed to diagnose the student body in order to pinpoint specific weaknesses and get an overall feel for how they approach math problems. We planned to use this information to create sample lectures to see what would be effective in improving these weaknesses.

## 3.1 Development of Project Direction

While performing our background research, our ideas and plans changed substantially, and their development deserves a note. Initially we had planned to take a strong hand in Holy Name's reorganization, and be rather forceful in the implementation of our ideas. We thought this would be the best way to have a lasting effect on the department, as well as being the most effective way to address the task we were given. However, we eventually shied away from this, as there is a huge amount of risk involved in changing the entire math department all at once. Such an undertaking is riddled with unpredictable variables involving the teachers and students. In retrospect, it was not feasible.

We then decided on a more modest, controlled set of goals, in which we would essentially build from the ground up a single math class to be taught at Holy Name. This class would serve as a guideline for other classes at the school. There would be minimal detailed research done on the educational theory side of the project, with more of a focus on educational practice instead. The more immediate results would allow Holy Name to rapidly make any needed adjustments to the IQP team's course guidelines. However, we decided that one course is too minor to really have any far-reaching effects. In particular, after the diagnostic test revealed the extent of the students' problems in reasoning and thought, we decided that this option was not sufficient to fix the problem.

Our final decision was more macroscopic and far-reaching, but at the same time cautious and minimal risk. We planned to develop a set of guidelines for Holy Name, based on observations, research into educational theories, and feedback through student and teacher interaction. We finally decided to choose this method because we realized that one year is not a sufficient length of time in which to bring about the required change.

## 3.2 Chosen Plan

The primary phase towards implementing this plan was to administer a diagnostic test to determine where performance suffered, analyze the data, and create some sample math lessons based on the results. These lessons functioned essentially as experiments to verify our research (and potentially help the students understand math better, though this is a rather ambitious goal over the course of one class). During these lessons, we administered surveys at the beginning and end of the period, which provided data that helped decide if the tested educational theories were successful in improving students' general approach to math, rather than their performance on specific problems. Attributes from the most successful lessons were submitted to the faculty of Holy Name for further consideration and ultimately implementation.

A large part of our efforts towards improving the student's math skills involved not only traditional methods of teaching the subject, but also problem solving skills that helped them succeed in class and on the SAT. There are many methods of achieving improved logic skills, one of which is to relate a programming language to their studies in a practical way. Such methods have been proven to help immensely in other environments [4]. Another nontraditional method of improving mathematical ability is to stimulate interest in the subject. As most students are far more inclined to succeed in a subject they are passionate about, presenting the material in a stimulating manner is extremely important. A generally successful and generic method is to utilize applied math problems (which can easily be tied into the general problem solving exercises). Such problems are widely available and require a deeper understanding of the material. Students must think about what they are doing and cannot simply 'go through the motions' that they are taught in standard math classes.

# 4 Diagnostic Test

## 4.1 Motivation

In order to gauge Holy Name students' overall performance on SAT/ACT math questions, we decided that administering a test designed to assess and pinpoint weaknesses would be instrumental in the development of our project. The test would include various questions from previous SATs, PSATs, and ACTs, varying in difficulty and topic. We would then administer it to Holy Name upperclassmen, and thus gain some measure of how effective the school's math curriculum is.

## 4.2 Creation of Test

This test addressed all the areas of math that high school students should cover using questions taken from previous years of PSAT and ACT tests. Each question was chosen based on its ability to test the student in a specific area of math, such as algebra or geometry, and its difficulty. When administered, the test also asked the students to answer basic questions about their previous test scores, grades, and history of mathematics education and Holy Name high school. These latter questions were planned to help the analysis by providing background data on the students and overall grade information.

The SAT and ACT tests are both used by colleges to judge the abilities of a student when deciding on admission. Having two tests with similar concepts to choose from would offer the team a broader range of questions. The SAT and ACT test the students in different ways [15], both in material covered and the types of questions asked. In general, SAT questions tend to be more problem and logic oriented, but the material is at a lower level than that on the ACT. Because of this, the team decided to include questions from each test. Having questions that are different could help identify the type of difficulties that the students are having. For example, a student could know how to mathematically approach a problem but get caught up in the wording. In other words, they do not understand what they are being asked to solve. Including ACT test questions was also helpful because it required basic knowledge of trigonometric functions that is not needed on the SATs.

Each diagnostic test consisted of 27 questions, 12 from the SATs and 15 from the ACTs.

Two different tests were made so that there would be a more diverse set of questions to analyze. The SAT questions were defined as either Number, Algebra, Geometry or Data. The ACTs were likewise defined as Algebra, Geometry, Trigonometry, Data, and Number. Each category was then sub-classified as easy, medium, or hard. This allowed us to judge the level of understanding within each topic. The SAT questions were already split into these topics, while the ACT needed to be sorted. We reviewed and categorized each question based on whether it was from the SAT or ACT, along with its topic and difficulty. Once the questions were split up, the team picked two questions from each category based on which ones covered the subjects the best, and were different from those already chosen. We note that based on the national average scores on the 2002 PSAT, taken from a student's score report, students should be able to get, very roughly, 75% of the easy questions correct, 50% of the medium, and 30% of the hard.

While making the tests, a theory on how well the students would do was created. We discussed amongst ourselves the expectations for the test as a whole as well as how each factor would affect the students' ability to answer a question. The team expected that certain difficulties and categories would present more of a challenge to the students. We also expected the students to fair better on the SAT section of the test over the ACT because it is generally coached or taught more frequently in schools. Within the categories of questions, the team thought that the students would find algebra and basic geometry the easiest while trigonometry would be the hardest. This was because trigonometry is a subject that is not traditionally emphasized in classes and especially SAT preparation, while algebra and geometry are more stressed in the curriculum. We also believed that the version of the test they took would not make a difference, as we had specifically chosen the questions to be as similar as we could.

The students were also asked questions regarding their past experiences with mathematics and the calculator being used on the test. The questions were chosen so that the student would not need a calculator to solve them. We did not expect much of a difference in most of the questions because a student was using a higher functioning calculator. However, with questions regarding graphing, trigonometry or a lot of computation there would be a difference. We also asked the students what their previous and current math classes were. We expected those students in higher level math classes to perform better on these tests, as people in higher level classes tend to have more drive or natural aptitude with math.

However, all the students would have completed all the classes necessary to know how to do each question. Each student was asked their final grade in their previous math class as well as what they scored on the math section of the SATs if they had taken them. Those students who had previously performed better were expected to have an easier time with our test. The idea of having these factors was to gauge whether or not they mattered at all, based on the significance of their correlation to performance, and to help correct the data analysis if they did. Unfortunately, a common misunderstanding was that students thought we asked for the year that they took their previous math course (i.e., $10^{\text{th}}$) rather than how well they did, so the correlation between this factor and performance could not be analyzed.

## 4.3 Data Analysis

### 4.3.1 Quantitative Analysis

After compiling the data from both tests into a master Excel file, we wrote a MATLAB program to analyze it. Both the Excel file and the MATLAB code can be seen in Appendix D.1. The first step was to strip off the qualitative columns of current class, previous class, grade in previous class (due to the aforementioned misunderstanding) and SAT score. The effects of these factors is discussed below in 4.3.2. We also ignored an index that was used to track progress while grading individually and removed those tests which were suspect. A test was flagged as suspect by the grader if it was felt the student gave it absolutely no serious consideration, perhaps leaving the entire test blank or drawing pictures rather than reading and answering the questions. Although we expected a moderate number of tests to need to be flagged, as the test was not part of the students' grade, very few flags were actually needed. When this initial trimming of the Excel file was completed we were left with 80 responses for test form A and 80 for test form B, down from only 89 and 82, respectively.

We next calculated one of our major quantities of interest, the percentage correct on each individual test. Although the aim of the diagnostic was to identify those areas in which students had strengths or weaknesses, we first needed to account for all the other differences between tests. This meant analyzing the score on each test with respect to which test form was taken (either form A or form B), what type of calculator was available to the student, and how long the student had been at Holy Name. Within the excel file these were input as coded sets of numbers. For the test column a '1' represented form A while a '-1' represented form B. The calculator column was the integers '-2' through '2', with '-2' representing the 'no calculator' extreme while '2' represented the 'TI-89 or equivalent' extreme. The 'schoolstart' column contained a number corresponding to the grade in which a student entered Holy Name, down to a resolution of 0.5 for 'half-way through the year' responses. The MATLAB program normalized the calculator and schoolstart columns to a -1 to 1 scale centered at 0 for the purposes of the analysis.

In order to test for the significance of these three factors a type of analysis known as ANOVA (short for ANalysis Of VAriance) was performed using the MATLAB command 'anovan'. This type of analysis takes all the deviation present in a distribution of scores and breaks it into components due to each of the factors being tested and random error. A comparison of the error due to each factor with respect to the random error is then made via an F-test. The p-value is then found for each factor and compared to the desired level of significance (in our case 0.05) to determine if the factor has a significant effect. The ANOVA table for the first three factors is seen in Table 1.

Table 1: ANOVA data, background parameters.

| Source | Sum of Squares | Degrees of Freedom | Variance | F-Statistic | p-value |
|---|---|---|---|---|---|
| Test | 0.00936 | 1 | 0.00936 | 0.27 | 0.6058 |
| Calculator | 0.23142 | 4 | 0.05786 | 1.65 | 0.1639 |
| Time at HN | 0.40868 | 8 | 0.05109 | 1.46 | 0.1765 |
| Error | 5.10674 | 146 | 0.03498 | x | x |
| Total | 5.82715 | 159 | x | x | x |

As all the p-values were above 0.05, we knew that none of the factors had a significant impact on test performance. This meant that we were free to analyze the performance on each individual question as a function of the difficulty level of the question, the source of the question (SAT or ACT) and the type of question being asked, all without having to worry about a potential confounding effect due to the test being taken, the calculator being used or the time the student spent at Holy Name. In order to look at the effects of these three new factors we computed our other major quantity of interest, the percentage correct over all students of each individual question. For this portion, since we had proven the test category had no effect, we were able to consider 54 questions answered by 80 students a piece, rather than the actual two sets of 27 questions given to 160 students total.

The test for significance of these factors again involved performing ANOVA in MATLAB, yielding the Table 2.

Table 2: ANOVA data, test parameters.

| Source | Sum of Squares | Degrees of Freedom | Variance | F-Statistic | p-value |
|---|---|---|---|---|---|
| Difficulty | 0.83567 | 2 | 0.41783 | 13.59 | 0.0 |
| SAT/ACT | 0.06472 | 1 | 0.06472 | 2.11 | 0.1536 |
| Type | 0.58238 | 4 | 0.1456 | 4.74 | 0.0028 |
| Error | 1.41414 | 46 | 0.03074 | x | x |
| Total | 2.83413 | 53 | x | x | x |

The p-value for the source of the question indicated that there was no significant discrepancy in performance between SAT questions and ACT questions. There was, however, found to be (with extremely high certainty) an effect due to the difficulty of the problem and the type of the problem. All the ANOVA test tells us, though, is whether there exist differences between the various averages, not the specific nature of those averages. In order to determine this specific nature, we first looked at the actual values of overall percentage correct both by difficulty and by type. These percentages were:

| | | | | | |
|---|---|---|---|---|---|
| Difficulty | Easy : 51.94% | Medium - 40.78% | Hard - 21.81% | | |
| Type | Num : 47.45% | Dat - 42.55% | Alg - 41.72% | Geo - 32.97% | Trig - 14.17% |

The natural question to ask here is whether the each of these percentages is in fact greater than the percentage below it, or if the statistical error present obscures the gap. To determine this we utilized a series of pair-wise z-tests. To perform these, the mean (those percentages already calculated above) and standard deviation (as known for binomial distributions) for each difficulty and type were calculated. The number of samples for each was quite high due to the implicit presence of 80 repetitions within each individual question, validating the use of z-tests via the Central Limit Theorem. We again obtained a p-value associated with each gap (easy to medium, Dat to Alg, etc.) and compared this with our chosen significance level of 0.05 to see if the error was indeed too large or not. The p-values for all gaps save the Dat to Alg gap were negligibly small, indicating that there was a real performance difference between the categories, but that the Dat to Alg categories had statistically identical performance. The actual values can be seen by running the MATLAB program on the Excel file from Appendix B.

In summary, students performed better on easy questions than they did on medium, and substantially better on medium than they did on hard. They performed best on numerical questions, followed by algebraic and data-based questions. They showed more weaknesses in geometric questions, and significant weakness in trigonometry.

### 4.3.2 Qualitative Analysis

During testing, we had also asked the students to show their work on the test booklets in order to analyze their ability qualitatively, as well as assess whether or not they took the test seriously. Thankfully, based on the level of the work shown, the students did seem to put effort into doing well. This suggests that the students, on the whole, are quite motivated and hard-working, particularly considering that this test was not going to affect their grade.

In general, students showed that they were quite knowledgeable about facts learned from previous courses. Geometric properties seemed to be particularly well-remembered, and students on the whole seemed fairly comfortable with most of the basic definitions and properties expected in a typical high school math education. A notable exception was knowledge of trigonometry, which seemed to be sorely lacking. This is considerably troubling in light of the fact that most students reported taking Algebra II/Trigonometry last year, and we had not noticed any particular deficiencies in trigonometry based on the course syllabus.

We also noticed a heavy reliance on calculators for answering a few specific questions, which often led students astray. For example, one question asked "What is the maximum value of $4 \cdot \sin 3x$?" Many of the answers we read were similar to '3.85' or '3.92,' which implies that many students simply plotted the function in a graphing calculator and found what appeared to be the maximum manually, likely through the common 'trace' feature. Furthermore, there was very little evidence that students considered whether or not their answers made sense. One example of this was a simple trigonometry problem in which students were asked to find one leg of a right triangle, given another leg length of 45 meters and an angle of 27°. Many students simply multiplied 45 and 27, giving a nonsensical answer of 1215 meters for the leg. This might even indicate an attempt to utilize the concepts associated with measuring angles in radians; their methodology seems similar to finding the arc length of a section of a circle with an angle given in radians.

One facet we were particularly concerned with was the tendency of students to simply recall facts and formulas and use them incorrectly, often without credence to what the problem asked. For instance, in a geometry problem from the diagnostic test asking for the area of a right triangle, many students gave a hypotenuse. Initially this seemed to be a common misreading, but the high prevalence of the error suggests that students may have seen a right triangle and immediately computed the hypotenuse without thinking about the problem. Also, many students seemed to think that taking an average of a set of data points means summing up the points and dividing by two, regardless of the number of points. Errors such as these were quite common, indicating some trouble with applying factual knowledge.

Finally, we found that the math class students were taking this year did not make much of a difference in their overall performance. This was somewhat surprising, as it seemed that students currently taking Algebra II and Trigonometry did just as well as those in Pre-calculus. A notable exception to this was students in AP Calculus, who based on our observations, did quite a bit better on the test. The difference was not statistically significant, but our overall qualitative observations indicated a somewhat deeper understanding of the material.

## 4.4 Conclusions

Although students have a firm grasp on the facts of the subject material, the data analysis shows that there are large gaps in their problem solving abilities. Students on the whole did not approach problems logically, but rather seemed to somewhat blindly apply what they thought was an appropriate formula. We emphasize that which test was taken, how long the student has been at Holy Name, the class they are currently taking, and whether the question came from the SAT or ACT had no significant effect on their performance.

One significant drawback with this analysis is that we did not have a control (such as, for instance, giving the test to a local public high school). This means that our assessment of how well the students should be doing is quite subjective and speculative. However, when designing the test, we took these into consideration. As a reminder, based on national averages, students should be able to get roughly 75% of the easy questions correct, 50% of the medium, and 30% of the hard. Unfortunately, Holy Name students fell somewhat behind these expectations, presumably due to weaknesses in their problem solving ability, and the weakness we observed in their trigonometry. We concluded that, in order for students to do well on the SAT and in future math courses, we should focus on problem solving and logical thought development.

# 5 Sample Lectures

## 5.1 Motivation

Based on the analysis of the diagnostic tests, we felt that the greatest deficiency in Holy Name's math curriculum was not, per se, a weakness in mathematical pedagogy, but instead a weakness in the students' ability to think logically. Much of the responses showed little to no consideration of the plausibility of the given answers, and the work shown on the answer sheets indicated a lack of thoughtfulness in their approach. Students often recalled and tried formulas that seemed appropriate, with little consideration to relevance or accuracy.

We wanted to address these faults in their approach, but we also wanted to see which approaches to mathematical education works best for Holy Name students. In particular, problem-based learning, despite some of its flaws [10], seemed like a promising approach. We noted that students seemed to not have an effective approach to problem solving. We felt that problem-based learning could improve their ability to solve problems by focusing on both applications and building independent thought.

