Even Elementary Students Can Explore Algebra!:

Testing the Feasibility of From Here to There! Elementary,

a Game-Based Perceptual Learning Intervention

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Abstract

From Here to There: Elementary (FH2T-E), an interactive perceptual-learning (PL) mathematics intervention, was evaluated for effects on algebraic readiness in first and second-grade students (N = 242). In total, three studies were conducted: feasibility of intervention, intervention compared to control, and an investigation of potential learning mechanisms. Results showed feasibility success and student engagement, as well as significant learning improvement from pre to post comparison in relation to a traditional teaching control. Interaction analyses show that initial pretest performance (high/low) and gender of the student are not indicators of posttest improvement.

Keyword(s): Education Psychology, Elementary Math Intervention, Learning Sciences, From Here to There-Elementary

Even Elementary Students Can Explore Algebra!: Testing the feasibility of *From Here to There! Elementary*, a game-based perceptual learning intervention

Underperformance in mathematics by American children has been recognized as an impediment to our global competitiveness. Many researchers and political and educational leaders have identified overcoming American students' deficit in algebra proficiency as the key to closing the gap in math achievement (NMS, 2016). Deficits present in math understanding and performance abilities are deeply rooted in socioeconomic factors and problematic structures of mathematic standards and instruction found in early childhood education. However, this alone still cannot explain the continuing decline in student's math performance (Cortes et al., 2015). Such a trend is expected to persist if student's failure rates of math courses, in particular algebra, continue to increase. Researchers frequently mention that Algebra is the gateway to college (Welder, 2012), highlighting the importance of algebraic understanding upon its introduction in middle school, due to its correlation with high school and collegiate graduation rates, as well as employment earnings (NMAP, 2008).

Empirical data advances the idea that early childhood math competencies are a good predictor for later academic achievement (VanDerHeyden & Burns, 2009). National organizations have subsequently begun a push in elementary mathematics to provide early intervention programs as an initiative structured to improve student readiness in algebra, and decrease the number of students that are under-prepared (NCTM, 2011). Despite these initiatives, many students are still struggling when they reach middle school mathematics. It is likely that the decline in algebraic performance and formal math understanding stems from both lack of exposure as well as misconceptions that develop early, in critical windows where students form the foundations of math understanding.

The motivation for the current study derives from the current state of math education in American schools and the projected deficit in student mathematics performance that forms during elementary school. In this study, we examine the feasibility and effectiveness of earlier introduction of number sense and algebraic principles using a dynamic approach to mathematics instruction that has worked well for middle school children (Lins & Kaput, 2004; Carraher & Schliemann, 2007; Ottmar et al., 2015). Programs such as Project LEAP (funded by NSF - *National Science Foundation*) take a similar approach in their studies as they also recognize that the premise of early introduction of algebra is relatively untested. The intent of their research studies is motivated by the focus of early introduction of topics within the algebra domain; subsequently, the purpose of Project LEAP's research is to address children's developmental understanding of algebraic topics and the relative impact on understanding by using specific perceptual learning techniques compared to traditional instruction (NSF, 2009). Each program designed by the Project LEAP Foundation is tailored around the fundamental belief that the practice of early algebra education is critical to success in mathematics later in life (Lins & Kaput, 2004; Carraher & Schliemann, 2007). Therefore, similar to the Project LEAP programs, From Here to There (FH2T) was developed to provide a technology-based, self-paced program that allows students to dynamically transform, manipulate, and decompose numbers and operations to grasp the most basic mathematical concepts necessary for success (Ottmar et al., 2015). In this study, we seek to examine if the promising findings from a middle school perceptual learning intervention (From Here to There: Middle School, FH2T-MS) can also be seen in a younger cohort using an age-appropriate modified intervention program, From Here to There: Elementary (FH2T-E). In this paper, we report results from three studies. The first study served as a preliminary study to determine the feasibility of using FH2T in elementary classrooms. The second and third studies report findings from an experimental intervention condition testing the FH2T-E game compared to a business-as-usual (BAU - normal math instruction and lessons) control condition, as well as an assessment of student performance based on interaction with a non-gamified version of FH2T-E.

Literature Review

Early Readiness: The Importance of Building Conceptual Understanding and Number Sense

The ability to use symbolic representation in mathematics represents an important developmental milestone for children as they advance from number sense to more abstract algebraic thinking (Carr et al., 2011). However, algebra is not typically introduced to young children until they enter middle school, as this is seen as the transition from basic arithmetic to algebraic understanding (concrete to abstract techniques; Bay-Williams, 2001).

In kindergarten and first grade, math instruction is centered around the recognition of patterns within numerical expressions, and their ability to extend such rules/patterns to other math expressions (Lins & Kaput, 2004). Children in school are taught such patterns at an early age, but are not readily introduced to the purpose or flexibility within the patterns, but rather a memorized generalization of the skewed patterns. Students in elementary school first begin learning math as numbers by creating concrete representations (e.g., using concrete objects to model math problems), and often do not recognize the flexible potential or function of the digit (Carr et al., 2011). By second grade, children gain the capacity to cognitively represent such numbers, and begin doing so through abstract reasoning of numbers and their relations to one another (Carr et al., 2011). At this stage, children first acquire the ability to mentally represent numbers and operations. More broadly, there is evidence from the strategy-instruction literature indicating that when students are provided with ample opportunity to practice new strategies and understand the effectiveness of these new strategies then they are able to acquire and use these strategies independently (Bay-Williams, 2001; Carr et al., 2011).

In these first years of math learning, students' level of math proficiency is based on their ability to focus attention, test ideas, take risks, reason flexibly, try alternatives, perform self-regulation, and persevere (Copley 2010; as cited by the US Common Core, 2016). First graders often rely on concrete manipulations and picture references when gaining fluency in their mental math abilities. In large part, a child's understanding of math comes from discerning correct vocabulary, as well as appropriately using concrete and technological tools. By gaining access and exposure to numerous outlets to exercise such understanding, children are able to gain math proficiency by connecting patterns and structures to other mathematical concepts. Furthermore, by noticing such regularity and trends within problems and concepts, students can prepare for future tasks.

After learning numbers and patterns, children in the second grade begin developing logarithmic representation, which builds upon linear representation. Similarly, estimation tasks and numerosity estimation are presented at this age, and mastery of the topics has been linked to student's achievement test scores (Booth & Siegler, 2008; Carr et al., 2011). Linear representation of numbers helps students progress in their math understanding by reducing the likelihood for errors as children will appropriately acknowledge number distance and relation (about the number line). To test this, researchers have identified the importance of both verbal associations between the problem and solution, and the concept of number sense with the possible combination of numbers to manipulate expressions to find the solution (abstraction). Therefore, representation of numerical magnitude at an early age is critical to further math development in algebraic understanding.