## 5.2 Creation of Lectures

Once the results of the SAT/ACT diagnostic test were analyzed, the team determined that a more thorough evaluation was needed. To do this, we would teach sample lessons. These classes would be individualized for the level of math the students were taking. This meant we could choose material familiar to the students. Teaching to individual classes would help the team get a more personal evaluation of the students. We could see first hand how the students reacted to a subject with which they might not be completely comfortable, their problem solving methods and thought processes. This would also give us a chance to test our theories on education and the specific weaknesses of the current curriculum. We would be able to see what worked about our lessons, and what did not. Using this information, the team could adjust our recommendations for the school.

After it was determined which class we would be teaching, the team created a lesson plan that was catered to this level. We thought that breaking up the time into short sections would be the best way to keep the students attention. We started each class with a short speech about what we were doing with the school and how they were helping this. We emphasized that the group was there solely to get information and that they were not being graded. At the beginning of class, we would introduce ourselves and then pass out a quick survey for the students to fill out. In it, we would include two opinion questions: “Do you like math? Why or why not?” and “Do you feel that you are good at word problems?”. These were included to study students’ opinions of their own mathematical aptitude and enjoyment, which would add to our understanding of the student body. Then it asked them to complete three quick math problems that tested their logical reasoning skills and conceptual math knowledge.

Following the completed surveys, some quick examples of how to use logic to find an approximate answer or to check if an obtained solution is correct were run through by the lecturer. The class would then be split into groups to work on word problems. During this time, the team members went from group to group to listen to the students’ ideas on how to solve the problem as well as help answer questions they had. After about fifteen minutes, the class was brought back together and the problems were reviewed. At the end of each class, a second survey was handed out asking the same mathematical reasoning questions as the first. This was done to see if there was any difference in the way the question was completed or the answers. The students were also asked if they found our lecture to be helpful and why. In the end, we had taught two Algebra I classes, an Advanced Math class, and two Pre-Calculus classes.

## 5.3 Algebra I

### 5.3.1 Summary of Lecture

To start the class, the team introduced ourselves to the students and gave them a quick summery of what we were trying to accomplish. We then handed out the first survey and gave the students about five minutes to complete it. The first problem shown to the students was a question we got off the SAT/ACT diagnostic test that consisted of creating a ratio between different types of nuts in a bag to the total amount of nuts. Many versions of this problem were discussed as well as incorrect answers. By going over the wrong answers, the team hoped to get the students thinking about why a solution is right or why it makes sense. The next topic was meant to further this idea of checking an answer for feasibility. A linear equation was written on the board, and two possible answers were given. The students were then asked which of the two answers was most likely to be correct. Of the answers given, one would be logically impossible while the other answer, though not correct, would make sense. For example, the equation $156x - 32 = 50$ with solutions of $x = 2$ and $1/2$ was given. The answer of 2 can not be correct because multiplying 156 by anything over 1 would make it larger and thus farther from 50. After thoroughly discussing this with the students, they were split into groups and given a word problem.

The word problem asked the students to solve for the amount of time a person would need to pay off a \$200,000 loan from the bank with an income of $x$ dollars per month. We chose this problem because of the uncommon role the variable $x$ played in it. Students are most likely to see $x$ as the variable for which they are trying to solve, while in this problem it is an unknown constant and time must be found. This also means that the students would be given a chance to show that their equation made sense by choosing different values of $x$ and finding how much time it would take to pay off the debt. The equation should show that the more you make in a month, the less time is needed to pay off the debt.

Most students responded as we predicted, by assuming that they needed to find a value for $x$. However, once we asked them what they were solving for, they thought more into the problem and realized that the question was asking for time. Then, if the students were having more difficulty with the problem, we tried to relate it to something they had already done, or something with which they could personally identify. An example of this was asking the student how long they would have to work at a job that pays \$8.00/hr to buy a \$16.00 CD, which they were able to answer very quickly. With this new understanding, the students were able to set up the accurate equation. When the students were done setting up this equation, the lecturer asked them to justify it. Many of the students did not know how to do this so the lecturer began to plot a graph on the board by finding different points. This graph would be a variation of the $1/x$ graph, which the students had most likely never seen before. We explained that as the amount of money made approached debt, the time would approach zero, and while the amount made approached zero, time would go to infinity. The students seemed to understand what we were explaining as they were actively participating in the discussion.

Once this was completed, the final survey was distributed. With the time remaining after the students completed this, the most difficult survey problem was explained. However, there was not enough time remaining to complete this.

### 5.3.2 Results

Based on the pre-class and exit surveys, we substantiated some of our theories about the students and why they are having difficulties with mathematics. The first question on the quiz asked whether or not the students enjoyed math and why. Many of the students responded with answers saying that if they understood what was going on they liked it, or that they did not like it because it was too difficult to understand. They were then asked if they thought they were good at word problems. We found out after we taught the class that the students did not have much exposure to word problems. With that being understood, many of the students said word problems were harder and less enjoyable than other types of math. This was because the words confused them, or they lost the meaning of the problem within them, or thought they took too much effort. From these answers, the team hypothesized that the students did not have the confidence that they need to succeed in the subject.

Once the students were done answering these questions, they were asked a series of questions to test their understanding of basic math principals and their logical thinking skills. The first of these was an algebra problem where the student was shown the steps to solving for $x$ in two different ways. We asked them which of the ways was done wrong and why. The first problem shown was actually done incorrectly because the subtraction was executed wrong. The second problem was done correctly but in a different order than is usually taught to students; the first step that was shown was division and then subtraction. Most of the students caught the subtraction error. However, many students said that both of the problems were wrong because subtraction is supposed to be the first step. This reinforced the idea that the students just went through a process they were shown step by step and did not understand the reasoning behind what they were doing.

The next question was designed to test the student's ability to reason out a solution. It showed a picture, drawn to scale, of a triangle with the base and interior angle given. They were then asked if a given length for the other side was reasonable and why. Many of the students gave the correct answer and were able to say why the answer given did not make sense. This is helpful because it showed that the students can think about a problem logically, and see if a solution is reasonable. This led the team to believe that the students were not taught to think about an answer, but could do it once prompted.

The last problem given was the most difficult problem. It was a word problem that did not use any numbers which none of the students got correct. They seemed to not know what the question was asking for or how to setup an appropriate equation. For the most part, the survey questions were answered in the same way before and after class with the exception of this last problem. While the students still did not know exactly how to solve the problem, they seemed to have a greater understanding of what was being asked of them. This can be seen in the way the students started to set up equations or working through the problem and the increase of students attempting to solve it.

As an exit question, the students were asked if the lecture helped their understanding of the survey questions, or made them easier. While some students said that they understood everything before the lesson, or that they still did not know what was being asked, many said that they learned a lot from the lesson. For the most part, the students responded well and said that we helped them gain a better understanding of mathematics, and that they would now use our methods to check their answers.

## 5.4 Advanced Math Topics

### 5.4.1 Summary of Lecture

Having reviewed the results from the Algebra classes, and with the knowledge that we were going to teach an Advanced Math Topics class, we created another lesson plan. The format that was used on the previous class would be used again as it seemed to be successful. We started the class the same way we had previously, with a five minute opening survey. Because of the level of this class, we decided to cover some more advanced problems of math which still only used basic concepts. The survey would start with the same questions about how the students felt about math and word problems, and in the exit survey, our lecture. It would include the same basic algebra question used in the previous class, a question asking the students where the flaw in a proof was, and one where the students were asked to identify the shortest path across a rectangle out of three given solutions.

The lecture portion of the class first involved solving statistical problems involving different colored marbles in a bag which was comparable to the nut problem in the first class. This was done as a warm up exercise to get the students thinking about math topics that they might not have seen in a while. We also thought that it might help them with a problem that we were planning for the group work section of the class. We then went over questions similar to one which was on the SAT/ACT diagnostic. This question defined a function, "circle in a square", and then asked the student to solve a problem using this function. With this question, the team was hoping to get the students to think about using their math knowledge and to use it in a new way. We hoped to show the students that a basic understanding of math concepts can help them through many more complicated problems if used correctly. The lecturer asked the students questions as he was going along to engage the students, and also to get them thinking more.

Once the students showed that they understood both concepts that were being presented, the first group problem was explained which would be finding the ratio of circumference to area of a circle, and perimeter to area of a square. This problem was intended to show us the student's ability to set up ratios and then reduce them to get a simple, easy to understand equation and explain what it meant. Initially the students had a lot of trouble with this problem as it was unclear to them what was meant when we asked for a ratio. Once this was explained or reworded, the students had difficulty identifying what was being compared in the problem, or what they were being asked to find a ratio of. This was the most difficulty the group had seen the students have with a problem thus far. However, with guidance, the students were able to complete it.

After this was done, the students were given the same hardware store loan problem used in the Algebra I class. We repeated this question so we could look at the difference between how the classes answered and because it gave us a lot of information about the student's abilities. This problem was much easier for the students than the previous one. They still needed some hints on what the problem was asking them to solve for, but once this was done they had no problem solving it. The students came up with an equation, and then the lecturer went through the processes of making a graph of this and then analyzing it to justify their answer. The students responded well to relating different parts of math and explaining why a solution makes sense. Because the class went longer then expected, the exit survey was not administered until the next class.

### 5.4.2 Results

The first question the students were asked on the survey was whether or not they like math and why. The majority of students in this class answered that they did not like math because they did not understand it, it was too hard, there was too much to memorize, or it was boring. Some students said that they liked math as long as they understood what was going on, or if they had a good teacher. Then, the students were asked if they consider themselves good at word problems which most students said they were not. Some said that it was hard to figure out what a problem was asking when it was written out and not just a set of equations. Many students also said that they had difficulty setting up the equations. Overall, the students did not have a positive attitude towards math.

The first math question involved finding the error in a proof that we had set up for them. For our purposes, this problem was ignored because very few students answered it, and those who did gave incomplete answers or were unsure what was being asked. The next question was the algebra question used in the first class. The majority of students said that both solutions were incorrect because in the first one the subtraction was executed incorrectly and in the second one, the solution was reached by performing the operations in the wrong order. It was about an even distribution between students who said the first was done incorrectly and those who said the second exclusively. The last question the students were asked to complete, was to find the shortest path across a rectangle out of three options given. The answers for this problem were evenly distributed between the three options with about a quarter of the students not answering.

Unfortunately, none of these results changed with the exit survey. Even though the students did not respond well to our teachings on paper, we learned a lot from the class and were able to further substantiate our theories. We were also able to see which questions from each class were the most effective and use those to create a lesson plan for our final set of classes.

## 5.5 Pre-Calculus

### 5.5.1 Summary of Lecture

We were informed before we started creating a lesson plan, that the students were doing a lot of Algebra II review. Because of this, we decided that the test and class work would be more basic mathematical concepts similar to the first two lessons. These classes would have the same format as the last three; brief diagnostic entrance survey, a short lecture, group work, and then a final questionnaire. Many of the questions that were successful in the previous classes would be reused or modified slightly. This would not only give us questions at a level that the students were learning at, it would allow us to compare the answers for the upper and lower level students.

The opening survey had the algebra problem used in all the classes before, the shortest path question and a triangle geometry problem which would be a variation of the one used in the first class. It was changed to test the students not only of their logical thinking skills but also their knowledge of angle and common triangles. Most of the students needed the full five minutes given to finish and commented that the test was confusing or that they did not know how to answer the questions.

After the questionnaires were completed, the lecture portion began. It started with an interactive warm up lecture to get the students thinking about an answer, and thinking about how to solve problems that they had never seen before. We started by going over a problem that was on the SAT/ACT diagnostic test that was given out at the beginning of the year. The question defined a new function called "circle in a square" using $x$ and $y$. We then asked the students what this function would equal if 5 and 3 were used in place of $x$ and $y$. The students were hesitant to answer, but once they understood that "circle in a square" was a function just like multiplication or division, they answered correctly. A couple other new functions were defined and gone through quickly for the next few minutes.

Once this was done, the class was split into groups. We started this section of the class with the question that compares the ratio between circumference and area of a circle to the perimeter and area of a square. This problem showed weakness in the first class which is one of the reasons it was chosen for this class. The group wanted to see the difference in knowledge and understanding between the two classes. The students in this class had an easier time with this then the advanced math class however they still needed guidance. We went around to the groups of students and asked them questions that would force them to think about the problem and what they were solving for. Many of the students started by comparing circumference to area, for example, instead of the circle to the square but when they were asked what the question was asking them to compare, they said circle to square. After the students understood this part, they did not have a problem reducing the equations or the algebra.

The last question the students worked on was the hardware store loan vs. profit problem. Overall, this question had gone over well in the previous lectures, and gave us a good understanding of how the students were thinking and what they knew. The classes were ended with the same questions as the entrance survey. After the question was given, the students were given a hint of "distance equals rate times time". This hint was meant to help them relate the problem to other subjects that they might know and also to help them formulate the equation. Some time was given to the students to work on the problem before we went around asking if they had any questions. Some students had started writing down information and had ideas about how to start the problem. If they still were unsure about how to solve the problem, we asked them leading questions such as "what is rate, and what is it in this problem?" or "what is the distance in this problem?". This seemed to help the students understand the hint and the question better.

Once the students had formulated the equation on their own, we gave a short lecture going over ways to check this answer including the graph used in the other classes. They did not really know what we meant when they were asked if the graph made sense. However, once we said that it does because it shows that the more money you make the less time it takes to pay off the loan, they seemed to understand. We then derived an equation for total debt using $y = mx + b$. Each student was able to understand this formula, but many became confused when, in the context of this question, the slope was $x$ and the variable was $t$. However through some explanation and working through what each part of the total debt equation meant, they understood. We then graphed debt vs. time with various amounts of money being made every month. Changing the amount made monthly would change the slope of this graph, so that more money meant a steeper slope. The students had never seen a graph like this before, but we explained that this was another way to check your answer because as the more money was made, the closer the $x$-intercept or time would be to zero. For the last five minutes of the class, the students completed the exit survey.

### 5.5.2 Results

Between the classes and the surveys, the group once again substantiated our theory that the students memorized a process and did not know the underlying reason why. This is especially shown with the algebraic survey question. While there were some students who answered correctly, the majority of students said that both of doing the problem shown were wrong. Many realized that the second one had the correct answer, but said that it was arrived at incorrectly because division was done before subtraction. The next question on the quiz was finding the shortest path across a rectangle out of three given. Many of the students did this correctly which once again showed the students ability to reason out a problem if prompted. The last question asked the students if a triangle with two given sides and an interior angle was possible. About 60% of the students got this one correct and stated that the triangle was not possible because the two sides were equal, but the interior angle was not 45°. These answers did not change on the exit survey. The majority of students also did not think the questions any easier then before the class. Many of the students stated that they knew how to do the questions before, were still just as confused, or did not know how to apply what was taught in the class to the given problems.

## 5.6 Comparisons and Conclusions

After teaching five classes, we could validate certain things that we observed. The first issue that the team noticed was the method students were using to solve problems. The majority of students went through a problem step by step, in a specified order. However, when faced with a problem that was not presented in a way they had seen before or when asked to solve for something they had not solved for in the past, the students had trouble knowing what to do. Each class showed significant difficulty with the algebra problem in which an atypical method of solving the problem was presented. Another key troubling area that the team noticed was the student's confidence levels. Many of the student's initial response, when faced with these difficult problems, was to say they can not do it. However, once a member of the team went through the problem, the students were able to complete what was presented to them.

There were many influencing factors affecting the results from each class which we tried to take into account when reading through the surveys. One of these was the time of day that class was being taught. The classes held later in the day seemed to have less focus than the ones in the middle of the day. We theorized that as the day progressed, the students became more focused on getting out of school than participating in class. Another factor was the size of the class or group dynamics. The smaller the class was, the more comfortable the students seemed to be with one another, which helped and hurt equally. This helped while the students were doing group work because the students were able to talk out their ideas in a group of other students they knew well. On the other hand, the students were comfortable enough with one another that they would talk during the lecture time, and become completely unfocused at points.

The last factor we noticed that had a great impact on the class was the grade level of the students. The freshman students were more focused and interested in what we had to say. We thought that this might have been because they still have many years of math classes ahead of them and saw the value of learning mathematics, while the seniors were no longer invested in this aspect of their education. The seniors had already completed the SAT or ACT and were graduating soon, so they figured anything we were teaching them they did not need to know or would not be helpful. There was also a much smaller age difference between us and the students so they might not have had as much respect for our position.

# 6 Conclusions

## 6.1 Summary of Results

In order to make our final recommendations, we looked at the results from our diagnostic exams and sample lectures as a whole. The diagnostic exams were useful for pinpointing weaknesses in specific mathematical concepts. We were also able to determine the level of difficulty at which the students could perform. The sample lectures and the accompanying surveys were useful in observing the students' ability to think critically and apply simple concepts they already understood in nonstandard ways.