Research posits that the deficit in mathematical understanding begins to arise as students enter the transitional shift between concrete representation of numbers and abstract conceptualization. This likely occurs due to a lack of understanding of number sense and the ability to see the flexibility and fluidity of expressions through operations (Kalchman et al., 2011). Misconceptions and errors in algebra arise as early as number cardinality and notation. There are specific misconceptions and difficulties that students struggle with, namely, the overall understanding of order of operations, the use of parentheses within an expression, and the concept of equivalence (use of the equal sign) (Knuth et al., 2006; Welder, 2012; Ottmar, Landy, & Goldstone, 2012). First, students visibly struggle with conceptualizing order of operations at a young age. This misunderstanding likely derives from student's lack of organization understanding within an expression. For example, children often do not understand that parentheses also function as a multiplicative indicator as well as an organizational tool. To illustrate, 3x6 may also be written as 3(6), but more importantly may also separate an expression like this: (3+17) - 2. Children who do not have a solid understanding of the order of operations would likely struggle to determine the appropriate order in which they would solve the expression, making complex math expressions nearly unsolvable. Order of operations has been noted as a major area of confusion for students learning algebra (Welder, 2012). Decomposition allows students to see the correct method or best way for them to solve the problem (ex. 4 x 6 = 24 replace 6 to 4+2 to see 4 x (4 + 2) = 24, maintaining the same value about the equal sign). Current instruction readily focuses on teaching students to remember their order of operations through the memorized mnemonic P.E.M.D.A.S. (parentheses, exponents, multiplication, division, addition, and subtraction); however, without conceptual understanding of the operations, a student may believe that addition must be performed before subtraction, rather than seeing the purpose behind the expressions order (Welder, 2012). This concerning action could potentially lead students to view 2 + x as 2x (Welder, 2012). Second, the notation of equality and its role, is fostered in students understanding of the symbolism of equivalence, rather than as a directional symbol or one that separates problem from answer (Welder, 2012). This understanding becomes critical in algebraic understanding as students must be able to correctly interpret equal sign and view its relation and equivalence (Knuth et al., 2006; Welder, 2012). However, if provided early exposure to not only decomposition of the expressions numbers, but also flexibility about the equal sign, children will increase understanding as each technique possesses versatility. For instance, children when presented with 3 + 3 = 4 + 2 instead of 3 + 3 = 6 and 4 + 2 = 6, may begin to understand the flexibility of the equivalence notation rather than view it as a rigid obstacle (i.e. the number to the right of the equal sign does not need to be the expressions definitive answer). Equivalence in mathematics is noted as a rudimentary foundation of algebra, and relies on strong quantitative skills fostered in early elementary mathematics teachings (Knuth et al., 2006).

In attempts to understand alternative and beneficial ways of teaching, one should utilize the appropriate developmental trajectories for the expected age-range, and complement the intervention with suitable yet progressive math techniques (e.g., decomposition). Specifically, it is possible for algebraic interventions to rely on common mathematical concepts that are taught to students well before their first algebra class with a preliminary goal of understanding *where* and *why* children have difficulty with earlier content, so that further misconceptions may be prevented. Several specific barriers to tackling algebra have been identified in the sections above; however, questions in related literature address the overarching necessity for adaptable and alternative ways of learning that ensure self-explanation, understanding, and personal growth among students, rather than mindless memorization or deterrence to mal-rules. This study supports the aforementioned body of research, and promotes an alternative self-paced interactive learning environment. By creating ways to bridge the gap between children's knowledge and the concepts at hand, construction of self-guided knowledge rather than solution guided feedback (being told the answer) is allowed and facilitates growth of declarative knowledge -- as seen in the FH2T-E 'goal-state' (Aleven & Koedinger, 2002). Through introduction and exposure of critical algebraic reasoning and fundamental concepts at earlier ages, children are provided the necessary tools to succeed in algebraic and future math conceptualization.

Students who are successful in learning algebra progress through a series of conceptual steps that can be more precisely defined as number sense, logarithmic representation, fact families, and (most importantly) decomposition (VanDerHeyden & Burns, 2009). Interventions that emphasize techniques such as decomposition and teaching operations could contribute to promoting early readiness among elementary school students.

Decomposition and teaching operations approach to early readiness. Several programs, such as Clements and Sarama's *Building Blocks* (2007), highlight the current push for earlier introduction and precedence placed upon the initial techniques and tools introduced in early math learning. For example, decomposition is one technique that is found to be paramount to future algebraic understanding, yet is often explicitly introduced at varying ages/grades, in alternative context, and not readily associated to algebra or further math understanding. Decomposition as a math tactic is defined as the understanding that numbers are made of many different components, and may be rearranged in a way that makes the most sense to the student (Clements & Sarama, 2007). Within this study, we take a concept such as decomposition, and provide materials in which students can further understand the properties of a given number/expression. *Building Blocks*, a nationally funded program based on the work of Doug Clements is focused on incorporating technology to enhance the content of material given to children in early math education, as well as help children create a more solid foundation to reduce risk of later academic failure (Clements & Sarama, 2007). When considering decomposition, students begin with a single number and are asked to explore its properties, as they are asked, for example, "*what two numbers can make 10?*" Inclination and tactics of decomposition are taught as early as kindergarten, as teachers see the meaningful action behind children understanding grouping, relationships, and patterns. Acting as a springboard for children's math understanding at an early age, decomposition is imperative to understanding progressively more formal mathematical learning such as algebra.

Decomposition allows students to think forwards and backwards in a flexible way, in turn allowing them to manipulate expressions into something that they can more easily reason through. However, many precursors to algebra are taught in isolation, which allows minimal overlap in understanding between topics and reduced exposure for reasoning techniques. Therefore, it is possible that if decomposition of numbers was introduced at an earlier age, as a precursor to formal algebra learning, children could potentially gain exposure to increased understanding of mathematical reasoning and foundational concepts. Reasoning about and decomposing shapes and numbers is identified as a critical learning sector relative to First Grade Common Core Standards (USCC, 2016). By understanding congruence and symmetry of both shapes and expressions, children can break free from their initial solely concrete representation of numbers to a more abstract way of thinking. Therefore, current research posits that by introducing decomposition techniques at an earlier age, intervention strategies for introduction may result in an increased understanding of decomposition as students may begin to attend to visual patterns (Aleven et al., 2010; Brendefur et al., 2013).

The second year of formal schooling is strategically organized in a way to supplement the appropriate math development (USCC, 2016). By following the natural development of number sense and cardinality, math lessons begin with building a solid foundation of number sense and the concrete properties of numbers. It is upon this foundation that the learning of prealgebra and algebra is built in middle and high school. Such foundations and critical lesson plans are recently being introduced to classrooms via numerous learning intervention frameworks. Therefore, educational intervention programs possess learning frameworks that lend their success to the promotion of student engagement and increased performance in math education. Learning technologies offer an alternative approach for teaching and learning math content that differs from traditional math instruction and facilitates student acquisition of appropriate perceptual strategies.

Perceptual Motor Learning Frameworks in Mathematics Education

Programs that focus primarily on interactive engagement of content through visual cues and grouping, as well as dynamic interactions to make meaning of the interaction, utilize a Perceptual Learning (PL) framework. Such interventions introduce appropriate perceptual processes (visual cues, physical grouping, and dynamic interaction), specifically permitting such benefits when used in combination with mathematics instruction (Goldstone et al., 2011; 2017). Work in cognitive science and math education has found that perceptual learning is a valuable approach to teaching mathematics (Goldstone, Landy, & Son, 2010; Goldstone et al., 2011; Ottmar & Landy, 2017). By creating and utilizing perceptual learning techniques (within a PL framework) that lead to neural cognitive changes that are retained throughout one's life, meaning students not only gain an increased understanding but also *retain* an understanding. These techniques foster initiative towards using sensory and cognitive interaction, as reflected in increased student achievement and performance (Seitz & Watanabe, 2005; Kalyuga, 2009; Kellman et al., 2009; Goldstone et al., 2011; Cortes et al., 2015). Previous literature and findings expressed that PL frameworks offer increased meaningful exposure (when compared to conventional instruction), and include activities to uncover the fluency in extracting information from complex expressions through physical interaction (Goldstone et al., 2011). Students increase fluidity in expression recognition, as well as familiarity in expression transformation; all made possible through the fluid visual techniques in a Perceptual Learning framework.

Specifically, a perceptual learning framework may be broken down for further understanding. In particular, the theory, Pushing Symbols (PS), is a mathematical approach that embodies student-based discovery-learning techniques by displaying numbers as symbols and objects, thus making the innate structure of math symbols and expressions more explicit and visually determined (Ottmar, Landy, & Goldstone, 2012; Ottmar et al., 2015). Symbols are made into virtual objects that permit student manipulation to learn flexibility in numbers/expressions within the constraints of the natural laws of mathematics. PS stands alone as one form of true manipulation, through touch screen interfaces, that permits fluid visualization aligned with appropriate cognitive content (Ottmar, Landy, & Goldstone, 2012). Physical manipulation of mathematical symbols has been deemed intrinsically engaging, and offers a new yet natural way for children to understand symbolic constraints. This ‘play-like’ engagement offers puzzle-based situations rather than procedural steps. The Pushing Symbols (PS) theory is grounded in evidence supporting techniques of perceptual training to facilitate three fundamental algebraic perceptual processes (Ottmar, Landy, & Goldstone, 2012; Ottmar et al., 2015). First, *symbols are* *treated as physical objects.* By envisioning the process as a fluid motion (including destruction, creation, and flexibility) the symbols become physical objects susceptible to manipulation. Second, *perceptual grouping affects mathematical performance* (Seitz & Watanabe, 2005). When visualizing an expression, the placement of notation may lead the student to misunderstand the memorized standard order of operations. Third, *learning attentional tendencies is key to mastering mathematics.* Those who excel in math tend to place certain attention, or salience, on multiplication over addition. This understanding comes from appropriate attendance to specific components of an expression (Seitz & Watanabe, 2005; Carr et al., 2011; Welder 2012).