Through the analysis of the diagnostic test, we were able to answer our initial questions about where the students had weaknesses, and revealed several key points about the abilities of the Holy Name students. First and most importantly, the students performed significantly below national averages in every problem difficulty level and field of mathematics. The largest weakness by far however was trigonometry, with only 14.17% of all questions answered correctly. The easy and medium level questions had fairly similar results, with 51.94% and 40.78% respectively, answered correctly. The hard questions saw a steep drop in correct answers however, with only 21.81% answered correctly. Due to the nature of the hard questions, which generally apply math concepts in very nonstandard and deliberately confusing manners, we were able to deduce that the students had serious issues applying the equations and basic math concepts that they appeared to know fairly well. With the knowledge of the major weaknesses, we were equipped to do further diagnosis in the form of our sample lectures.

In forming our lectures, we tried to create math problems that we knew were based on material the students already knew. The entire purpose of the lectures was to see if the students could use this old material in new ways, as well as see which educational strategies worked best with Holy Name students. From the surveys we administered, we learned that the students did not have much previous experience with the types of problems we were utilizing. Upon reviewing the rest of the results of the surveys and making observations about the lecture portion, the lack of experience with nonstandard math problems became more apparent. Generally speaking, when left to think on their own, the students had trouble thinking through our math problems from beginning to end. When we gave them some hints as to how to approach the problem, they usually were able to complete them correctly. This suggests that the students are indeed knowledgable enough to succeed in mathematics, but they currently lack the confidence and experience to flourish.

## 6.2 Reflections and Limitations

Although we were satisfied with the outcome of this project, we are aware that there are clear limitations, as well as some things we certainly could have done differently. Knowing what we do now about the final direction of the project, we could have done more applicable background research. While the various educational theories, such as Learning Styles or Multiple Intelligence, were useful for our general understanding, only problem-based learning was directly used in the formulation of our project. In addition, our initial progress was slowed by communication difficulties between ourselves and Holy Name. We were unaware that Mrs. Mary Riordan had left her position as headmaster; because she was our primary contact, this lead to confusion on our part.

We feel that the diagnostic test was overall quite successful. We were able to get a wealth of information regarding the breakdown of students' strengths and weaknesses from the resulting analysis, and successfully eliminated many external factors. It did have its limitations, however. The students had no incentive to perform well on the test, as they knew ahead of time that it would not count towards their grade. In addition, we ordered the questions on the test in a manner logical to us in order to simplify the program running the analysis. Had we instead randomized the order of the questions, students without sufficient time to complete the entire diagnostic would have still gotten a representative sample of questions, rather than a sample primarily taken from the SAT.

Lastly, because of our time constraints, we were unable to explore as many methods of teaching as we would have liked. While we feel that we gained quite a bit of insight from the five lectures we did teach, there is certainly more work to be done. In particular, some of the questions in our lectures could have been either re-worded or eliminated completely. For instance, the 'false proof' question from the Advanced Math class (Appendix C) had unreasonable expectations of the students. It also would have been useful to do follow-up lessons with the classes we had already taught once.

Despite these limitations, we are overall confident in our findings. In a presentation to the Holy Name Math Department faculty the teachers noted that a lot of our observations were things they had previously noticed, and said that it was nice to have these observations independently and statistically corroborated. We also had broadly positive experiences with the experimental lectures; in some cases, the students made genuine leaps in understanding.

## 6.3 Recommendations

Based on our observations, study of educational theory, and discussions with Mr. Marcotte, we formulated a series of recommendations. We broadly divided these into short-term and long-term plans, with some of the long-term plans being dependent on how well-received the short-term plans are.

### 6.3.1 Observed Strengths

Throughout we emphasized cautiousness and small steps forward. There are plenty of positive aspects to Holy Name students' math comprehension which should be noted, commended, and preserved. We want to avoid compromising their strengths by being overly risky. Despite their weaknesses in formulating ideas and rationalizing mathematics, we noted that their intuition was quite strong. Supporting this observation is the students' overall strong performance on certain problems designed to test this, including the "shortest path" question. Although it is difficult to tell where in their education that they receive this intuition, we feel that it is an important skill that we should try to keep. Another positive, which we ascertained primarily from the diagnostic tests, is that the students know various mathematical facts quite well. Most students were familiar with things like the Pythagorean theorem, geometry, and basic properties which are key to mathematical understanding. They are clearly learning mathematics; we emphasize that the problem is that they have trouble applying what they learn. We also observed that the students, on the whole, worked quite well in group settings. These were all taken into consideration when creating these recommendations.

### 6.3.2 Short-Term Recommendations

Our first short-term recommendation comes directly from the questionnaire we gave to the three classes. Because a number of students seemed to get the "linear equations" question wrong, we feel that one positive change would be a slight change in the way that linear equations are taught. In particular, based on our own experiences and analysis of the students' responses, we note that students always learn to subtract constant terms first, then divide by the constant coefficient of the variable. Possibly because of this, students thought that reversing these operations was incorrect, despite being mathematically sound. Although this is not necessarily a practical thing to do, not realizing that the reverse operations are equally valid indicates a lack of mathematical understanding which can manifest itself beyond simple linear equations. By teaching students to think more broadly and universally, we hope that they will understand more fully. In particular, this might make solving systems of linear equations more straightforward. On the other hand, by extension of some of the arguments in [10], there is the valid criticism that teaching linear equations more universally may result in students being less capable of actually solving them. Students may get bogged down in abstraction, and not necessarily get a broad view of the method of finding a solution. Kirschner et. al. [10] note that self-learning puts additional stress and responsibilities on the student, which detracts from learning the problem at hand. We cannot fully say whether this will happen or not in the case of Holy Name. To gauge this, we encourage consistent examination and testing (not necessarily exams) to make sure that students retain practical problem-solving abilities.

Another recommendation is very simple, but we feel that it could be very important. Rather than solving for $x$ in nearly every equation, we recommend changing the variables around somewhat. We noticed that students were often highly confused when asked to solve for a variable that was not $x$.[3] This was made most clear by the hardware store problem, where we deliberately used $x$ as a constant to see how students would respond. By using non-standard characters for variables, we believe students will focus more on what they are actually solving for, and hence focus more on what they're actually doing.

On a related note, we also think that students can be engaged further by asking "leading questions" of them rather than solely telling answers. We do not mean merely asking the class questions about the material, but instead to make a conscious effort to have the students, as a whole, discover concepts on their own. As a simple example, one can show the students what an x-intercept is graphically and have them find out how to do it on their own. While this might be excessively demanding of the students, its gradual introduction may pay off by teaching students to think more independently. However, the risks are quite high; as mentioned above, something like this may reduce the students' actual technical competence.

[3] We note that this extends even to college math students.

We give this recommendation with the cognizance of the ill-fated “New Math,” [24] which showed educational researchers that an emphasis on fundamentals does not lead to increased mathematical skill. As satirist Tom Lehrer noted, “in the new approach, as you know, the important thing is to understand what you’re doing, rather than to get the right answer.” We therefore believe that caution and hesitation should be used here, but that the option is worth pursuing.

Our final short-term recommendation is to improve the students’ self-confidence. Often a student in a group would nearly solve the hardware store problem, but refrain from actually writing the solution due to uncertainty, and would only give the answer when encouraged. We do not feel that the students have particularly low self-confidence, but an improvement would give them the boldness and bravery required to make conceptual leaps. This not only might encourage self-learning, but we think it may also reduce student apathy and aversion towards mathematics. Hembree [7] wrote a paper detailing that increased mathematical self-esteem alone led to marked increases in test scores and mathematical understanding among high school students, particularly among males. Efforts to encourage students in their studies is a benefit we cannot exaggerate.

Depending on how students adjust to changing the variables and the linear equation changes, we think a related benefit might be introducing abstract characters in algebraic expressions, rather than concrete numbers; for instance, asking students to solve $ax^2 + b = c$ for $x$. We noted that the students had quite a bit of difficulty adjusting to this concept because (say, $c$) was “not a number.” Of course, it is a number, just not one explicitly given. We think that introducing more abstract notation, even if it might be a rough transition, will help students think more thoroughly about the mathematics involved, as it turns algebraic problems into something more thought-intensive than simply taking the (given) numbers away from $x$.

### 6.3.3 Long-Term Recommendations

One major change we propose is an ideological shift to an approach in the spirit of the problem-based learning theories. We noticed that, after a somewhat slow initialization period, most students seemed to fully understand the hardware store problem, even if they did not quite understand all of the math involved. Depending on how students adjust to the minor changes in keeping the variables free and asking leading questions, we feel that taking the problem-based approach could be beneficial. However, because a course fully emphasizing problem-based learning has been shown to lead to lower SAT scores in certain cases, we do not recommend a complete overhaul of the curriculum. Rather, we think that having a single day per week (or possibly biweekly) of problem-based learning could be a compromise that allows students to learn independent thought without sacrificing their actual mathematical knowledge.

We also feel that, should problem-based learning not be implemented in full, certain facets of it could be adapted and used to help improve student engagement and understanding. In particular, using real-life examples, known to aid student motivation, can also be more strongly emphasized. In particular, we think that having students build equations as solutions to word problems, rather than merely answers in the form of numbers, could be of enormous aid to their skill, despite being highly demanding of them. Furthermore, an equation can be analyzed much more readily and broadly than a single model, and it shows students word problems whose solutions do not boil down into simple arithmetic.

In our presentation to the Holy Name Math Department faculty we discussed some of these short-term and long-term recommendations. Overall, they were quite well received. One teacher in particular seemed very open to the ideas of programming classes as well as another IQP doing a control/experiment study of some of our methods applied to geometry courses. The PowerPoint presentation given is included in Appendix E, along with a further description of the meeting.

## 6.4 Future Plans

Overall, this IQP was very successful in terms of gaining knowledge about the school as well as creating a preliminary set of recommendations for improvement. However, the group realizes that there is still a lot of work that can be done. It will take time to implement any new idea within the math department as well as see if there are lasting results from this. Because of this, we have come up with some ideas for future WPI involvement as well as other ideas to continue our work.

Our first idea would be creating an introductory programming class that students would take in the early years of high school or even in middle school. It has been shown that learning a programming language helps students to think in a logical manor, and look at a process in steps and identify how to complete it [4]. We believe that the students would benefit from this, and also would learn a useful skill. Unfortunately, this would require that a completely new class be introduced, which might have to take the place of another class. The team thought that a possible solution to this would be having this class be taught for the students when they just enter the school instead of a math class. During the class, the math class that the students should have completed in their previous year could be incorporated for review. This would bring all students to the same level when they enter the math sequence of classes. This class could potentially be designed through another IQP, an idea enthusiastically accepted by the teachers at our final presentation.

Another idea that we had was to get another IQP team to continue our diagnostic and sample class work. Due to the limited time we had to teach, we did not gain a complete view of how effective our classes were. Students from WPI could use their time to create more lesson plans that would cover more areas or theories for improvement then ours did. Because the IQP would only include these lesson plans, the new team would have much more time to try different ideas, where as we only had time to test our final theory. They would also have time to gauge how the students responded, and see if there was any significant improvement by doing diagnostics throughout their work. Unfortunately, this has the downside of having to allot time within a course schedule for the WPI students to teach, allowing less time for the normal course work. This can be avoided, however, by incorporating the normal course work into the experimental lessons. Overall, while we feel significant inroads have been made regarding Holy Name High School's math department, there remain plenty of areas still worth examining, a task which could be fulfilled by various other IQP groups in the coming years.

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# Appendices

## A Educational Theory

There are numerous different types of theories in existence when it comes to education. Some of them oppose others quite directly, while many can be grouped into sets of similar theories sharing many viewpoints with one another. Through reading about a great deal of different theories, we chose several that both resonate with our views and could potentially be applied to Holy Name High School's Mathematics Department in our attempt to improve the understanding and aptitude of the students. While some types of theories are clearly unsuitable to a high school atmosphere, dealing with abstracted ideas of how the human mind absorbs and retains information, those which we have settled upon are primarily focused on the ways different individuals require information to be presented to them in order to best understand it. Here we attempt to produce a description of the major theories we have settled upon utilizing.

### A.1 Learning Styles

The information in this section is from, unless otherwise stated, Felder and Solomon's summary of learning styles [3].

The theory of learning styles is a label that has been applied to the work of a great many educational researchers. At the most fundamental level it refers simply to the idea that different individuals have different approaches to learning material. A common example is that one student may grasp a particular concept easily if some form of illustration accompanies it, while another may better succeed if presented with an abstract discussion alone. The particular theory of learning styles discussed here refers to categorizing individuals based on four axes. These dichotomies are referred to by their extremes, although each student typically has values that lie somewhere in between. These four categories are as follows:

- Active/Reflective Learners
- Sensing/Intuitive Learners
- Visual/Verbal Learners
- Sequential/Global Learners

The meanings of each of these will now be discussed each in turn and in detail. It is important to remember that these do not sort students into being better at a given subject in school, but rather indicate how students would best learn in any subject. Just because someone is a verbal learner does not make them unfit for Mathematics and destined only to study English, but rather indicates that a given mathematical concept must be presented in a different manner from a visual learner to gain the same level of understanding.

### A.1.1 Active/Reflective Learners

Active learners are people who, when presented with a new body of information, will immediately begin to manipulate and do things with it. They tend to want to try things out rather than merely ponder their implications. Group work will generally appeal more to active learners and will likely find it very hard to deal with a lecture-only style classroom setting. If placed in such a classroom they will likely find it beneficial to study for things in large groups and discuss the what-ifs of an upcoming test.

Reflective learners are, as one would suppose, of opposite tendencies. They prefer to take newly learned information and think about it on their own before doing anything about it. When given a new problem, these learners analyze the details and think for a while before supplying an answer. They will often have difficulty in a very fast moving class where they are left without time to sit and think, but can make up for this by going through notes and pausing frequently while studying.

As a result of these differences, active learners will tend to make more mistakes than reflective learners, not because they are less intelligent, but simply because they voice their thoughts as they reach them. Conversely, reflective learners will often take longer to complete a task due to the time they spend thinking and planning before beginning.

### A.1.2 Sensing/Intuitive Learners

Sensing learners are people who are more comfortable when material is concrete and grounded. They prefer learning facts and are most comfortable when presented with well defined routes to follow for a given type of problem. They are usually practically minded and seek connection because course material and the real world of everyday experience.

Intuitive learners will be much more comfortable with abstraction and prefer possibilities and relationships over factual data. They typically latch onto new concepts faster and will be less adverse to being tested on material not explicitly covered in the course of a class. They dislike rote memorization of fixed steps to follow and would prefer innovation be required of them.

Similarly to an active/reflective learning design, a course structure might be devised that relies heavily on step-following, causing intuitive learners to perform below their potential, or conversely tends to reserve the 'A' range of grades for those people who were able to make the extra insightful leap on an exam, something which disfavors sensing learners.

### A.1.3 Visual/Verbal Learners

This is perhaps the most common of the four measures of learning style discussed herein. Visual learners are easily described as those individuals who benefit from graphical aid to material being presented. In the context of mathematics this is easily recognized as graphs, charts, diagrams, etc. These people will often learn better if they see and construct graphical representations of ideas learned in class, visually linking related concepts.

Verbal learners do not receive as much aid from such devices, and require written and spoken explanations to fully understand concepts. In mathematics this can manifest itself as a teacher speaking the words corresponding to algebraic symbols as they write formulas on the board or putting words to every facet of a figure when one is presented. These learners will tend to benefit from writing summaries of what they've learned or working in groups so as to both listen to their fellow classmates explain different interpretations and practice putting a voice to their own.

### A.1.4 Sequential/Global Learners

Sequential learners take material one step at a time, moving methodically and logically toward a solution. They can find it potentially troublesome if too many steps are skipped at one time in the course of a problem. On the other hand, global learners pick up information in bits and pieces until a moment of sudden realization is achieved. They can sometimes solve a complex problem very quickly and do interesting new things once this realization has occurred, but then find it difficult to explain how or why they did what they did. Global learners might benefit from gaining an overview of an upcoming chapter by skimming the material before the lesson is begun.

These descriptions can make many people incorrectly conclude that they are global learners, however, as most everyone has experienced a time when after a period of groping with half-learned facts the final understanding suddenly snaps into place. It is important to realize that what makes someone a global learner is their inability to manipulate those same half-learned facts until the revelation has occurred. Table 3 summarizes the four categories.