It is here that the pivotal action and understanding of *decomposition* becomes imperative. Decomposition, as seen in the context of the Pushing Symbols theory, reinforces the importance of early readiness and preparation for future learning. Such understanding of physical transformations and appropriate mathematical laws is impractical without first creating a solid basis of understanding surrounding the decomposed properties and flexibility of numbers (Ottmar, Landy, & Goldstone, 2012). Decomposition is made possible as math symbols become virtually movable objects and student thinking shifts towards a more flexible and dynamic way of learning.

From Here to There! (FH2T)

From Here to There! (FH2T) is a mathematics utility game that uses perceptual based interventions to introduce foundational algebraic concepts (Ottmar et al., 2015). The guided instructions used in FH2T mirror those set forth by the United States Common Core Mathematics Standards for middle and high school (CCMS). Aligned with the progression in the Common Standards, the modules in the game allow students to slowly increase complexity at a subjective level (individual level). This was done by assessing the United States First Grade Common Core Standards (USCC 2016) to then understand the scaling of conceptualization within each mathematical construct, and thus align it to the classroom curriculum.

From Here to There! is an intuitive program that relies on self-paced interaction, and functions as an introduction for students to mathematical content through discovery-based puzzles that engage perceptual-motor systems (Ottmar et al., 2015). The original intervention program was designed to increase student familiarity with algebraic notations as manipulative objects. The hope of this intervention was to increase student confidence and ease with performing certain expressions. This innovative game displayed on a touch-screen interface allows both physical and dynamic manipulations of the expressions by students, providing a powerful source of perceptual-motor experiences, which in turn lead to increased acquisition of appropriate operation parameters (Ottmar et al., 2015). With this tool, instead of simply applying memorized procedures, students are able to directly interact with numbers as objects.

From Here to There meets the criteria for a high quality mathematical program with appropriate expectations and assessments (NCTM, 2017). Beyond the game's intuitive and engaging nature, it also possesses alignment to the necessary developmental trajectories of its target audience, and provide an interactive and comfortable environment in which students may explore math topics. In detail, the first fundamental guiding principle set forth for the state of Massachusetts mathematical programs places *learning* paramount. Learning is described as "exploration in ways that stimulate curiosity, create enjoyment of mathematics, and develop depth of understanding" (USCC, 2016). Programs that follow a perceptual learning framework, such as *From Here to There,* offer a dynamic and interactive outlet to promote student engagement, as well as academic achievement (Seitz & Watanabe, 2005; Ottmar et al., 2015).

FH2T specifically offers a unique game software accessible on both a touchscreen and personal computer interface. The game's universe-like module progression allows students to ‘play’ the game by increasing complexity, and levels, each time a construct (i.e. subtraction, addition, order of operation) is understood. An image from the game’s home screen can be seen in Figure 1.

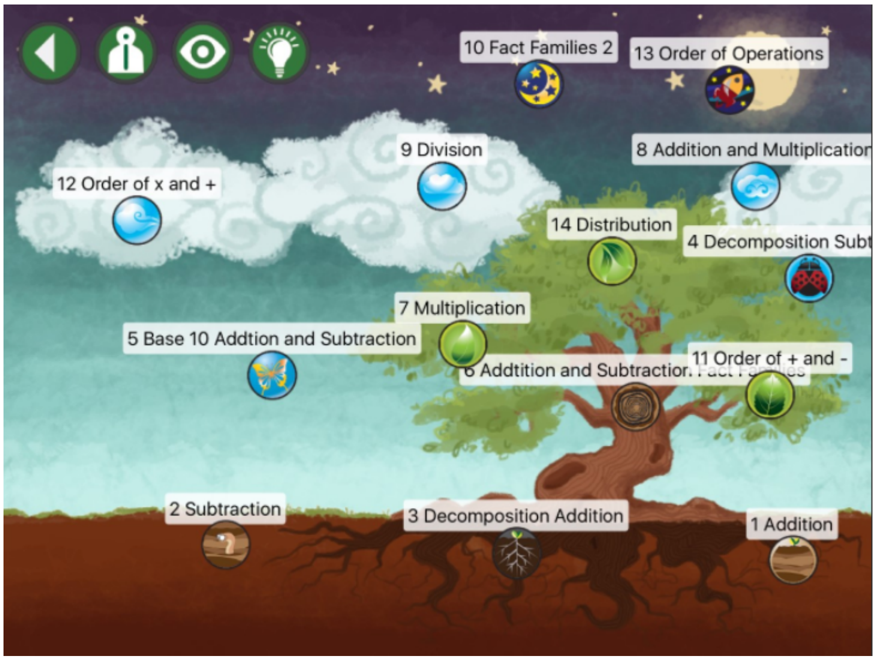

Figure 1. FH2T-E World View. The front image of the game includes the 14 worlds within the FH2T-Elementary Universe. Students begin at the first world, and the subsequent worlds become available as the progress through each level.

This underscores the necessary balance of procedural and conceptual understanding in a previously tested thought-provoking and engaging environment. Such engagement elicits perseverance through cognitively difficult tasks (extended learning) and promotes mathematical reasoning developed through maturation of abilities and skills (Carr et al., 2011; Belenky & Nokes-Malach, 2012; Ottmar et al., 2015).

The process of decomposition and promotion of early readiness is key to the empirical success of *From Here to There!*. Like other mathematics intervention based programs/games, the main focus has been to increase student understanding, with attempts at increasing such understanding to promote early readiness (Ottmar et al., 2015). However, the unique features of FH2T such as its goal-state 'solution', as seen in Figure 2, provide a suitable environment for students to engage in trial-and-error decomposition while remain within the confines of natural math law.

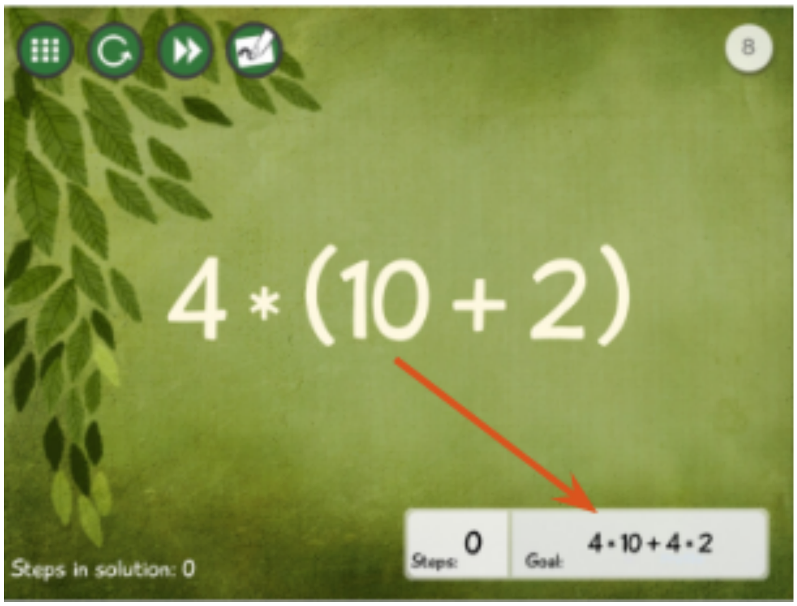

Figure 2. FH2T-E Sample Problem. The image above displays the designated "goal-state" for each problem. This creates puzzle-based expressions pushing students to rearrange the expression to match the goal.