Table 3: Learning Styles Summary.

| | Left extreme | Right extreme |
|---|---|---|
| Act/Ref | Immediately manipulate new information, try out new ideas, enjoy working in groups | Thinks on new concepts before applying hem, analyze details of problem before responding, prefer slower class with time to think |
| Sens/Int | Prefer concrete/grounded material better at learning facts and dealing with well defined routes of problem solving | Better with abstract ideas, grasp new concepts faster and prefer being required to use innovation to solve a problem |
| Vis/Verb | Benefit from pictures, charts, diagrams, etc., being used when learning | Benefit from written or spoken explanations rather than graphical aid |
| Seq/Glob | Take things one step at a time, move through problems methodically and logically | Gather information in pieces until a realization is made and everything comes together |

## A.2 Structure of Intellect

Structure of Intellect (SI) theory takes the concept of intelligence and breaks it down into three groups of classifications: operations, contents and products. It further provides five operations, five contents, and six products, resulting in a total breakdown of intelligence into 150 separate components [23]. This is likely unnecessarily complicated for the situation of Holy Name High School, as any given combination would apply to a very small number of students simply due to the sheer number of combinations possible, making targeting of specific types logistically impossible. Regardless, it serves to lead into another theory yet to be addressed, and is thus worth briefly describing. In general, the theory can be thought of as using one of the operations to address one of the products in terms of one of the contents.

The five operations SI theory names are as follows:

- Cognition
- Memory
- Divergent Production
- Convergent Production
- Evaluation

To elaborate, cognition is the act of recognizing or discovering something. Memory refers simply to recall, as the name would indicate. Divergent production is taking a problem and producing a large number of solutions for it while convergent production seeks the single ideal solution to a problem. Evaluation consists of making such determinations as positive or negative, good vs. bad, etc. These are intended to entail the distinct ways in which a person can approach and contemplate new concepts [19].

The six products put forth by the theory are:

- Units
- Classes
- Relations
- Systems
- Transformation
- Implication

Within products, a unit refers to a singular entity, and can be such things as a number, letter or word in addition to things like balls or a person. Classes are higher order groupings of units, for example positive numbers, sentences, or people. Relations are the connections between concepts. Transformations are restructurings of contents and implications inferences made from separate pieces of information. These are supposed to represent all the types of things on which a person can operate [19].

The final category, contents, is broken up as:

- Visual
- Verbal
- Symbolic
- Semantic
- Behavioral

Visual and verbal are similar to the definitions of the learning styles, except that here they refer to the properties of the products that can be experienced through those senses. Symbolic refers to those things we experience in terms of numbers, letters or symbols, while semantic is things we experience through ideas or meanings. Finally, behavioral is things we experience through actions and expressions of the people around us [19].

These many combination can be made concrete with examples like Evaluation of Semantic Units being the determination of which objects on a list fit a certain description or Divergent Production of Symbolic Relations being the generation of formulas linking two numbers. In this way any conceivable action fits into its own category. As noted above, though, this theory can grow quite cumbersome due to its size, and serves best to introduce the next theory.

## A.3 Multiple Intelligence

The Multiple Intelligence (MI) theory is in many ways a simplified and generalized version of SI theory (Sec. A.2). The two agree on the point that intelligence cannot be measured by a single, stand-alone metric, but rather has multiple independent factors. However, while SI theory structures intelligence into the 150 combinations touched on in Section A.2, MI theory first attempts to set forth a set of criterion for each of the subdivisions. In addition, MI theory focuses less on the idea of measuring each facet of intelligence with standardized tests and more on accepting that intellect takes many forms and judging intelligence more subjectively.

The criterion for something to be considered a type of intelligence is set by MI theory as [5]:

- The existence of a distinct developmental history for a capacity

- The existence (or lack thereof) of correlations between capacities
- Individuals who are prodigies, idiot savants, or exhibit learning disabilities
- Ethnographic records of how a given intelligence is fostered or discouraged by different cultures
- The existence of symbol systems to encode certain kinds of meanings
- Biological evidence that a given intelligence is located in a specific portion of the brain
- Evidence of a distinct evolutionary history for a given intelligence

Based on these criterion an original list of seven intelligences was devised [22] [11]:

**Spatial Intelligence** This corresponds to what previous theories have named visual. These individuals think strongly in pictures and retain information in vivid images.

**Linguistic Intelligence** Linguistic intelligence is what the other theories discussed have named verbal. People who tend toward this type of intelligence will excel with material presented in the context of words and descriptions.

**Logical-Mathematical Intelligence** Although often considered to go hand in hand with spatial intelligence, MI theory asserts that it is different. Logical-mathematically intelligent students will tend to be very logical in their thought processes and form patterns easily. They will often be good problem solvers and abstract thinkers.

**Bodily-Kinesthetic Intelligence** These types of people will frequently express themselves through movement and tend to have excellent hand-eye coordination and balance. They will learn a concept best if there is some means for them to interact with it physically and are often good at constructing things with their hands.

**Musical Intelligence** Like the logical-mathematical individuals, musically intelligent people also have an affinity for patterns but add to this an affinity for sounds and rhythms. They will frequently be quite sensitive to any ambient noise in their learning environment and respond positively to music they find pleasing. Although these individuals are obviously musically inclined, concepts in mathematics can be potentially made clearer through analogy, such as fractions to musical rhythms.

**Interpersonal Intelligence** People with this intelligence type often relate very well to those around them. They will try to empathize with those around them and usually respond well to working in groups.

**Intrapersonal Intelligence** The final type of intelligence refers to those individuals who manage to think introspectively very well. They will often try to understand themselves, their own strengths and weaknesses, and their relations to others.

Since MI theory was first put forth, an eighth type of intelligence has been added [5], naturalist intelligence. These people tend to be very sensitive to the world around them and how they fit in with it. They frequently have an affinity for caring for plants or animals and will learn better if the subject matter can be tied to the natural world. This is a good example of the criterion for an intelligence type set out by the theory as it was added later due to the discovery of a portion of the brain dedicated to recognizing natural objects and differentiating them from man-made ones. A ninth existential intelligence has also been proposed, dealing with the human nature of raising fundamental questions about existence or life and death, but is yet to be accepted pending the discovery of a portion of the brain relating to such things.

Given the breakdowns MI theory affords it is tempting to simply split everyone into groups according to which intelligence they exhibit the most and then merely teach each group according to that intelligence. The theory itself takes issue with this approach, though [5]. First it advises strongly against doing so on the grounds that MI theory is a tool to improve methods of education rather than an educational goal in and of itself. Once other goals have been established it should be used to aid the different types of students towards achieving those goals by incorporating different facets into the lesson that address multiple types of intelligence. Second it says that not every type of intelligence directly applies to every field. In mathematics, for example, it is harder to conceive of teaching methods which would incorporate both inter- and intrapersonal intelligence into the curriculum. MI theory also says that while everyones distributions among the intelligences are different, everyone also has a certain measure of every kind. Therefore, just because someone is very inter- or intrapersonally focused should not preclude them from being able to absorb mathematics knowledge from a curriculum which doesn't address those types as strongly as others.

In addition to these points the theory warns of confusing intelligence with things such as wisdom or morality. While certain intelligences might lend themselves more towards very wise or moral behavior, this does not make them the same thing. It also cautions not to expand the theory into a device aimed to prescribe behavioral norms to people so as not to trivialize it through over-simplification, for example, teaching math to musically intelligent students by having them set it to song or listen to enjoyable music while studying [5].

## A.4 Aptitude Treatment Interaction

Aptitude Treatment Interaction (ATI) most generally refers to the idea of looking at the performance of a student or group of students as a function of the various teaching options available [21]. This is based on the cognitive styles possessed by the student or students who make up the group. This theory builds off of all the previous theories and serves to demonstrate some ways in which they may be applied to actual classrooms. The cognitive styles of ATI are the different ways in which people can learn, and can be taken as the very same learning styles of the previously mentioned: Active/Reflective, Sensing/Intuitive, Visual/Verbal and Sequential/Global [14]. Also, as will be discussed shortly, ATI seeks to find a teaching option that best enhances the performance of the students. The SI and MI theories would both imply that each intelligence type has a certain teaching style that would work best for those students, thus fulfilling the criterion set by ATI.

The first step in developing an ATI involves finding some measure of performance that you wish to maximize. In the case of Holy Name High School it would be conceivable for this to be math SAT scores. The outcome of performance for a variety of different types of teaching is then measured for many students. Trend lines can then be fit for those students who follow a given learning style or have a certain intelligence type, and with them a graph of performance versus teaching types made. Two such representative graphs would be in Figure 1 [14]:

A graph of the left variety would seem to indicate two teaching styles that favor two opposed learning styles. Treatment 1 clearly favors Aptitude 2 while Treatment 2 favors Aptitude 1. At this stage several options are available. One option is for the aptitude groups to be put in separate classes and each taught with the appropriate treatment. This can potentially lead to problems with an insufficient number of teachers, though. Alternatively, one groups education can be neglected, and the teaching style set to maximize the other,

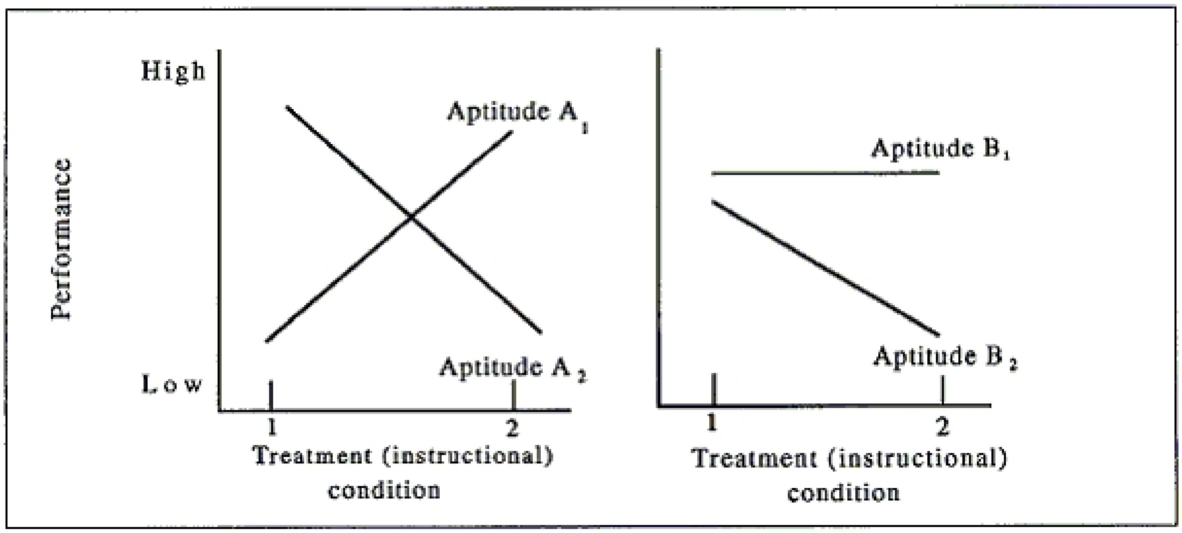



Figure 1: Two types of aptitude by treatment interactions.

clearly an unacceptable situation. Finally, some type of mixture of the two treatments can be attempted, resulting in the best overall performance from both groups in a single classroom. The right graph is easier to interpret, as group 1 is unaffected by treatment type, the treatment which maximizes group 2 should clearly be selected.

One complication with the ATI approach is that the learning styles vary not only from person to person, but sometimes also within a person depending on what is being done or their environment. Also, the level of structure imposed by the environment will affect those with higher ability and those with lower ability differently [21]. Finally, it is important to emphasize that when this model discusses ability, it does not refer to the innate overall intelligence of the student, merely the effectiveness of a given type of teaching on that student.

## A.5 Problem-Based Learning

A problem-based mathematics curriculum entails teaching mathematics in the context of problem solving. For instance, rather than solving the equation of a line with slope $b$ passing through the point $(c, d)$, a student may be asked to find a model for a stores total savings if they make $b$ dollars a day and had $d$ dollars on the $c^{\text{th}}$ day of the year. This isn't intended to be a simple word problem; students collaborate amongst themselves without ever expressly being told that the model is linear. The power of problem-based teaching is that it directly addresses a common concern among students regarding how useful their math education truly is. Furthermore, problem-based learning has been shown to improve student flexibility in their approach to problem solving, and to improve a student's intuition of mathematical concepts. Gains in SAT scores seem to be, at best, modest, but this approach to curriculum reform seems well-suited to a math program which fails to engage the students [12].

An overview of problem-based learning (in general, not specific to mathematics) is given in Reference [8]. Basically, problem-based learning entails replacing both assignments and lectures with a general, somewhat complicated problem which is worked on in groups. The key is that deficiencies in knowledge are identified by the students, who then go to the teacher as a reference and a guide, rather than a lecturer. The teacher is "no longer the repository of knowledge, s[he] is the facilitator of collaborative learning." Figure 2 [8] illustrates the process with a flowchart.

The problem is given by the teacher. Students analyze this problem by identifying what they know from both the situation and their prior knowledge, then hypothesize a possible solution. If they can't do so, or if they feel that their current solution is not satisfactory (which may be at the discretion of the teacher), they realize that they need new information, and obtain it themselves. They then apply this information to refine their hypothesis. After solving the problem, the students then reflect on what they learned. The important distinction between this form of learning and traditional techniques is that the students actively seek and apply knowledge on their own initiative, without passively writing notes as in a normal class. According to Hmelo-Silver[8], this improves a student's ability to learn successfully, as well as identifying their own learning style. Furthermore, it is suggested that this doesn't only apply to skilled learners, but even less-skilled students in the class. However, this data is still somewhat incomplete, being based only on a few case studies.

Reference [6] give an example of problem-based learning in the context of high school mathematics. The problem is analysis of linear data. The students are introduced to a scatterplot of cord length vs. weight of a bungee cord (Fig. 3 [6]), and asked questions such as "What is the length of the unstretched cord?" and "How much does one ounce of weight stretch the cord?" By teaching concepts of slope and y-intercept with natural physical ana-

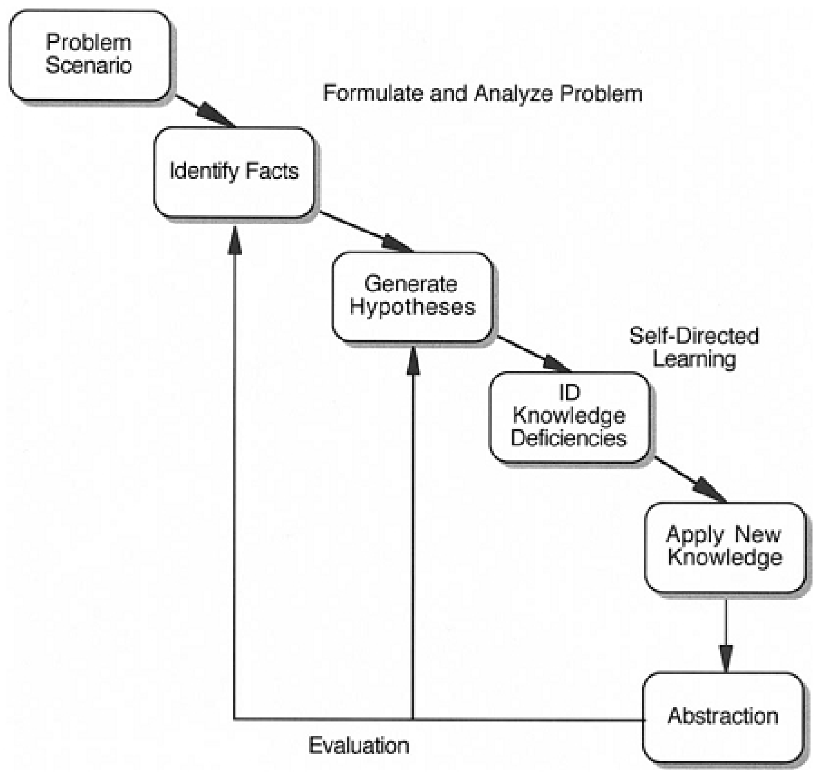



Figure 2: The problem-based learning cycle.

logues, students gain intuitive understanding of the concept, as well as an appreciation of its uses.

The concept is further extended to examining relationships between linear graphs, tables, and equations more abstractly, followed by more examples (rubber bands of same length, but different thickness so that the slopes of the stretch v. weight graphs are different). Not only are the students asked to explain the physical significance of the data, but also to explain the mathematics behind it. The teacher does not impart information directly, but prods the students appropriately so that they come upon the information on their own accord.

One study[10] analyzed the effectiveness of problem-based mathematics curricula in three Californian high schools. A detailed analysis is given in their paper, but generally SAT math scores stayed about the same: for two schools, problem-based students were 22 points higher (or 2.75 standard deviations) in one school and negligibly higher in the other. However, marked improvements were shown in students self-confidence, appreciation of mathematics,

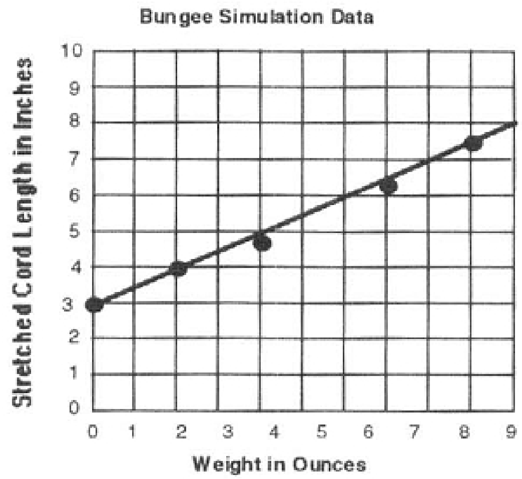



Figure 3: The bungee simulation data.

and (perhaps most significantly) propensity to reason problems out using words and diagrams rather than numbers.

# B Meetings at Holy Name

- 9/7/07 - Phone Contact: Found out there was new headmaster and head of math dept.
- 9/18/07 - Meeting with Mr. Marcotte, head of Mathematics Department: Discussed the project specifics such as background research, the final goals and how we are going to accomplish these.
- 10/5/07 - Meeting with Mr. Marcotte: Discussed administering a diagnostic test to the juniors and seniors and getting past SAT results from CollegeBoard.
- 11/7/07 - Meeting with Mr. Marcotte: Discussed the test outcome, and how the project will progress after they are analyzed.
- 11/28/07 - Meeting with Mr. Marcotte: Discussed what the first lesson plan should include, who we would teach and what we wanted to show them.
- 12/12/07 - Meeting with Mr. Marcotte: Showed him our lesson tentative lesson plan and received feedback on how to make it more effective.
- 12/19/07 - Sample Lecturer: Taught two Algebra I classes.
- 1/30/07 - Meeting with Mr. Marcotte: Discussed how the first classes went, and teaching more classes.
- 2/4/07 - Sample Lecture: Taught one Advanced Math class.
- 2/8/07 - Sample Lecture: Taught two Pre-Calculus classes.
- 2/27/07 - Final Presentation: Made a presentation to the entire math department discussing what we did over the course of our project, what our final recommendations were, and answered any questions that the teachers had.

# C Diagnostic Tests and Lecture Materials

The following section contains the various materials used at Holy Name High School. The first page is the cover page for both diagnostic tests. This is followed the diagnostic tests, each 7 pages in length with Test A first and Test B second. After the diagnostic test materials are the pre- and post-lecture questionnaires administered to the Algebra I, Advanced Math, and Pre-Calculus (in that order) classes.

What type of calculator are you using? (Please circle one)

a. TI-89 or similar
b. TI-83, 84 or similar
c. Scientific
d. 4 function (basic calculator: +, -, ÷,×)
e. None

When did you start at Holy Name High School? (Please circle one)

a. Beginning of middle school
b. Beginning of high school
c. Other
   Please specify ____________________________________________

What math class are you currently taking?

___________________________________

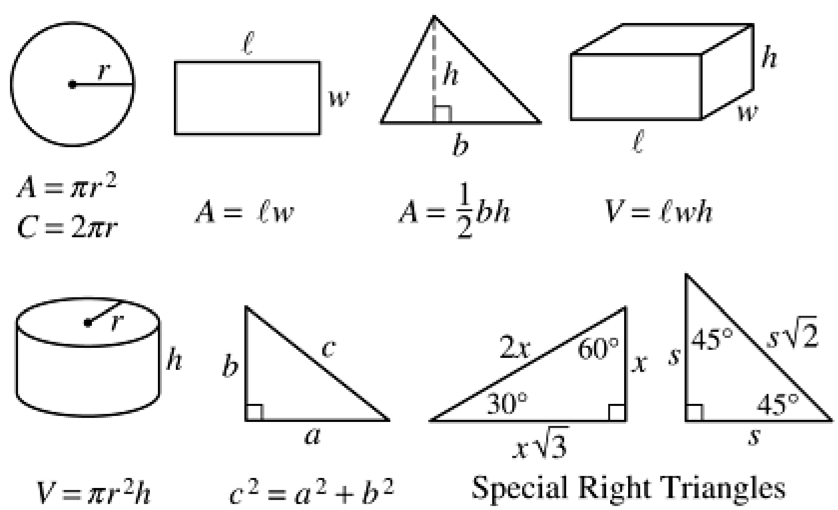
Which math course did you take last year?  What was your grade?

___________________________________

If you have taken the SATs before, what score did you receive on the mathematics section? (Optional)

________________________

Reference Information




The number of degrees of arc in a circle is 360.
The sum of the measures in degrees of the angles of a triangle is 180.

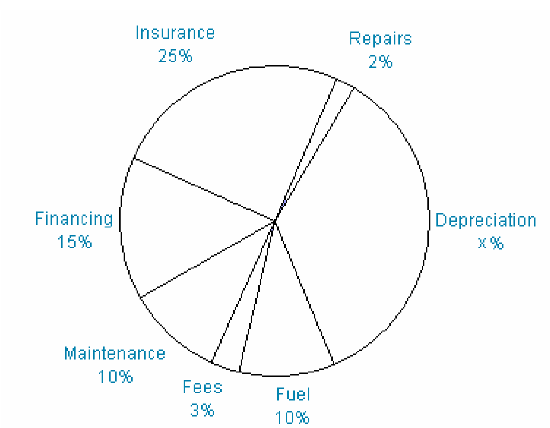
1. If | x – 3 | > 3, which of the following could be a value of x?

A. -1
B. 0
C. 2
D. 3
E. 6

2. If x = 10, what is the value of $\frac{x(x-1)+x(x+1)}{x}$?

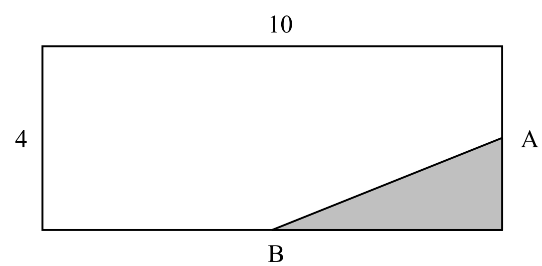
3. According to the graph, what percent of the total 5-year ownership cost of a new automobile is depreciation?

THE TYPICAL 5-YEAR OWNERSHIP COSTS OF A NEW AUTOMOBILE




Total Cost: $36,000

4. If A and B are the midpoints of the two sides of the rectangle below, what is the area of the shaded region?



5. If $x$ and $y$ are integers and $xy = 24$, then $x + y$ could be equal to which of the following?

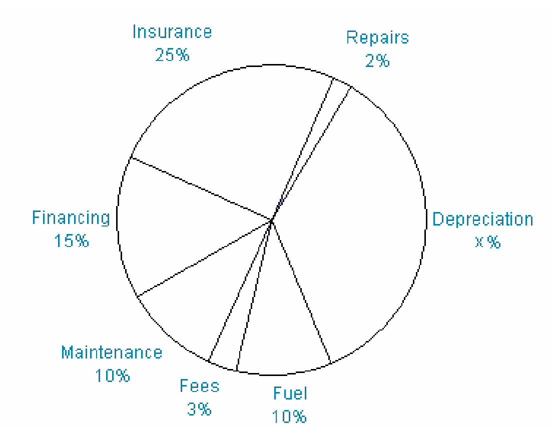
I. 10
II. 11
III. 12

A. I only
B. II only
C. I and II only
D. I and III only
E. I, II and III

6. For all positive numbers x and y, let $x$ ◙ $y = 1 - x/y$. What is the value of 3 ◙ 5? Simplify your answer.

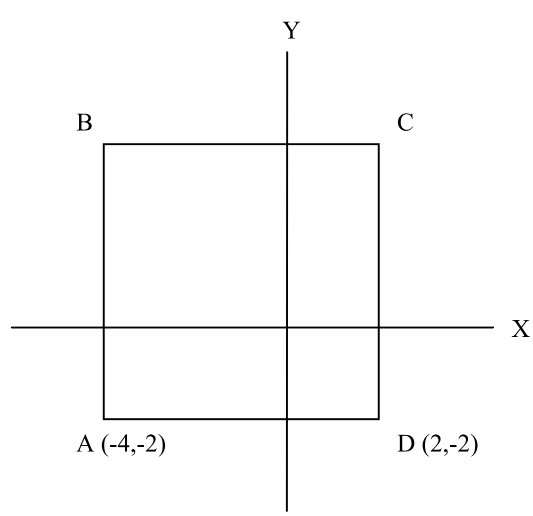
7. Tracy purchased a new automobile. For the first 5 years, she is responsible for all costs except insurance, which her parents are paying. If all her costs are typical, as shown in the graph, then maintenance represents what percent of the costs for which she is responsible.

THE TYPICAL 5-YEAR OWNERSHIP COSTS OF A NEW AUTOMOBILE




Total Cost: $36,000

8. In the figure below, ABCD is a square. What are the coordinates of point B?



9. If X and Y represent digits in the correctly written addition problem below, what is digit X?

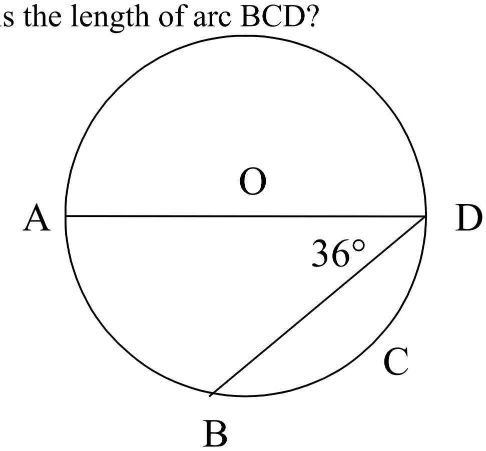
```
  XY
+YX
 1X3
```

10. If $(x + y)^2 - (x - y)^2 = 84$ and $x$ and $y$ are positive integers, which of the following could be a value of $x + y$?

A. 10
B. 12
C. 14
D. 16
E. 18

11. If the average (arithmetic mean) of $q$, $r$, $s$, and $t$ is 10, what is the average of $s - r$, $3r + t$, $10 - r$, and $6 + q$?

12. In the figure below, AD is a diameter of the circle with center O and AO = 5. What is the length of arc BCD?



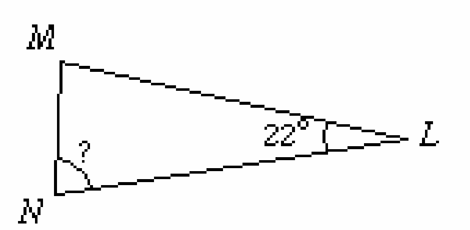
13. $3 \times 10^{-4} = ?$

A. -30,000
B. -120
C. 0.00003
D. 0.0003
E. 0.12

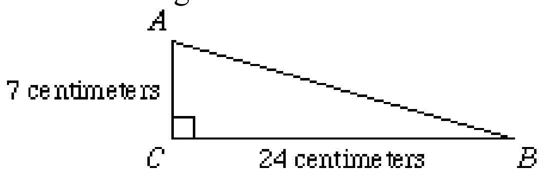
14. Simplify (x)(x)(x)(x), for all x.

15. A rock group gets 30% of the money from sales of their newest compact disc. That 30% is split equally among the 5 group members. If the disc generates $1,000,000 in sales, how much does one group member receive?

16. The triangle below is isosceles and is drawn to scale. What is the measure of $\angle N$?



17. In right triangle $\Delta ABC$ below, what is the sine of angle A?




18. In a shipment of 1,000 light bulbs, $\frac{1}{40}$ of the bulbs were defective. What is the ratio of defective bulbs to nondefective bulbs?

19. For all $x>0$, $\frac{2x^2+14x+24}{x+4}$ simplifies to:

A. $x+3$
B. $x+4$
C. $2(x+3)$
D. $2(x+4)$
E. $2(x+3)(x+4)$

20. A particle travels $1\text{x}10^8$ centimeters per second in a straight line for $4\text{x}10^{-6}$ seconds. How many centimeters has it traveled?

21. A circle with center (-3,4) is tangent to the *x*-axis in the standard (*x*,*y*) coordinate plane. What is the radius of this circle?

22. Over all real numbers *x*, what is the maximum value of $4 \cdot \sin(3x)$?

23. What is the largest possible product for 2 even integers whose sum is 34?

24. What are the values of *a* and *b*, if any, where $a|b-2|<0$?

A. $a<0$ and $b\neq2$
B. $a<0$ and $b=2$
C. $a\neq0$ and $b>2$
D. $a>0$ and $b<2$
E. there are no such values of *a* and *b*.

25. There are $n$ students in a class. If, among those students, $p$% play at least 1 musical instrument, which of the following general expressions represents the number of students who play NO musical instrument?

A. $np$
B. $.01np$
C. $\frac{(100-p)n}{100}$
D. $\frac{(1-p)n}{.01}$
E. $100(1-p)n$

26. In the standard $(x,y)$ coordinate plane, the graph of $(x-2)^2+(y+4)^2=9$ is a circle. What is the area enclosed by this circle, expressed in square coordinate units?

27. If the angles $\angle X$ and $\angle Y$ each measure between 0° and 90°, and if $\sin X = \cos Y$, what is the sum of the measures of the angles $\angle X$ and $\angle Y$ ?

THIS SPACE INTENTIONALLY LEFT BLANK.

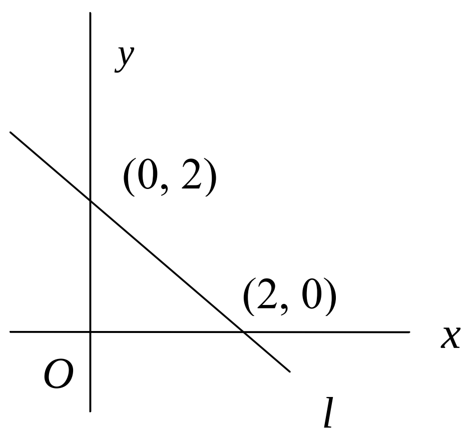
1. If 1 pound of walnuts is mixed with 4 pounds of cashews, what fraction of this mixture, by weight, is cashews?

2. If $n$ equally priced items cost a total of $d$ dollars, what is the cost, in dollars, of $z$ of these items?

3. The relationship between the temperature expressed in Celsius degrees (°C) and Fahrenheit degrees (°F) is given by $F - 32 = \frac{9}{5}C$. According to the table below, how many continents have had temperatures above 125° Fahrenheit?

| Continent | Temperature |
|---|---|
| Africa | 58°C |
| North America | 57°C |
| Asia | 54°C |
| Australia | 53°C |
| Europe | 50°C |
| South America | 49°C |
| Antartica | 15°C |

4. What is the slope of line $l$ in the figure below?



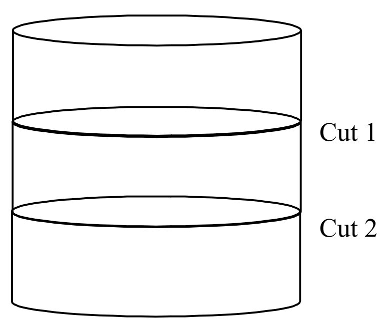
5. Sheng counts by 4's to put herself to sleep at night. If she counts from 300, the first number, and falls asleep after 464, the *n*th number, what is the value of *n*?

6. If $3(x - 3) = 9$, what is the value of $\frac{1}{x - 3}$ ?

7. Set P is the set of numbers that belong to both set X and set Y below. Set Q is the set of numbers that belong to set X but not to set Y. What is the set of numbers that belong to either set P or Q?

Set X {1, 2, 4, 7}
Set Y {2, 5, 7, 8}

8. The height of the cylinder shown below is 8 inches and the radius of its base is 6 inches. If two horizontal cuts are made as shown through the cylinder, dividing it into three cylinders of equal volume, what is the volume in cubic inches, of each of the resulting cylinders?



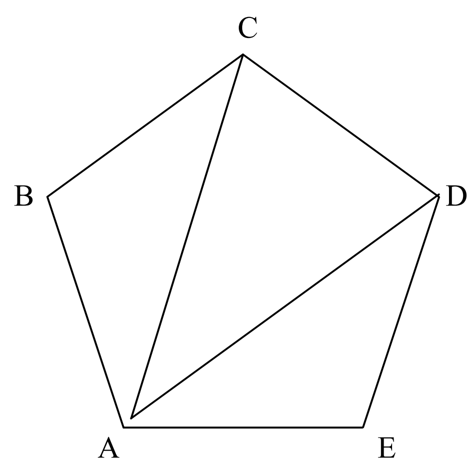
9. If $x^{10} = 5555$ and $\frac{x^9}{y} = 5$, what is the value of $xy$?