In detail, From Here to There (FH2T) is an individualized, self-paced, and engaging application that exposes young students to mathematical concepts and techniques through discovery-based interaction. Through a puzzle-like approach, students when interacting with the game do not apply their learned procedures by rewriting different expressions, but rather this technology allows students to physically and dynamically manipulate algebraic expressions. By providing a potentially useful and beneficial outlet for perceptual-motor experiences, preparation for future learning among a younger cohort of students may increase. The promising effects can be seen in previously conducted feasibility experiments, as well as the contributions of early readiness and exposure. Below we describe the design theory, features, and goals of the program.

As an intervention, the *From Here to There* framework provides mathematical equations and notations that align with the mathematical standards appropriate for that age range. FH2T-E is designed to possess 14 modules that focus on different mathematical concepts, including the distributive property, order of operations, basic operations, solving linear equations, and factoring. Each module presents a series of puzzles, organized into one to three "levels" per module. These puzzles have various goals including calculating a stated expression, transforming an expression so that it matches a goal, and stating whether two expressions are equivalent. Rather than simply solving the program, students are asked to make the given expression look like an equivalent expression that was specified in the goal. To achieve this goal, students are required students to perform a series of dynamic interactions, including decomposing numbers (8=5+3 or 11-3), performing operations to combine terms, rearranging terms to apply the commutative and associative properties, and adding terms to both sides of an equation.

Studies performed in the middle school population show promising results, as measured by age-appropriate measures of pre-algebraic and early algebraic skills (Ottmar & Landy, 2015). Although promising, this approach has not been used in elementary school. Therefore, an assessment of feasibility and early introduction was conducted, and the subsequent success of the feasibility study analyzed for further expansion.

Current Study

The primary aim of the overall project was to test the feasibility of earlier introduction of pre-algebraic concepts, the differences in use of modalities (iPad/Computer), and the subsequent efficacy of FH2T and improved learning when introduced at an earlier age. The successfully scaled-down program introduces fundamental mathematical strategies in a way that younger children may benefit from the experience of visual reasoning and physical interaction with symbols as objects. Within the study, there were three fundamental research questions that derive from the studies theoretical framework. The first question surrounded the feasibility of introduction of a scaled-down version of the intervention at an earlier age. We hypothesized that through the game's iterative design and unique goal-state 'solution' that elementary aged students would be able to conceptualize the patterns and strategies of pre-algebraic concepts such as decomposition.

In order to assess the success of the intervention once introduced at an earlier age, a larger controlled study focused on the results of pre and post assessments of conceptual understanding to determine transfer and retention. Empirical evidence from the subsequent middle school introduction of the intervention supports the expectation of increase ability, achievement, and overall math understanding/comfort. Finally, the third research question will help determine the benefits of gamified features present within an educational game. Gamification is the process by which game features are incorporated into electronic learning and assessment tools in order to enhance student motivation and engagement (Aleven et al., 2010; Belenky & Nokes-Malach, 2012). Interestingly, students are to be drawn into "stealth learning" by using a format with which most students are familiar, that of video games. Educational video games have been found to have results superior to conventional teaching and learning formats for a number of different critical skills. When considering the information concert, one must consider the possibility that such gamified features my supplement connections made to the material, thus solidifying understanding (Aleven et al., 2010). For educational games, one must consider the possible learning mechanism by which learning and engagement are impacted, and determine potential qualities that may deter from learning. There is potential that such game-like aesthetics in elementary games would promote future learning and increase engagement. However, it remains to be determined whether the game-like features within an elementary program would take away from learning and/or remain a superfluous detail neither adding nor taking away from the educational value of the game. Specifically, we hypothesized that the iterative and intuitive design of the game itself will allow for motivation and increased learning results when compared to the results of the game features gamified aspects; therefore, if there is not an evident difference, we presumed that increase achievement would result from physical manipulation within the game, regardless of the aesthetically pleasing features.

Method

Project Participants

In total, 10 teachers were recruited for this overall project (3 studies). In total, two hundred forty-two students participated from three different elementary schools in Massachusetts (118 female, 124 male). There was one first grade classroom (12 female, 11 male), and nine second grade classrooms (106 female, 113 male). Teachers participation in the study was voluntary. Private/Student Investigators visited with local schools to present the FH2T game and supporting information for inclusion in their classroom. Interested teachers were then provided an overall description of the game and study protocol, a study timeline, as well as information and demo videos of the game before committing to participation. Once teachers agreed, they were asked to send investigators a class roster detailing the first name, last initial, and gender of all students who would be participating. Parental consent forms were not needed as the study was determine exempt by the IRB.

Project Design

Study 1: Establishing the feasibility of FH2T-E. The first feasibility study included twenty-three first graders (6-7 years old; 11 males, 12 females) from one classroom in Massachusetts. Study 1 utilized a between-participants experimental design, meaning that half the students were randomly assigned to play FH2T-E on an iPad, and the other half played the game on a laptop computer. Therefore, conditions were based on modality: iPad or Computer. Study 1 addressed the following goals:

1. Establish feasibility of FH2T-E for introducing students to early mathematics concepts
2. Examine improvements in mathematics performance scores after the intervention
3. Determine differences in the effectiveness between iPad and computer versions.

Study 2: Comparison to control condition & analysis of group differences. The purpose of the second study was to conduct a prospective, controlled efficacy study to measure the impact of FH2T-E on math learning in elementary students. The primary goal of the efficacy study was to extend and incorporate the findings from the feasibility study in a larger population of elementary students spread over several different schools and classrooms, with a set of students receiving conventional math education (control condition). This study addressed two research questions:

1. Does FH2T improve math performance when compared to control classrooms?

2. For whom does FH2T improve learning?

   a. Is more improvement seen in boys than in girls?

   b. Is more improvement seen with high-performing vs low-performing students?

Study 2 included a between-classroom experimental design, with two conditions.

Classrooms that volunteered to participate in the experimental condition (playing FH2T) were provided the necessary materials, while teachers that opted out of the experimental activity but volunteered as a control classroom completed the pre and post study worksheets while continuing to receive traditional instruction, and only performing math worksheets. To maintain a between-classroom design, classrooms were randomly placed in either the FH2T-E condition, or the traditional instruction control group. Classrooms remained in their respective conditions for the remainder of the study. After addressing any major differences in the modality used in Study 1, the primary research focus for this study addressed learning outcomes and achievement between-classrooms, in comparison to the control condition. By addressing the differences between-classrooms in the experimental condition in comparison to a control condition, we were able to address learning gains present after interaction with the FH2T-E game.

Study 3: Analysis of gamification as potential learning mechanism. The purpose of the Study 3 was to investigate possible learning mechanisms (promoting the pre to posttest performance improvement). This study also included a between-classroom experimental design, with two conditions. Classrooms that volunteered to participate were randomly assigned in one of two experimental conditions: gamified versus non-gamified (explained below in materials). Classrooms remained in their respective conditions for the remainder of the study. Although different students interact with different aesthetic features of the game, the math content in each version is exactly the same. By addressing the differences between-classrooms in the experimental conditions, and analyzing student achievement, we were able to address a possible mechanism by which *From Here to There: Elementary* is leading to learning gains. Specifically, the gamified version of the FH2T-E intervention is intended to reinforce the elements of decomposition and perceptual learning with elements of competition and reward of appropriate behaviors. Therefore, the scope of this study had one research goal:

1. Determine if the learning benefit is due to the gamification aspects of FH2T-E, or from the conceptual design and intrinsic educational benefit.

Materials

Technological device condition & game. All computers used in the study were laptops and/or iPads purchased by the schools or provided by the researchers; all programs were run using Google Chrome as the internet host. In all experimental conditions, the FH2T-E intervention was played through either an iPad or computer interface, while the control condition received their daily math activities from their teachers and basic math worksheets.

The game is self-contained when running on an internet host, and compiles all data (such as gesture approach, frequency of restart, number of steps to solution) within its internal storage system. The web-version of the game was provided to teachers as a link, and is available here: fh2t-mqp.herokuapp.com.