10. The product of three different prime numbers $p$, $r$, and $s$ is less than 200. If $2 < p < r < s$, what is the greatest possible value of $s$?

11. Five cities, A, B, C, D, and E lie along a straight road, not necessarily in that order. Cities A and E are at opposite ends of the road. Some of the distances between the cities are given in the chart below. For example, the distance between Cities B and C is 30 miles. How many miles is City A from City E?

| | City A | City B | City C |
|---|---|---|---|
| City A | 0 | | 20 |
| City B | | 0 | |
| City C | | 30 | 0 |
| City D | | 20 | |
| City E | | 10 | |

12. Pentagon ABCDE in the figure below has sides of equal length and the five marked angles have equal measure. The measure of $\angle ACD$ is $72^\circ$. If the measure of $\angle DAE$ is $x^\circ$, what is the value of x?



13. Which of the following is divisible by 3 (with no remainder)?

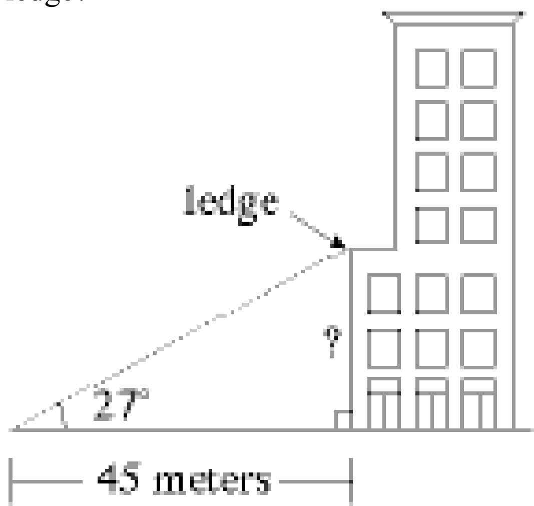
A. 2,725
B. 4,210
C. 4,482
D. 6,203
E. 8,105

14. If $7y=2x-5$, then $x = ?$

15. How many yards of material from a 24-yard length of cloth remain after 3 pieces each $3\frac{1}{2}$ yards long, and 5 pieces, each $2\frac{1}{4}$ yards long, are removed?

16. When graphed in the $(x,y)$ coordinate plane, at what point do the lines $x + y = 5$ and $y = 7$ intersect?

17. From a point on the ground the angle of elevation to a ledge on a building is 27°, and the distance to the base of the building is 45 meters. How many meters high is the ledge?




18. Which of the following is equal to $\sqrt{20}$ ?

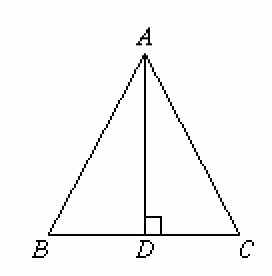
A. $2\sqrt{5}$
B. $2\sqrt{20}$
C. $4\sqrt{5}$
D. 10
E. $10\sqrt{2}$

19. How many solutions are there to the equation $x^2-15 = 0$ ?

20. When getting into shape by exercising, the subject's maximum recommended number of heartbeats per minute ($h$) can be determined by subtracting the subject's age ($a$) from 220 and then taking 75% of that value. This relation is expressed by which of the following formulas?

A. $h = .75(220 - a)$
B. $h = .75(220) - a$
C. $h = 220 - .75a$
D. $.75h = 220 - a$
E. $220 = .75(h - a)$

21. In the figure below, $AB = AC$ and $BC$ is 10 units long. What is the area, in square inches of $ABC$?




A. 12.5
B. 25
C. 25.2
D. 50
E. Cannot be determined from the given information.

22. In ΔABC, if $\angle A$ and $\angle B$ are acute angles, and sinA = $\frac{10}{13}$, what is the value of cosA?

23. Phillip charged $400 worth of goods on his credit card. On his first bill, he was not charged any interest, and he made a payment of $20. He then charged another $18 worth of goods. On his second bill a month later, he was charged 2% interest on his entire unpaid balance. How much interest was Phillip charged on his second bill?

24. A scuba diver often sends up a balloon-type marker. The marker starts out fairly small and gets larger as it approaches the surface. The chart below shows the marker's volume at multiples of 33 feet below the surface of the water. Which of the following equations fits these data?

| | | | | | | |
|---|---|---|---|---|---|---|
| $D$ | Depth in feet | 0 | 33 | 66 | 99 | 132 |
| $V$ | Volume in liters | 1 | $\frac{1}{2}$ | $\frac{1}{3}$ | $\frac{1}{4}$ | $\frac{1}{5}$ |

A. $V = \dfrac{33}{d+33}$

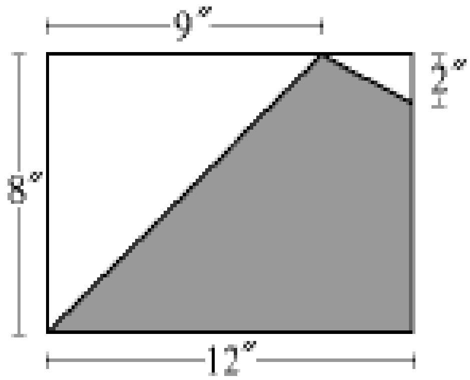
B. $V = \dfrac{d-33}{33}$

C. $V = -\dfrac{d}{66}+1$

D. $V = \dfrac{132-d}{d}$

E. $V = \dfrac{d-33}{\frac{d}{33}+1}$

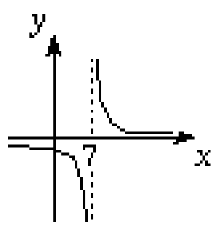
25. Lengths are shown in inches on the drawing of the rectangle below. What is the shaded area, in square inches?



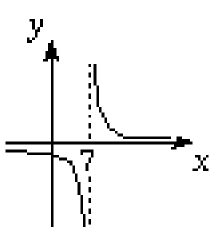

26. Which of the following is equivalent to $\frac{\sin x \tan^2 x + \sin x}{\tan x}$ for 0°<$x$<90° ?

A. $\sin x + \cos x$
B. $2 \sin x \tan x$
C. $\sec x$
D. $\cos x$
E. $\frac{\sin^2 x}{\cos^3 x}$

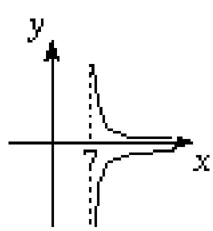
27. The graph of $y = \frac{7}{x-7}$ is shown below.



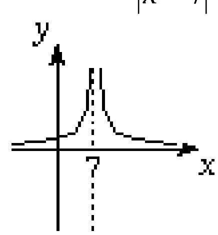

Among the following, which is the best representation of $y = \frac{7}{|x-7|}$?



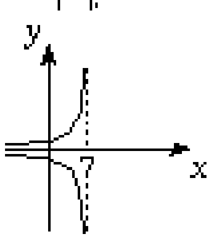

A.



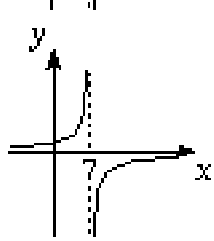

C.




E.




B.




D.

1 - Do you enjoy math? Why or why not?

2 – Would you consider yourself good at word problems? If not, what part do you have trouble with?

3 - Suppose two students were asked to solve the equation $3x + 6 = 24$. One student did this:

$3x + 6 = 24$

$3x = 12$

$x = 4$

And the other student did this:

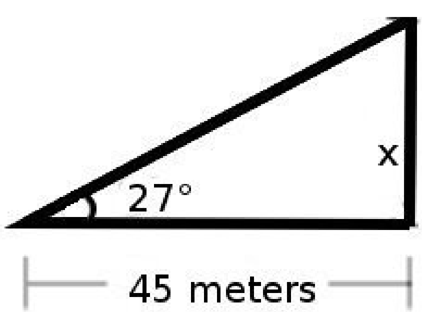
$3x + 6 = 24$

$x + 2 = 8$

$x = 6$

Which student did the problem incorrectly (it's also possible that both students made a mistake)? Indicate where the mistake was.

4 – Consider the correctly drawn right triangle below. A student was asked to solve for the length of the side of this right triangle labeled by "x," and found x = 215 meters. Does this answer make sense? Explain why or why not.




5 – If *n* equally priced items cost a total of *d* dollars, what is the cost of *z* of these items?

1 - Suppose two students were asked to solve the equation 3x + 6 = 24. One student did this:

3x + 6 = 24

3x = 12

x = 4

And the other student did this:

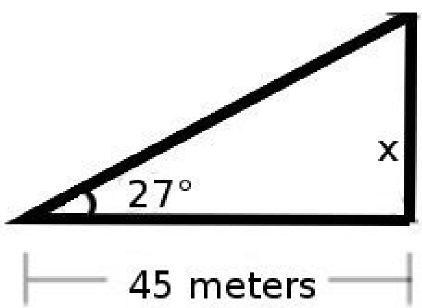
3x + 6 = 24

x + 2 = 8

x = 6

Which student did the problem incorrectly (it's also possible that both students made a mistake)? Indicate where the mistake was.

2 – Consider the correctly drawn right triangle below. A student was asked to solve for the length of the side of this right triangle labeled by "x," and found x = 215 meters. Does this answer make sense? Explain why or why not.




3 – If *n* equally priced items cost a total of *d* dollars, what is the cost of *z* of these items?

4 – Did you find the questions easier after the lesson? If so, why? If not, why not?

1) Do you enjoy math? Why or why not?

2) Do you consider yourself good at word problems? Explain.

3) The following statement is clearly false, but we "proved" that it was true. Find the flaw in the proof.

Theorem: $2 = 3$.

Proof: Suppose the theorem is true. Then $2 = 3$ means that $3 = 2$, so we can add either 3 or 2 on both sides of any expression. So, $2 + 3 = 3 + 2$, which means $5 = 5$. This is true, so it must be true that $2 = 3$.

4) Suppose two students were given the problem $3x + 6 = 24$. One student solved the problem like this:

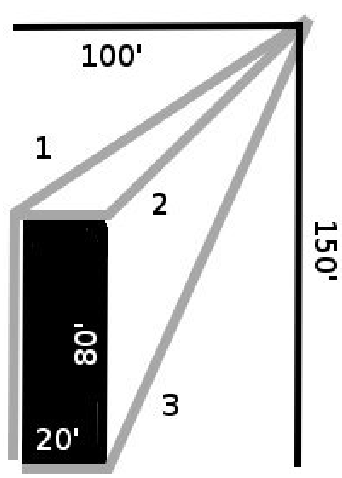
$$\begin{aligned} 3x + 6 &= 24 \\ 3x &= 12 \\ x &= 4 \end{aligned}$$

The other student solved the problem like this:

$$\begin{aligned} 3x + 6 &= 24 \\ x + 2 &= 8 \\ x &= 6 \end{aligned}$$

Which student made a mistake? It's also possible that both students did the problem incorrectly. Point out where the mistake was, and explain why at least one student had to make a mistake.

5) In order to get from the bottom left-hand corner to the top right-hand corner, which of the following three paths (the grey lines) is the shortest? The figure is not drawn to scale, but the measurements (in feet) are accurate.



1) The following statement is clearly false, but we "proved" that it was true. Find the flaw in the proof.

Theorem: $2 = 3$.

Proof: Suppose the theorem is true. Then $2 = 3$ means that $3 = 2$, so we can add either 3 or 2 on both sides of any expression. So, $2 + 3 = 3 + 2$, which means $5 = 5$. This is true, so it must be true that $2 = 3$.

2) Suppose two students were given the problem $3x + 6 = 24$. One student solved the problem like this:

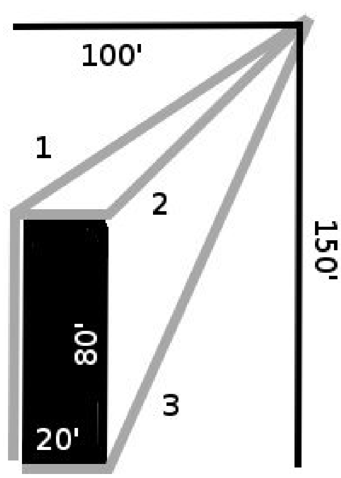
$$\begin{aligned} 3x + 6 &= 24 \\ 3x &= 12 \\ x &= 4 \end{aligned}$$

The other student solved the problem like this:

$$\begin{aligned} 3x + 6 &= 24 \\ x + 2 &= 8 \\ x &= 6 \end{aligned}$$

Which student made a mistake? It's also possible that both students did the problem incorrectly. Point out where the mistake was, and explain why at least one student had to make a mistake.

3) In order to get from the bottom left-hand corner to the top right-hand corner, which of the following three paths (the grey lines) is the shortest? The figure is not drawn to scale, but the measurements (in feet) are accurate.




4) Did you find the questions easier after the lecture? If so, why? If not, why not?

1) Do you enjoy math? Why or why not?

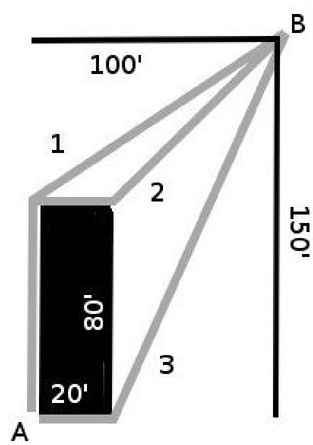
2) Do you consider yourself good at word problems? Explain.

3) Suppose two students (Joe and Jimmy) were given the problem $3x + 6 = 24$.

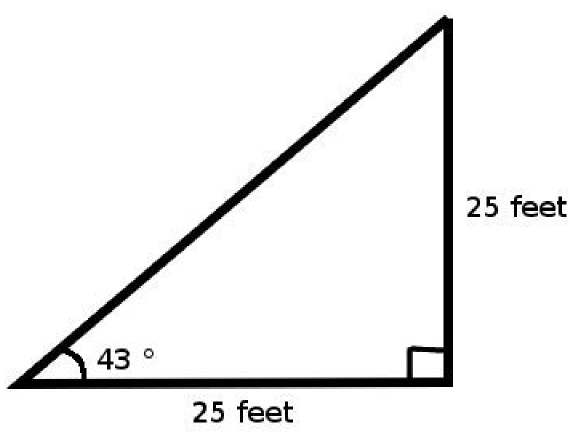
$$\begin{array}{c|c} \text{Joe} & \text{Jimmy} \\ 3x + 6 = 24 & 3x + 6 = 24 \\ x + 2 = 8 & 3x = 12 \\ x = 6 & x = 4 \end{array} \tag{1}$$

Which student made a mistake? It's also possible that both students did the problem incorrectly. Point out where the mistake was, and explain why at least one student had to make a mistake.

4) In order to get from the bottom left-hand corner (labelled 'A') to the top right-hand corner ('B'), which of the following three paths (the grey lines) is the shortest? The figure is not drawn to scale, but the measurements (in feet) are accurate.




5) Does the triangle below make sense; i.e., is it a possible triangle? Explain your answer.

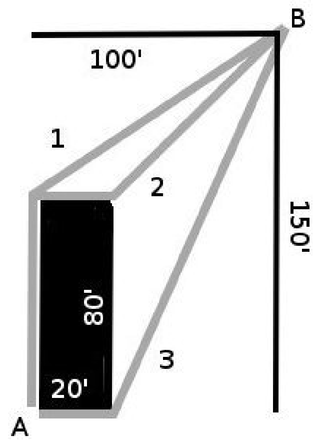


1) Suppose two students (Joe and Jimmy) were given the problem $3x + 6 = 24$.

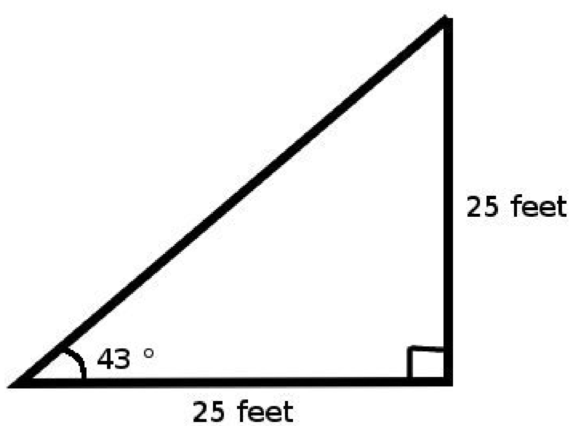
$$\begin{array}{c|c} \text{Joe} & \text{Jimmy} \\ 3x+6=24 & 3x+6=24 \\ x+2=8 & 3x=12 \\ x=6 & x=4 \end{array} \tag{2}$$

Which student made a mistake? It's also possible that both students did the problem incorrectly. Point out where the mistake was, and explain why at least one student had to make a mistake.

2) In order to get from the bottom left-hand corner to the top right-hand corner, which of the following three paths (the grey lines) is the shortest? The figure is not drawn to scale, but the measurements (in feet) are accurate.




3) Does the triangle below make sense; i.e., is it a possible triangle? Explain your answer.




4) Did you find the questions easier after the lecture? If so, why? If not, why not?

# D Diagnostic Test Miscellany

## D.1 Data from Diagnostic Test

In presenting this table, the following abbreviations are used for the column headers and contents: I = index, T = Test, C = Calculator, SS = SchoolStart, TY = This Year, LY = LastYear, G = Grade, Ss = SATscore, F = Flag, Co = Comments, A1 = Algebra I, Ge = Geometry, TA = Algebra II/Trigonometry, PC = Introduction to Analysis (Pre-Calculus), Ca = Calculus, AP = AP Calculus, AM = Advanced Math Topics, I3 = Integrated Math III. As the comments were too wide to display, a placeholder number is listed, with Table 5 detailing what each represents. Note that this data already has the flagged tests removed, as they do not effect the analysis (being disregarded for various reasons already). This table is thus the exact Excel spreadsheet designed to be operated on by the MATLAB function in Section D.2.

Table 4: The data from the diagnostic test for use with the code in Section D.2.

| I | T | C | SS | TY | LY | G | Ss | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | F | Co |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 7 | AM | TA | 1 | 410 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 |
| 2 | 1 | 1 | 9 | AM | TA | 1 | 370 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | |
| 3 | 1 | 1 | 8 | AM | TA | 1 | 440 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 2 |
| 4 | 1 | -1 | 9 | AM | AM | 0 | | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | |
| 5 | 1 | 1 | 9 | AM | TA | 1 | | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 3 |
| 6 | 1 | 1 | 7 | PC | TA | 2 | | 1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | -1 | 1 | |
| 7 | 1 | 1 | 7 | PC | TA | 0 | | 1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | |
| 8 | 1 | 2 | 9 | PC | TA | 1 | | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 4 |
| 9 | 1 | -2 | 9 | TA | Ge | -1 | | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 5 |
| 10 | 1 | 1 | 7 | PC | TA | 2 | | -1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 4 |
| 11 | 1 | 0 | 9 | TA | Ge | 1 | | 1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | 1 | |
| 12 | 1 | 0 | 7 | TA | Ge | 0 | | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 4 |
| 13 | 1 | 1 | 7 | AP | PC | 1 | 510 | 1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | |
| 14 | 1 | 1 | 9 | TA | Ge | | | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 6 |
| 15 | 1 | 1 | 7 | AP | PC | 1 | 430 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | |
| 16 | 1 | 1 | 9 | TA | Ge | 2 | | -1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | |
| 17 | 1 | 1 | 7 | TA | Ge | 1 | | -1 | -1 | 1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | 4 |
| 18 | 1 | 1 | 10 | AP | PC | | | -1 | -1 | 1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | |
| 20 | 1 | 1 | 9 | PC | TA | 1 | | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | |

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| I | T | C | SS | TY | LY | G | Ss | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | F | Co |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 21 | 1 | 1 | 7 | PC | TA | 2 | | 1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | 1 | 1 | |
| 22 | 1 | 1 | 10 | PC | TA | 1 | | 1 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | 7 |
| 23 | 1 | 1 | 7 | PC | TA | 1 | | -1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | |
| 24 | 1 | 1 | 10 | PC | TA | 2 | | 1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | |
| 25 | 1 | 1 | 7 | PC | TA | 1 | | -1 | -1 | 1 | 1 | -1 | 1 | 1 | -1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | 7,8 |
| 26 | 1 | 1 | 8 | PC | TA | 2 | | 1 | -1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | 7 |
| 27 | 1 | 1 | 9 | PC | TA | 1 | | 1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | |
| 28 | 1 | 1 | 7 | PC | TA | 1 | | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 9 |
| 29 | 1 | 1 | 11 | PC | TA | 1 | | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | -1 | 1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | |
| 30 | 1 | 1 | 10 | PC | TA | | | 1 | -1 | 1 | -1 | 1 | 1 | 1 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | 1 | 1 | |
| 32 | 1 | 1 | 7 | PC | TA | 1 | | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | |
| 33 | 1 | 1 | 7 | PC | TA | 0 | 510 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | |
| 34 | 1 | 1 | 7 | PC | TA | 0 | 430 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | |
| 35 | 1 | 1 | 7 | PC | TA | 2 | 350 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | |
| 36 | 1 | 1 | 10 | PC | TA | | | -1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | |
| 37 | 1 | 2 | 9 | PC | TA | | | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | |
| 38 | 1 | 1 | 7 | PC | TA | | | -1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | |
| 39 | 1 | 1 | 9 | PC | TA | | | 1 | -1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | |
| 40 | 1 | 1 | 9 | AP | PC | | 510 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | 7,9 |
| 42 | 1 | 1 | 10 | AP | PC | 2 | 700 | 1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 10 |
| 43 | 1 | 1 | 11 | AP | PC | | 730 | -1 | -1 | 1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | 1 | 1 | -1 | 1 | 1 | |
| 44 | 1 | 1 | 9 | AP | PC | | 650 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | |
| 45 | 1 | 1 | 7 | AP | PC | | 540 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | 1 | 1 | |
| 46 | 1 | 1 | 7 | AP | PC | 2 | 680 | -1 | -1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | -1 | 1 | 1 | 1 | 7,10 |
| 2 | 1 | -2 | 9 | TA | Ge | 1 | | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | |
| 3 | 1 | 1 | 8 | TA | Ge | | | 1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | |
| 6 | 1 | 1 | 9 | PC | TA | 0 | | 1 | 1 | 1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | 1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | 1 | |
| 7 | 1 | 1 | 9 | PC | TA | 1 | | 1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | |
| 8 | 1 | 1 | 9 | PC | TA | 1 | | 1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | |
| 10 | 1 | 1 | 9 | TA | Ge | 2 | | 1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | |
| 11 | 1 | 1 | 7 | TA | Ge | | | 1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | |
| 12 | 1 | 1 | 7 | TA | Ge | 0 | 375 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | |
| 13 | 1 | 1 | 9 | TA | Ge | | | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | |
| 14 | 1 | 1 | 10 | TA | Ge | 1 | | -1 | -1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | |
| 15 | 1 | -2 | 7 | TA | Ge | | | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | |
| 18 | 1 | 1 | 8 | PC | TA | | 620 | 1 | -1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | |
| 19 | 1 | 1 | 9 | PC | TA | 1 | | 1 | -1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | |
| 20 | 1 | 1 | 7 | PC | TA | 1 | 410 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | |
| 21 | 1 | 1 | 9 | PC | Ge | 2 | | 1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | |
| 22 | 1 | 0 | 7 | AM | I3 | 1 | | 1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | |

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| I | T | C | SS | TY | LY | G | Ss | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | F | Co |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 23 | 1 | 0 | 9 | AM | TA | 0 | 460 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | |
| 24 | 1 | 0 | 7 | AM | TA | 0 | 340 | 1 | -1 | 1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | |
| 25 | 1 | 1 | 9 | TA | Ge | 1 | | -1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | |
| 26 | 1 | 0 | 7.5 | TA | Ge | 2 | | -1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | |
| 27 | 1 | 1 | 10.5 | TA | Ge | 0 | | 1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | 1 | 1 | |
| 28 | 1 | 1 | 9 | TA | Ge | | | 1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | -1 | 1 | -1 | 1 | |
| 29 | 1 | 1 | 7 | TA | Ge | 0 | | 1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | |
| 30 | 1 | 1 | 7 | TA | Ge | | | -1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | |
| 31 | 1 | 1 | 7 | TA | Ge | | | -1 | -1 | 1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | |
| 32 | 1 | 2 | 12 | PC | TA | 2 | | 1 | -1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | |
| 33 | 1 | 1 | 7 | PC | TA | 1 | | 1 | -1 | 1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | |
| 34 | 1 | 1 | 10.5 | PC | Ge | 2 | | 1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | |
| 35 | 1 | 1 | 7 | PC | TA | 1 | | -1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | |
| 37 | 1 | 1 | 9 | AM | TA | | | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | |
| 38 | 1 | 1 | 7 | AM | I3 | 1 | | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | 1 | |
| 39 | 1 | 1 | 7 | AM | Ge | | 360 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | |
| 40 | 1 | 0 | 7 | AM | TA | 1 | 410 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | |
| 41 | 1 | 0 | 8 | AM | I3 | 1 | | -1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | |
| 42 | 1 | 1 | 7 | AM | TA | 1 | 480 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | |
| 44 | 1 | -1 | 9 | AM | TA | | | -1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | |
| 39 | 1 | 2 | 7 | PC | TA | 0 | | -1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | |
| 1 | -1 | 1 | 7 | PC | TA | | | -1 | 1 | -1 | -1 | 1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | 1 | 11 |
| 2 | -1 | 1 | 7 | TA | Ge | 2 | | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | |
| 4 | -1 | 1 | 11 | TA | Ge | 2 | | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | |
| 5 | -1 | 1 | 7 | PC | TA | 1 | 520 | -1 | 1 | 1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | |
| 6 | -1 | 1 | 10 | PC | TA | | | -1 | -1 | -1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | |
| 7 | -1 | 1 | 7 | PC | TA | 2 | | 1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 12 |
| 8 | -1 | 1 | 9 | PC | TA | 1 | | -1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | |
| 9 | -1 | 0 | 9 | TA | Ge | 1 | | 1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | |
| 10 | -1 | -2 | 9 | TA | Ge | | | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | |
| 11 | -1 | -1 | 7 | TA | A1 | 1 | | -1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | |
| 12 | -1 | 1 | 7 | TA | Ge | | | -1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | |
| 13 | -1 | 1 | 9 | TA | Ge | 0 | | 1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | |
| 14 | -1 | 0 | 9 | TA | Ge | 0 | | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 13 |
| 15 | -1 | -2 | 9 | TA | Ge | 1 | | -1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | 13 |
| 16 | -1 | 1 | 9.5 | TA | Ge | | | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | |
| 17 | -1 | 2 | 8 | TA | Ge | | | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | |
| 18 | -1 | -1 | 9 | TA | Ge | | | -1 | 1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | |
| 19 | -1 | 1 | 7 | TA | Ge | 1 | | -1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | |
| 20 | -1 | 1 | 7 | PC | TA | | | -1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | |

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| I | T | C | SS | TY | LY | G | Ss | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | F | Co |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 21 | -1 | 1 | 7 | PC | TA | 1 | 550 | -1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | 1 | |
| 22 | -1 | 1 | 9 | PC | TA | | | 1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | |
| 23 | -1 | 1 | 7 | PC | TA | 0 | 350 | -1 | 1 | -1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 13 |
| 24 | -1 | 1 | 10 | PC | TA | -1 | | 1 | 1 | -1 | -1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 13 |
| 25 | -1 | -2 | 9 | PC | TA | 2 | | 1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | |
| 26 | -1 | 1 | 12 | PC | TA | | 460 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | -1 | 1 | |
| 27 | -1 | 1 | 8 | PC | TA | 1 | | -1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | |
| 28 | -1 | 1 | 8 | PC | TA | 2 | 480 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 12 |
| 29 | -1 | 1 | 7 | PC | TA | 1 | | 1 | 1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | |
| 30 | -1 | 1 | 9 | TA | Ge | | | 1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | 1 | 12 |
| 31 | -1 | 1 | 7 | PC | TA | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | |
| 32 | -1 | 1 | 7 | PC | TA | 1 | | -1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | -1 | 1 | 1 | |
| 33 | -1 | 1 | 9 | PC | TA | 2 | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | -1 | 1 | -1 | 1 | 1 | |
| 34 | -1 | 1 | 9 | TA | Ge | 1 | | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | |
| 35 | -1 | 2 | 9 | AP | PC | 2 | 570 | 1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | 13 |
| 36 | -1 | 1 | 7 | TA | Ge | 1 | | -1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | 1 | |
| 37 | -1 | 1 | 7 | TA | Ge | | | -1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | 1 | 12 |
| 38 | -1 | 1 | 9 | TA | Ge | 2 | | -1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 12 |
| 1 | -1 | 1 | 11 | PC | TA | | | 1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | -1 | 1 | |
| 2 | -1 | 0 | 9 | AM | | -1 | | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | |
| 3 | -1 | 1 | 10 | AM | I3 | 2 | 420 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | |
| 4 | -1 | 1 | 7 | AM | TA | 0 | | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | |
| 5 | -1 | 1 | 7 | PC | | 2 | 450 | 1 | 1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | |
| 6 | -1 | 1 | 9 | TA | Ge | 0 | | 1 | 1 | 1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | |
| 7 | -1 | 1 | 9 | PC | TA | | | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | 1 | |
| 8 | -1 | 0 | 7 | TA | Ge | 0 | | -1 | 1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | |
| 9 | -1 | 1 | 9 | TA | A1 | 1 | | -1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | |
| 10 | -1 | 1 | 9 | AP | PC | 2 | 550 | -1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | |
| 11 | -1 | 1 | 10 | TA | Ge | 2 | | 1 | 1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | |
| 12 | -1 | 1 | 8 | AP | PC | | 630 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | -1 | 1 | |
| 13 | -1 | -2 | 9 | TA | A1 | | | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | |
| 14 | -1 | 2 | 7 | TA | Ge | | | -1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | |
| 15 | -1 | 1 | 9 | AP | PC | 1 | | 1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | 1 | |
| 16 | -1 | 1 | 9 | TA | Ge | | | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | |
| 17 | -1 | 1 | 9 | TA | Ge | 1 | | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | |
| 18 | -1 | 1 | 9 | PC | TA | 2 | | 1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | |
| 19 | -1 | 1 | 9 | PC | TA | 0 | 430 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | 1 | |
| 20 | -1 | 1 | 7 | PC | TA | | | 1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | |
| 21 | -1 | 1 | 7 | PC | TA | 1 | | -1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | |
| 22 | -1 | 1 | 9 | PC | TA | | 410 | 1 | 1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 23 | -1 | 1 | 7 | PC | TA | 2 |  | -1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 |  |
| 24 | -1 | 1 | 7 | PC | TA | 1 |  | -1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | 1 | 1 |  |
| 25 | -1 | 1 | 9 | PC | TA | 2 | 570 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | 1 |  |
| 26 | -1 | 1 | 9 | PC | TA | 2 | 540 | 1 | 1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | 1 | 1 |  |
| 27 | -1 | -2 | 9 | PC | TA | 2 | 640 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | 1 | 1 |  |
| 28 | -1 | 1 | 7 | PC | TA | 2 | 530 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 |  |
| 29 | -1 | 1 | 9 | PC | TA |  | 410 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 |  |
| 30 | -1 | 1 | 7 | PC | TA | 1 |  | 1 | 1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 |  |
| 31 | -1 | 1 | 11 | PC | TA | 0 | 640 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 |  |
| 32 | -1 | 1 | 7 | PC | TA | 1 | 480 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 |  |
| 33 | -1 | -2 | 7 | PC | TA |  | 480 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 |  |
| 34 | -1 | 1 | 9 | PC | TA |  | 370 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 |  |
| 35 | -1 | 1 | 9 | PC | TA | 2 | 490 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 |  |
| 36 | -1 | 1 | 9 | PC | TA |  |  | -1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 |  |
| 37 | -1 | 1 | 10 | PC | TA | 1 |  | -1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 |  |
| 38 | -1 | 1 | 9 | AP | PC | 1 | 590 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 |  |
| 39 | -1 | 1 | 7 | AP | PC | 2 | 620 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 |  |
| 40 | -1 | 1 | 9 | AP | PC | 2 | 680 | 1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | -1 | 1 | 1 | 1 |  |
| 41 | -1 | -2 | 11 | AP | Ca |  | 750 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |  |
| 42 | -1 | 1 | 7 | AP | PC | 2 | 680 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | 1 |  |
| 44 | -1 | -2 | 12 | AP | PC |  |  | 1 | 1 | -1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | 1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | 1 |  |

Table 5: Key for comment column of Table 4.

| Key | Comment |
|---|---|
| 1 | Stops after 16 |
| 2 | Didn't use Calcluator |
| 3 | Stops after 21 |
| 4 | Answered 1,215 for #17 |
| 5 | #8 - Right Procedure |
| 6 | #8 - Misread |
| 7 | Answered 1110.778 for #9 |
| 8 | Dropped '-' on #16 |
| 9 | #23 - off by factor of 10 |
| 10 | Missing pages |
| 11 | #12  angle given, not arc length |
| 12 | Answered -3.95 on #22 |
| 13 | #23 - right numbers, computed sum |