Gamified version (FH2T-E). As part of the gamified condition within the study, students played through the existing version of the game that possess game-like features. Gamification relates to the presence of prizes, bonuses, sounds, stars, etc. (i.e., any feature making the game more aesthetically pleasing rather than just the presence of math expressions). Aligned with these levels, FH2T is composed of 14 modules, each focusing on one primary math concept (See Figure 3).

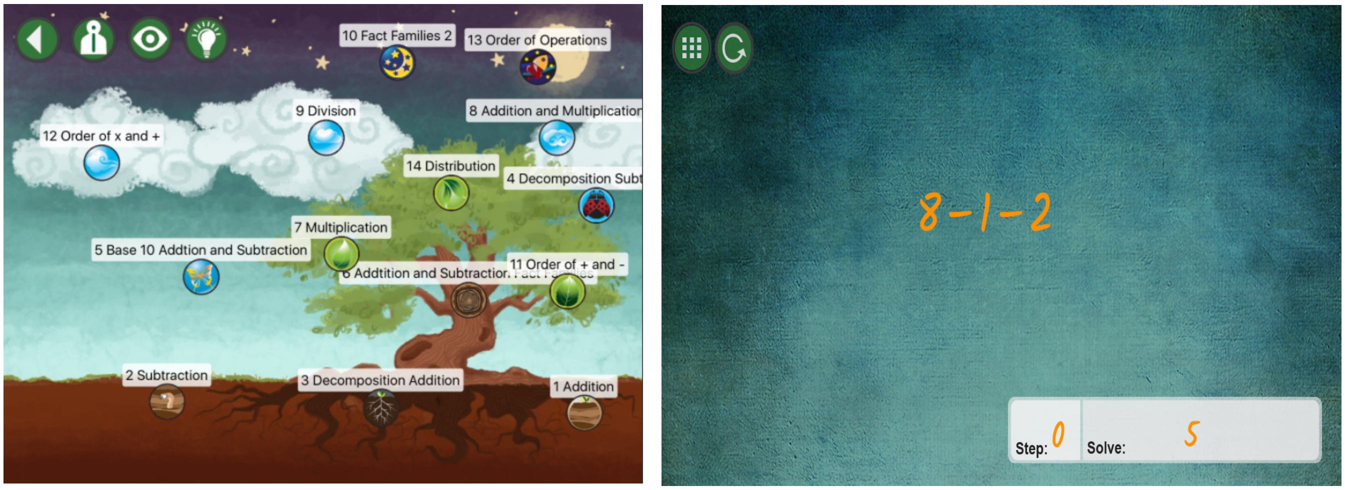

Figure 3. FH2T-E Gamified Version Display Screen. The images above display the home screen layout of the FH2T game (left), and a sample expression (with game like features present) (right). The 14 bubbles indicate the 14 worlds present within the game.

Each lesson contains a subset of problems, organized to increase in difficulty as the level progresses. Each question/puzzle/problem has been designed to include calculations, transformation of the expression, and comparison of expressions. A crucial component to FH2T is the goal-state solution. Rather than providing students with a specific solution (e.g. x=4), the game provides students with a desired goal state (2+ 2) that forces students to use the flexibility in their knowledge to reach the given state. Students were asked to mimic the given expression, and performed dynamic interactions such as decomposition. Furthermore, students were asked to perform operations of combining and breaking up terms (addition/subtraction), as well as rearranging problems to embody both commutative and associative properties.

Each new world within the modules required students to use both new operations and alternative rules to match their previously acquired procedures and skills. In the game, each new procedure is presented to the students as a motivational puzzle, increasing in difficulty, until the level is completed and the procedure is ‘unlocked’.

Non-gamified version (plain FH2T-E). The non-gamified version of *FH2T-E* has been stripped down to obtain only the 18 math problems within each level. As students play through this version, there is no recognition of level completion or rewarded clovers for accuracy and efficiency. This lack of aesthetic features and reward-based prizes creates a condition to assess the degree at which the generated results stem from the advantages that gamification or the intuitive/interesting nature of the FH2T game provide. However, the mathematical problems, levels, and goal-states within the game have not changed. Students must still master one topic/problem before moving on to the next, with each problem increasing in difficulty. These changes can be seen in Figure 4.

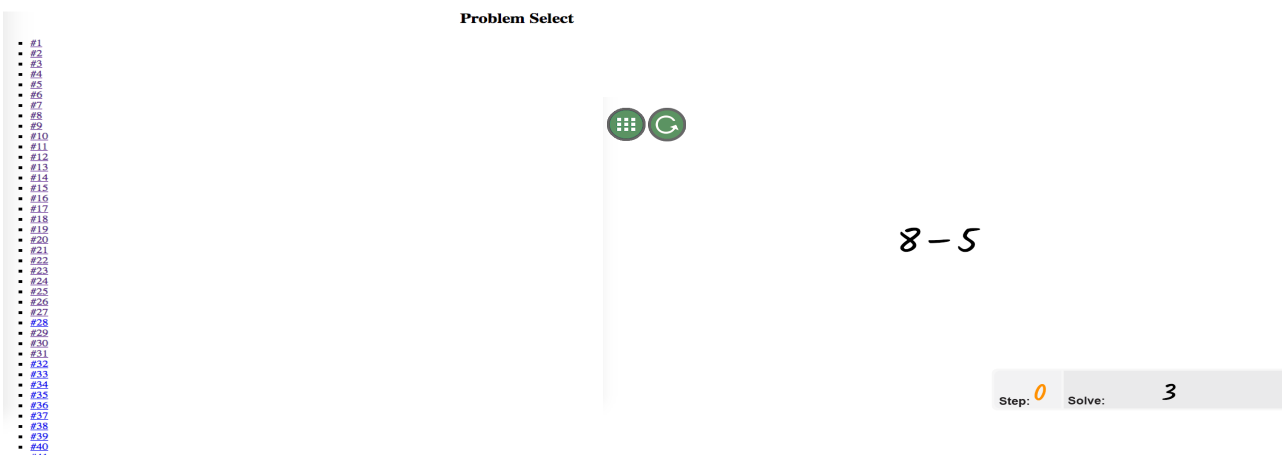

Figure 4. FH2T-E Non-Gamified Version Display Screen. Above is the problem menu (on the left) and the sample problem (on the right). The non-gamified version of the FH2T-E game possesses the same mathematical content; however, the aesthetic game-like features present in the original FH2T-E have been stripped for the experiment.

Therefore, if differences between the experimental conditions is statistically evident, results may highlight one possible mechanism by which FH2T leads to gains and increased achievement (e.g., gamification through embodied touch) or provide evidence for engagement as a result of the game's intuitive puzzle-like design.

In short, although the math content on both version of the games remained the same, students playing the FH2T gamified version viewed the original colorful universe settings, were presented awards for correct completion, and experienced the aesthetically pleasing aspects of the game. The only differences in the game are the visual material.

Measures

Data collection for this project will include a combination of student scores on pre- and post- study worksheets, naturalistic observation of the students interacting with the game, and teacher interviews to gain perceptions and opinions regarding the FH2T-E game.

Pre and post assessments. The assessments were created to analyze achievement outcomes, that highlight student improvement relative to interaction with the game during the experiment. The assessments contain 15 questions and have been tailored to mirror first and second grade math standards set forth by the United States Common Core (See Appendix A) (USCC, 2016). Questions within the pre and post assessments were kept similar, as the only changes made included formatting of the order of the problems, and the numbers within each expression. This way, achievement was measured based on percentage correct improvement on both assessments (See Appendix B for sample assessment).

Naturalistic observation and teacher interviews. To obtain quantitative data relative to student's perceptions of the game and how the overall game was received within the classroom. One-on-one informal interviews were only conducted with the teachers who participated in the experimental conditions in this project, and student observations were taken by the student investigator. From the interviews, teachers were asked to describe the experience, student's perceptions, pros and cons of the game, as well as general comments relating to the feasibility and rationale supporting the game.

Project Procedure

The proposed protocol for each of the three studies in this project followed the same format (See project protocol timeline in Appendix C). After teachers were provided information relative to the game, they were asked to follow the detailed instructions.

Prior to the introduction to the game, students completed a 15-item pre-study worksheet to assess their current math abilities. This assessment was made to be compatible not only with first and second grade mathematics standards set forth by the United States Common Core (USCC, 2016), but also with the participating teacher's current difficulty level of assignments and activities within this class's primary math curriculum (See Appendix B for pre/posttest examples). These questions mirrored baseline understanding of decomposition, operational strategies, and basic notation, as well as understanding of mathematics vocabulary. If questions posed too much difficulty for children, the teacher was available to provide assistance and feedback; however, we asked that solutions not to be given, only motivational or instructive comments.

Completion of the pre-assessment was done one week before interaction with the game. Before playing, students then watched the demo instructional videos to introduce the game and potential misunderstanding of the game tactics. Students interacted with the game for four, 20 minute sessions spread out over two weeks, while those in control classrooms continued business-as-usual math instruction and did not interact with the game during the study timeline. After the four sessions were completed, students completed the post-study worksheet in the following week. The problems and expressions on the posttest were similar to those found on the pretest; however, the numbers within each problem have been changed, and the order has been rearranged to offer variety in problems. After completion, students and teachers (including the control groups) were thanked for their participation, and were able to keep/given access to the game to use as they please. Students then continued their business-as-usual math instruction with their teachers and respective math content.

Results

Study 1 (Feasibility Study)

In order to determine if the use of the elementary version of FH2T-E was appropriate in elementary school grades, the FH2T-E tool was used in 23 first grade students, all within a single classroom with a single teacher. 11 students used a computer screen and 12 students used an iPad. We examined whether there was improvement in math performance using a two-tailed t-test on the pre and post assessment mean scores. As shown in Figure 5, test scores, vindicated by the total percentage of items correct, improved significantly from 63.6% to 71.8%, comparing pre- ($M= 0.636$, $SD=.26$) to post- worksheets ($M= 0.718$, $SD= .24$). There is an approximate difference of 8.1% ($SD=.33$, *ɲp*$^{2}= 0.33$, $t\ (1) = 1.717$, $p = 0.05$).

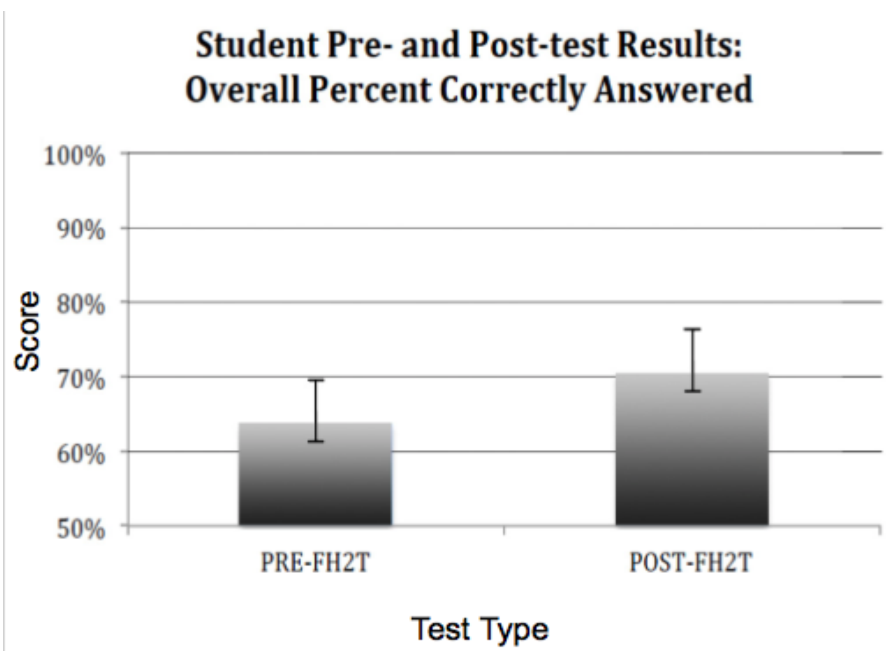

***Figure 5*.** FH2T-E Feasibility Study Change in Student Improvement. The above graph displays the improvement in student performance among both conditions using FH2T-E in the first-grade feasibility study.

Linear regression was also used to examine contributions of pretest performance, gender, and delivery device (modality) on mathematics performance. Pretest scores (*M=0.636, SD=0.256, β=-0.189*) for individuals (on both modalities) significantly predicted posttest scores (*M =0.718, SD=0.243)*, accounting for 0.6 of the posttest difference, (*β=0.607, t (22) =14.21, partial* $eta^2 = 0.33$, $R^2 = 0.036$, *p=0.05, two-tailed test*). No gender differences were observed; girls and boys performed similarly on pre/post-assessments. Analyses failed to reveal significant differences between computer and iPad groups, indicating the utility of both delivery device modalities.

Overall, the intervention was feasible for implementation by the students and the teacher, both of whom offered positive comments at the end of the intervention. These results cohesively support our initial overarching hypothesis that the game had increasing potential for early elementary introduction, and that students at a younger age could reason through and enjoy the material. In the absence of a randomized design, one cannot conclude that the gains were due to the intervention; however, the presence of strong gains in a short duration are indicative of the program's potential efficacy.

Study 2 (Efficacy Study)

To directly measure the efficacy of FH2T-E in elementary school students, 146 students participated after nine teachers/classrooms were recruited from 3 different elementary school classrooms in Central Massachusetts. Six classrooms were enrolled in the experimental group and three in the control group. Comparison of pretest and posttest scores (overall percent of items answered correctly) is shown in Figure 6 based on condition.

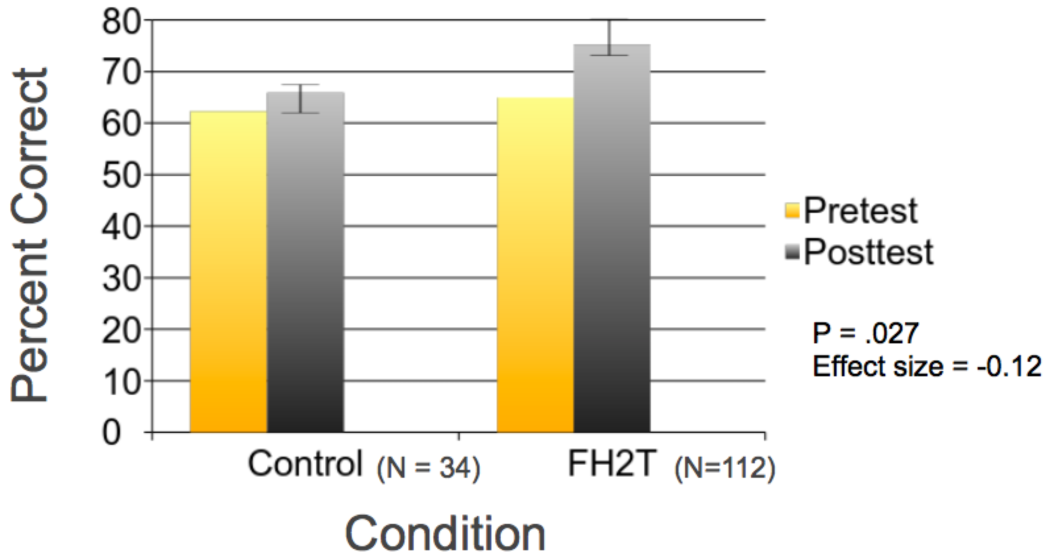

Figure 6. FH2T-E Efficacy Study. The above graph displays the efficacy of FH2T-E for increasing math performance, as compared with conventional math teaching (Control vs. Exp.).

As shown in Figure 6, math performance was improved in both groups. However, the students using FH2T-E had a 7.06% significant improvement from pretest (*M=65.88, SD=27.69)*, to posttest ($M=75.24$, $SD= 24.59$, $t\ (4) = -2.23$, *partial* $eta^2 = -0.12$, $R^2 = 0.611$, $p=0.027$, *two-tailed t-test*), demonstrating a statistically significant greater efficacy of FH2T-E as compared with conventional elementary school math teaching.