## D.2 MATLAB Code for Analysis of Diagnostic Test

```
function IQP(x)
%This m-file is designed to operate on the excel spreadsheet IQP.xls in
%order to determine which factors of the testing affected the resulting
%scores and then to order the significant factors by those categories in
%which students performed the best down to the worst with significant
%pairwise differences.

data = xlsread(x);
[n,m] = size(data);
data = data(2:n,2:m);
data = data(:,[1:3 6 8:35]);
data(:,2) = data(:,2)./2;
data(:,3) = (data(:,3)-9.5)./(2.5);
data(:,4) = data(:,4)./2;
data(:,5:31) = (data(:,5:31)+1)./2;
%Above this sets up initial data matrix scaled between -1 and 1

%This section creates two data matrices, one for each test
dataA = zeros(80,32);
dataB = zeros(80,32);
for i=1:80
    dataA(i,:) = data(i,:);
end
for i=1:80
    dataB(i,:) = data(i+80,:);
end

%This section re-orders the columns in test A so that the type is the same
%as test B for a given problem number
dummy = dataA(:,13);
dataA(:,13) = dataA(:,14);
dataA(:,14) = dummy;
%above is that labeled "switch", below is that labeled "reorder" in the
```

```
%sheet which we will scan shortly
dummy = dataA(:,31);
dataA(:,31) = dataA(:,30);
dataA(:,30) = dataA(:,29);
dataA(:,29) = dataA(:,28);
dataA(:,28) = dummy;
data = [dataA;dataB];

%Below this is the response vector of percentage correct along with mean
%and standard deviation for each question (summing down the columns)
colrespA = zeros(27,1);
colrespB = zeros(27,1);
[n,m] = size(dataA);
for i=1:27
    for j=1:n
        colrespA(i) = dataA(j,i+4) + colrespA(i);
    end
end
%Commented out 6 lines as they appeared unused.
%colmeanA = colrespA;
colrespA = colrespA./n;
%colstdevA = sqrt(n.*colrespA.*(1-colrespA));
[n,m] = size(dataB);
for i=1:27
    for j=1:n
        colrespB(i) = dataB(j,i+4) + colrespB(i);
    end
end
%colmeanB = colrespB;
colrespB = colrespB./n;
%colstdevB = sqrt(n.*colrespB.*(1-colrespB));

colresp = [colrespA;colrespB];
%colmean = [colmeanA;colmeanB];
```

```
%colstdev = [colstdevA;colstdevB];
%End of the 6 lines commented out.

%The following loops generate the difficulty vector
difficulty = zeros(27,1);
for i=[1:4 13:17]
    difficulty(i) = -1;
end
for i=[9:12 23:27]
    difficulty(i) = 1;
end
difficulty = [difficulty;difficulty];

%This defines the test vector
test = data(:,1);

%This defines the calculator vector
calculator = data(:,2);

%This defines the schoolstart vector
schoolstart = data(:,3);

%This defines the SAT/ACT vector
for i=1:12
    satact(i,:) = 'SAT';
end
for i=13:27
    satact(i,:) = 'ACT';
end
satact = [satact;satact];

%This defines the type vector
for i=[1 5 9 13 18 23]
    type(i,:) = 'NUM';
```

```
    type(i+1,:) = 'ALG';
    type(i+2,:) = 'DAT';
    type(i+3,:) = 'GEO';
end
for i=[17 22 27]
    type(i,:) = 'TRG';
end
type = [type;type];


% Now we can do the stats itself.  Will test effect on resp due to
% difficulty, satact, and type.  Then will test sensitivity to test,
% calculator and schoolstart.

%ANOVA for the Test, Calculator and Schoolstart factors
rowresp=zeros(160,1);
for j=1:160
    for i=1:27
        rowresp(j) = data(j,i+4) + rowresp(j);
    end
end
rowresp = rowresp/27;

anovan(rowresp,{test calculator schoolstart});

%ANOVA for the Difficulty, SAT/ACT, and Type of question factors

anovan(colresp,{difficulty satact type});

%The above determined that difficulty and type had significantly different
%averages, now a ranking must be determined (between significantly
%different pairings)

%First we find the average percentage correct for each individual
```

```
%difficulty level and each individual type.

%Difficulty

easy = find(difficulty == -1);
medium = find(difficulty == 0);
hard = find(difficulty == 1);
e = colresp(easy);
m = colresp(medium);
h = colresp(hard);

meane = mean(e);
meanm = mean(m);
meanh = mean(h);

diffmean = [meane meanm meanh]

%Type

Num = strmatch('NUM', type);
Alg = strmatch('ALG', type);
Dat = strmatch('DAT', type);
Geo = strmatch('GEO', type);
Trg = strmatch('TRG', type);
num = colresp(Num);
alg = colresp(Alg);
dat = colresp(Dat);
geo = colresp(Geo);
trg = colresp(Trg);

meannum = mean(num);
meanalg = mean(alg);
meandat = mean(dat);
meangeo = mean(geo);
```

```
meantrg = mean(trg);

typemean = [meannum meanalg meandat meangeo meantrg]

%Since we now know that on the difficulty scale, easy was better than
%medium was better than hard, and on the type scale, num was better than
%dat better than alg better than geo better than trg we can compare
%adjascent means to determine significant rankings.

%We briefly define the normal distribution for use is getting p-values

syms x real
normal01 = 1/sqrt(2*pi) * exp(-x^2/2);

%Difficulty
%n = 18 for all levels, but we multiply this by 80 to reflect the fact that
%each of the percentages correct were derived from 80 right/wrong's.  This
%allows us to use a z-test with ease.

stdeve = std(e);
stdevm = std(m);
stdevh = std(h);

%We want to test if meane > meanm, so we have a one sided z-test

zem = (meane - meanm)/sqrt(stdeve^2/(18*80)+stdevm^2/(18*80));
pvalem = 1 - int(normal01,-inf,zem);
pvalem = vpa(pvalem,4);

%Next is meanm > meanh

zmh = (meanm - meanh)/sqrt(stdevm^2/(18*80)+stdevh^2/(18*80));
pvalmh = 1 - int(normal01,-inf,zmh);
pvalmh = vpa(pvalmh,4);
```

```
%And thus the combined vector

diffpval = [pvalem pvalmh]

%Type
%n = 12 for num, alg, dat and geo and n =  6 for trg, again all multiplied
%by 80 for the same reason as before.

stdevnum = std(num);
stdevalg = std(alg);
stdevdat = std(dat);
stdevgeo = std(geo);
stdevtrg = std(trg);

%Test meannum > meandat

znumdat = (meannum - meandat)/(sqrt(stdevnum^2/(12*80)+stdevdat^2/(12*80)));
pvalnumdat = 1 - int(normal01,-inf,znumdat);
pvalnumdat = vpa(pvalnumdat,4);

%Test meandat > meanalg

zdatalg = (meandat - meanalg)/sqrt(stdevdat^2/(12*80)+stdevalg^2/(12*80));
pvaldatalg = 1 - int(normal01,-inf,zdatalg);
pvaldatalg = vpa(pvaldatalg,4);

%Test meanalg > meangeo

zalggeo = (meanalg - meangeo)/sqrt(stdevalg^2/(12*80)+stdevgeo^2/(12*80));
pvalalggeo = 1 - int(normal01,-inf,zalggeo);
pvalalggeo = vpa(pvalalggeo,4);

%Test meangeo > meantrg
```

```
zgeotrg = (meangeo - meantrg)/sqrt(stdevgeo^2/(12*80)+stdevtrg^2/(6*80));
pvalgeotrg = 1 - int(normal01,-inf,zgeotrg);
pvalgeotrg = vpa(pvalgeotrg,4);

%And thus the combined vector

typepval = [pvalnumdat pvaldatalg pvalalggeo pvalgeotrg]
%{
This all leads to the final conclusion that only difficulty and type of
problem were significant.  Moreover, each category was broken down into
significant effects as:
easy > medium > hard
and
num > data = alg > geom > trig
%}
```

# E Presentation to the Holy Name Math Department

On February 28, 2008, we went to Holy Name to present our results. We were given as much time as we needed at a meeting of the entire math department. We first gave a brief introduction about the IQP program at WPI and explained our main objectives as a team. We then summarized the methodology behind and results of the diagnostic test we administered. One of the teachers professed concern regarding the number of exam samples we gathered, but Dante assured him that our data was indeed statistically sound. The next part of the presentation explained how and why we formed our lesson plans. Having presented all the necessary background information, we were able to more easily explain our recommendations to the faculty. One teacher seemed to find some of our phrasing confusing, but the response overall was very positive. Most of the teachers agreed with our observations and recommendations and seemed glad that they had some statistical evidence to support their own findings. At the end of our presentation, we mentioned the idea of bringing in additional IQP teams in the future, which was received very well by the faculty. The acceptance of our project marked an important step in our IQP because the overall success of our project is largely dependent on how valuable our research and advice is to Holy Name.

# WPI IQP Math Assessment

## February 28, 2008


**Dante Amoroso**
**Mike Barone**
**Nick LeCompte**
**Laura Rosato**

# What is the IQP?

- Interactive Qualifying Project
- Non-major related
- A group based project to address the needs of a community
- Designed to incorporate science and technology with social concerns

# Our Project

- Working with Holy Name to enhance standardized test scores in math
- Diagnosing students' strengths and weaknesses
- Substantiating conclusions via student interactions

# What We Did

- Researched educational theories and standardized tests
- Administered diagnostic test based on the SAT & ACT
- Created and administered sample lesson plans

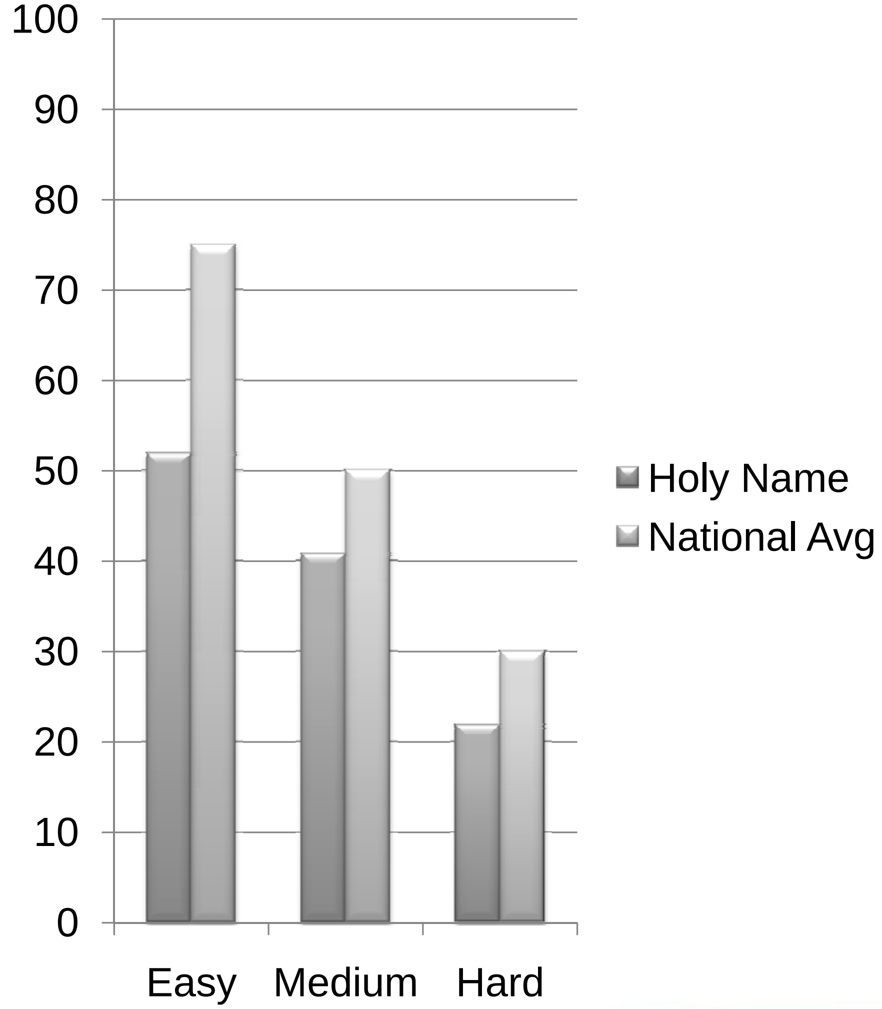
# Test Results

- Eliminated all factors except for difficulty and type of problem
- Looked at qualitative as well as quantitative data

# Difficulty of Problems

**Easy, medium, and hard classifications for ACT questions based on existing SAT designations**

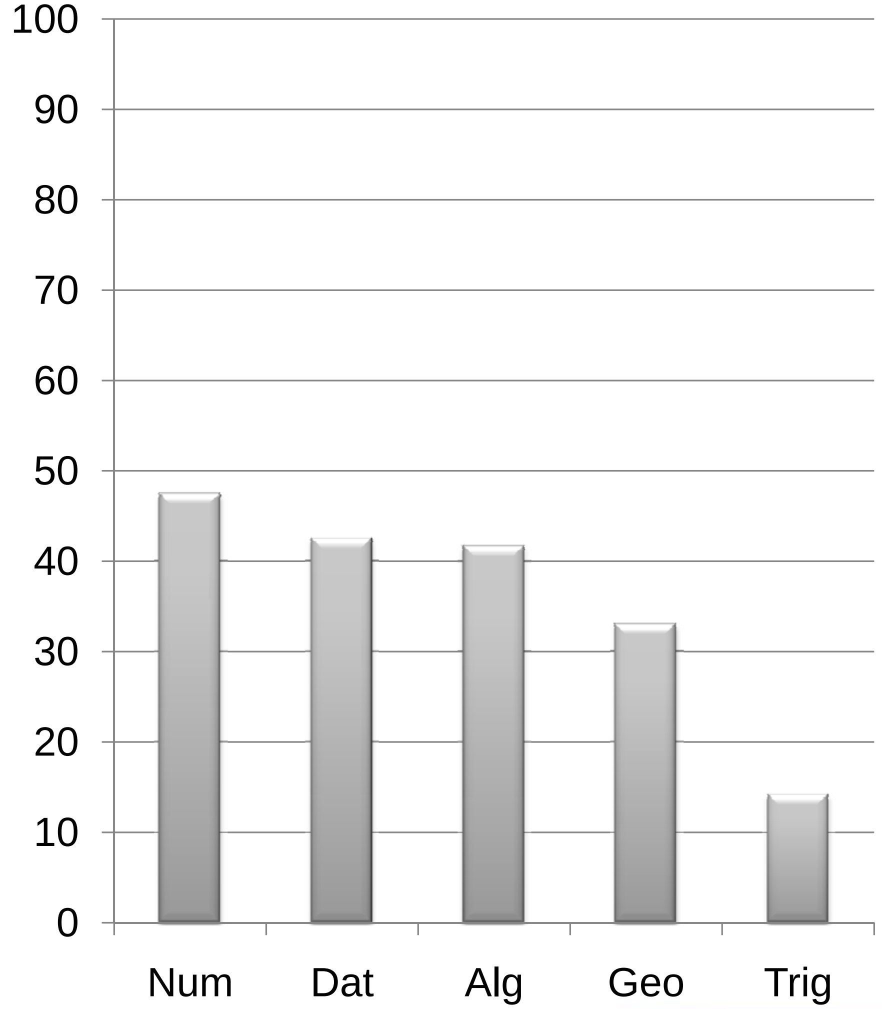
**National averages taken from past PSAT records**



# Types of Problems

**Numbers, Data, Algebra and Geometry classifications taken from SAT guidelines**

**Trigonometry questions taken from ACT only**



# Qualitative Results

- **Students remembered mathematical facts and formulas well**
  - **i.e. Pythagorean theorem, quadratic formula**
- **Difficulty in using facts properly**

# Lessons

- Five lessons total
  - 2 algebra I
  - 1 advanced math
  - 2 precalculus
- Primary focus on teaching logical thought over new mathematical concepts

# Recommendations

- Room left for further investigation
  - Different types of lesson plans stressing alternative methods
- Improve students' self confidence
- Introduce varied notation
- More application based word problems (models, not answers)
- Ask leading questions

# Future Plans

- Further WPI involvement (another IQP)
- Implement an introductory class on programming
- Continue diagnostics during implementation phase

# Thanks!


- Students, faculty, and staff of Holy Name for your cooperation and time
- Mr. Marcotte and Mr. Decoteau for their guidance and use of their classes
- Mrs. Riordan for initiating the project

# Questions?