Next, we sought to determine whether the student's pretest scores or gender accounted for the percent improvement in math performance by running regression analyses. Shown in Tables 1 and 2, regression analyses were conducted using the total percent correct (pre/post) (preTotalpercCorrect is equivalent to the student's worksheet scores) and the students gender. Regression analyses were performed to determine any potential group differences, and showed no significant differences in the regression analysis evaluating for the contribution of each of these variables.

Table 1.
Co-variable analysis determining whether pretest scores of students affected posttest improvement.

	Unstandardized Coefficients		*Standardized Coefficients*		
Model	**B**	**Std. Error**	***β (Beta)***	**t**	**Sig.**
Constant	30.346	4.263	-----	7.119	.000
Gender	-2.958	2.696	-0.058	-1.097	.274
preTotalperc Correct	.707	.059	.741	11.918	.000
Control	-14.113	7.739	-0.234	-1.824	.070
*pre**X**control*	.743	.744	.131	.999	.320

This regression test initially analyzed pretest scores as a significant indicator of posttest improvement. The results of this first regression indicated that pretest ability did not significantly predicted posttest improvement (preTotalprecCorrect *(β = .743, p=.320)*. Furthermore, the results of the first regression indicated the predictor (pretest score) explained 61.1% of the variance (*R^2 = .611, F (3,142) = 74.3, p<.01*). The P-value for the analysis is 0.320, non-significant, meaning pre-intervention ability is not a strong indicator of posttest improvement.

Second, the same regression analysis was conducted to determine a potential interaction between pretest score and the student's condition assignment. The results indicated the two predictors (pretest ability/condition) explained that pretest ability and the condition assignment did not significantly predict posttest improvement (preTotalprecCorrect (β = .740, p<.01). The results of this regression analysis indicated the two predictors explained 61.4% of the variance (*R^2=.614, F (4,141) = 55.98, p<.01*).

Table 2.
Co-variable analysis determining whether gender of students affected posttest improvement.

	Unstandardized Coefficients		*Standardized Coefficients*		
Model	**B**	**Std. Error**	***β (Beta)***	**t**	**Sig.**
Constant	29.736	4.825	-----	6.163	.000
Gender	-6.051	7.115	-0.119	-0.851	.396
preTotalperc Correct	.715	.071	0.749	10.089	.000
Control	-7.093	3.171	-0.118	-2.237	.027
*pre**X**gender*	.319	.676	.073	.473	.637

The results of the third and final regression analysis indicated the two predictors (pretest ability, gender) explained that pretest ability and gender did not significantly predicted posttest improvement (*pre**X**gender* (*β = .319, p=.637*). Furthermore, the results of the second regression analysis indicated the two predictors (pretest ability/gender) explained 61.2% of the variance ($R^2 = .612, F(4,141) = 55.49, p < .01$). The P-value for the analysis is 0.637, non-significant, meaning gender is also not a strong indicator of posttest improvement.

Taken together the outcome analyses from study 2 indicates that FH2T-E has preliminary evidence of efficacy in the elementary school population and that this efficacy is not significantly affected by prior mathematics ability nor by the student's gender.

Study 3 (Gamification Analysis)

In order to determine whether the gamified features of FH2T-E were the primary cause of improved performance, we conducted an independent samples t-test to compare the pretest to posttest performance (overall percent of items answered correctly) between the "stripped down" or non-gamified version of FH2T-E and the gamified version. The results indicate that students performed equally well if they used the non-gamified version (*M*= 72.69, *SD* = 23.04) or the gamified version (*M= 75.24, SD = 24.60), t (183) = 0.705,* R^2 *= 0.50, p =0.482, two-tailed),* with no significant difference. The results are shown in Figure 7.

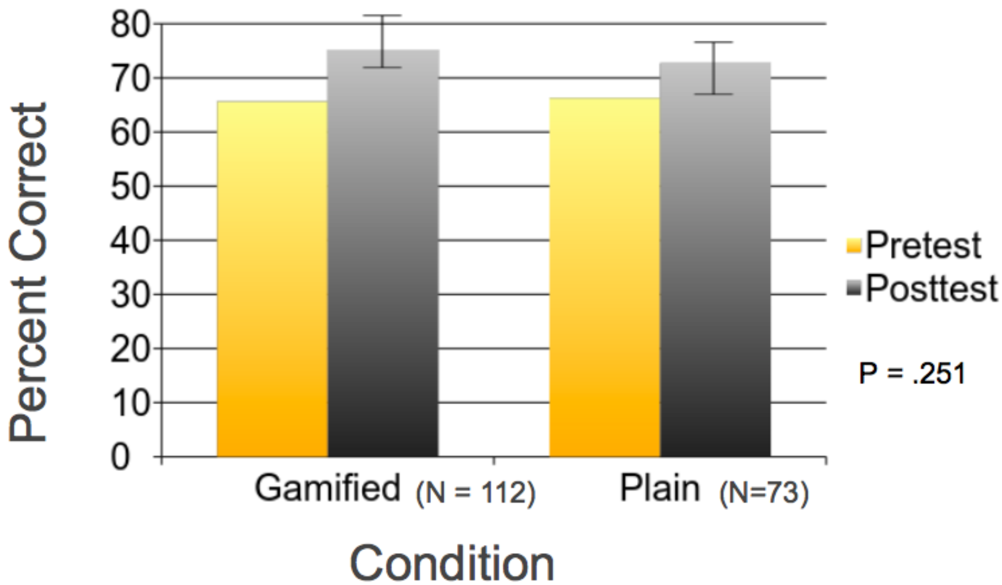

Figure 7. FH2T-E Gamification Analysis for Potential Learning Mechanism. The above graph displays the efficacy of the non-gamified version of FH2T-E as compared with efficacy of the gamified version.

Discussion

This study tested the hypothesis that a perceptual learning (PL)-based on-line tool (FH2T-E) will improve learning of algebraic principles in an elementary school population more effectively that standard instruction. The study demonstrated the feasibility of using FH2T-E in this population. It also showed that FH2T-E resulted improved math learning compared to conventional math instruction. Furthermore, this research revealed that the benefits of the game are more in the way students learn in the intervention rather than the game-like features provided in the game. Overall, these findings reinforce the concept that PL helps improve mathematical reasoning. This outcome specifically extends the evidence for the PL approach to math instruction into the elementary school-aged population. Furthermore, there were no significant interactions between gender or prior achievement. This suggests both a broad applicability of the findings and evidence that there is no difference between the two versions in their effects on learning outcomes.

We created the developmental framework behind the PML intervention and constructed two experimental designs to highlight potential impacts and increase of student performance and engagement. The inclusion of Common Core Standards (USCC, 2016) and end of the year assessments put together by teachers outline the aforementioned appropriate developmental trajectories of the students in the target audience. The FH2T-E intervention is based on the theoretical concepts of decomposition and perceptual learning. In publications set forth by The National Council of Teachers of Mathematics (NCTM) findings indicate that once a solid foundation of decomposition of numbers is understood, students are then able to successfully engage with higher-order formal math problems (NCTM, 2016).

Following these findings, future research directions should include studies to expand and generalize the FH2T-E approach within this age range and to develop additional versions of PL-based interventions designed for even younger students (Lins & Kaput, 2004; Clements & Sarama, 2007). If the PL model is generalizable, and has the potential to scale up or down, one would predict that the benefits of FH2T-E would likewise be generalizable to many other settings. Beyond these growing factors present within the United States, future studies should address whether FH2T-E will benefit students with different demographic characteristics (racial, ethnic, linguistic, cultural, etc.) than those in the study population. When considering other potential learning mechanism, or the potential value of gamification in a replicated study, future studies could include outcome measures reflecting differences in student engagement, motivation or strategy. This data can and will be obtained from the in-app data logged for each individual student's "game-session". The findings from this data may lend support to approach tactics within a perceptual learning framework (Lins & Kaput, 2004; Aleven et al., 2010; Ottmar et al., 2015). Finally, within this in-app data, the FH2T-E program has the capability to record all errant attempts made by students as they approach solving various items. Thus, it enables the researcher to visualize both the effective strategies used by students and the maladaptive approaches that the fall into. This sort of data could be used to intervene earlier and more effectively in poorly performing students as approach methods are tracked and feedback is immediate.

Another beneficial component to the FH2T-E program and perceptual learning framework is the endless potential for creating interactive activities from content. The FH2T-E is an example of a technology-based program that is available online. This accessibility will not on only help students continue their math practice during the school year, but it may also bridge the gap over summer break when students often lose ground in content understanding. This flexible and accessible nature of the FH2T program supports the creation of new games in the future that can introduce physical interaction with content via a technological interface. The games and concepts are made to be easily introduced and cost effective making introduction into school classrooms more realistic (Ottmar et al., 2015).

The potential limitations in these studies should be noted. To begin, the sample size for each study could have been more equivalent, in particular the control condition. Due to circumstances beyond our control, there was an uneven number of teachers and students in the experimental and control conditions. This was largely due to control teachers opting out of the study once it had started due to other constraints and priorities. The most evident limitations within the study design arose as an inevitable consequence of interacting with young children in schools with various academic schedules and instructors with diverse teaching styles. This differential attrition between groups may indicate that teachers see more value in the FH2T program compared to traditional worksheet style assessments. Furthermore, although we sampled a number of teachers in three elementary schools in varying school districts, we were unable to control for potential variables and sources of bias (e.g., varying ability, teaching styles, expectations, socioeconomic standards, etc.). In the future, replications of this study could look further into the potential interactions and results that arise when confounding for student race, gender, socioeconomics, and math anxiety. For example, stereotype threat is an interesting field as research as such threat has been shown to have significant effects on student performance (Schmader et al., 2008). It is possible that such factors may influence student performance and improvement when interacting with the game. Another limitation to this study was the time allotted for the overall project; subsequently, more time for the data collection would have produce more sufficient data. With working in a shorter timeline, study materials were streamlined and simplified to include basic appropriate math content; however, the testing materials could be further improved and validated in the future. For studies that include multiple grade levels (this study included 1st-2nd overall), it is necessary to operationalize and pilot test one's study measures to represent the appropriate material of the target audience. Also, the worksheets in this study could have been shortened as some students seemed to struggle to complete the sheet in the designated 20 minutes. These limitations are changes to consider when expanding this study design to include more elementary grades.

Such a comprehensive approach could ultimately lay the groundwork for an entirely new experience for students in the acquisition of algebraic literacy. If PL eventually replaces rote memorization as the foundation for mathematical thinking in the earliest years of formal education, it may be possible to move all students along faster and effectively toward math literacy, ultimately improving their occupational skill level within a future math-savvy work force. Such a transformation of American early-grade mathematics education has the potential to close the “math gap” between American children and those in higher-performing European and Asian nations potentially within a single generation (Hanushek et al., 2010). This could likewise improve the global competitiveness of our nation in global and digitally-connected economy of the 21st century.

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Appendix A
Common Core Curriculum Alignment

Common Core Standards		From Here to There: Elementary Worlds												
		Addition	Subtraction	Decomposition Add.	Decomposition Subtr.	Base 10 Add. & Subt.	Add. & Subt. Fact Families	Multiplication	Add. and Multiplication	Division	Fact Families 2	Order of + and -	Order of Operations	Distribution
Understand the relationship between numbers and quantities; connect counting to cardinality.	K.CC.B.4	*	*	*	*	*								
Compare two numbers between 1 and 10 presented as written numerals.	K.CC.C.7	*	*	X	X	*	X							
Decompose numbers less than or equal to 10 into pairs in more than one way	K.OA.A.3	*	*	*	*	*	*				X			
For any number from 1 to 9, find the number that makes 10 when added to the given number	K.OA.A.4	*		*										
Apply properties of operations as strategies to add and subtract.	1.OA.B.3	*	*	*	*		X							
Understand subtraction as an unknown-adde	1.OA.B.4	*	*	X	X									
Relate counting to addition and subtraction	1.OA.C.5	*	*	*	*	*	*		X		X	X		
Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false.	1.OA.D.7	*	*	X	X	*	X	X	X					X
Represent and solve problems involving addition and subtraction	2.OA.A.1	*	*	*	*		X					*		
Fluently add and subtract within 20	2.OA.B.2	*	*	*	*	*					X	X		
Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones	2.NBT.A.1	X	X	X	X									
Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction	2.NBT.B.5	*	*	*	*	*	*				X		*	
Add up to four two-digit numbers using strategies based on place value and properties of operations	2.NBT.B.6	*										*	*	
Interpret products of whole numbers	3.OA.A.1							X	X					
Interpret whole-number quotients of whole numbers	3.OA.A.2							*	*	*				
Determine the unknown whole number in a multiplication or division equation relating three whole numbers	3.OA.A.4							*	X	*			X	*
Apply properties of operations as strategies to multiply and divide	3.OA.B.5							*	*	*		X	*	
Understand division as an unknown-factor problem	3.OA.B.6									X			X	X
Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division	3.OA.C.7							*	X	*	X			
Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations	3.OA.D.9										X	X	*	X

X : The specified Common Core Standard is closely aligned to this world
***: The specified Common Core Standard is used in and/or related to the world**

Appendix B
Sample Second Grade Pre/Post Study Assessment

1. Can you make **17** in two ways?

   ____**-**____ = **17**

   ____**-**____ = **17**

2. How can you make **30**?

   ____ + **17** = **30**

   65 - ____ = **30**

3. Which is **<u>NOT</u>** equal to **<u>24</u>**? Circle it!

a. 8 x 3

b. 4 + 8 + 8 + 4

c. 4 x 4 x 1

d. 20 + 4

4. If you have 31 + **94** + 25, can you change **94**?:

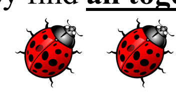
   ↙↘

   31 + ____ + ____ + 25 = 150

5. Find the missing number(s)!

1 **x** 2 **x** 5 = _____	15 **-** _____ + 1 = 11
9 **-** 3 + _____ = 12	4 **x** 2 **-** 1 = _____

6. **<u>5</u>** students are looking for ladybugs in the garden! Each student found **<u>2</u>**! How many ladybugs did they find **<u>all together?</u>**



____ ladybugs

7. Marcus has **7** stars, but he wants **35** for his poster. How many more stars does Marcus need?

7 + ____ = **35**

8. Solve!

15 + 22 + 75 = ___	16 = ___ **x** 4

9. Are these equal? circle **<u>YES</u> or <u>NO</u>** ________________ 31 + 17 = 8 x 6

Appendix C
Detailed Study Timeline

Before the Study:

Before engaging with the game, tutorials will be made available and shown to the students (e.g., sign in procedure, necessary tactics within the game, troubleshooting, etc.). Teachers will be provided a short 15-item worksheet to have their students complete in order to assess their current math ability levels. Students should be given 20 minutes to complete the worksheet.

Study Day 1:

On the first day of the study, students will begin actively playing the game. Students will be asked to sign onto their device and instructed to begin independently playing for **20 minutes**. The 20 minutes may occur at the teacher's' discretion and convenience. Classrooms in the control condition will be provided math worksheets to complete over the four sessions. The four total sessions (4 days of 20 minute interaction per day) should occur during regular school hours. When done, students exit the game and the system will record the progress and time.

Day 2:

Students will continue the same process and play for **20 minutes** on the intervention game. Observations taken by teachers will be noted; however, students will interact independently with the game for the four days of the study.

Day 3-4:

Students will continue the same process and play for **20 minutes** on the game. Day 4 will be the last day of game interaction. The student participants will have played ~80 minutes on the intervention game over the span of four days.

After Study:

At the conclusion of the study, a post- study worksheet will be administered to all participating classrooms. This 15-item worksheet will replicate the initial pre-study worksheet with number and order alterations made. Post-study worksheets will be collected in person.