Mathematical Education, Activities and Creation

An Interactive Qualifying Project
submitted to the Faculty of
WORCESTER POLYTECHNIC INSTITUTE
in partial fulfilment of the requirements for the
degree of Bachelor of Science

By
Yihong Zhou
Mengwen Li

Date:
May 3, 2016

Report submitted to:
Professor Brigtte Servatius
Worcester Polytechnic Institute

Acknowledgement

Table of Contents

List of Figures

List of Tables

Chapter 1: Introduction

Mathematics is a popular course for basic education and high-level education globally. From kindergarten and heuristic education to professional studies, math is always being focused on as a subject. Pure mathematics can be simple but also difficult. It contains topics of quantity, structure, space and change, most of which have easy to understand representations and difficult to understand proofs. Pure math is also the first area children touched as they learn everything about math starting from counting integers. Applied mathematics uses mathematical methods to solve problems in other subjects such as engineering, science and business. In college, mathematics students can learn applied math which requires in other fields such as finance, medical science and social science. Problems in applied mathematics have a natural motivation and their solution is of general interest. But even in pure mathematics there are some popularized notoriously difficult problems like "P versus NP", "Hodge conjecture" and "Riemann hypothesis" (Keith Devlin, *The Millennium Problems: The Seven Greatest Unsolved Mathematical Puzzles of Our Time* New York: Perseus Books Group (2002)). All of these make mathematics an attractive subject. The human struggle in conquering these problems makes headlines, such as Andrew Wiles' proof of Fermat's last theorem. The mathematical theory developed in the centuries long solution process produces a wealth of new problems.

Traditional classroom teaching is the main source of knowledge acquisition in mathematics as well as other subjects taught in US schools. Other teaching methods such as online education, self-teaching and home-schooling provide alternate sources for learning mathematics. Students emulate the approaches that different teachers used to analyze the mathematical problems and gradually, as part of the creative process, form their own ways of thinking through the practice and feedback.

In Massachusetts high school students form relatively stable patterns of math learning through their previous 8 years of mathematics education. We received our primary education in China.

To examine the differences in the curriculum system and education methods and their effect on students learning and the resulting abilities to creatively apply mathematical knowledge to solve problems in the US versus China, we looked at textbooks, course curricula, and standardized tests for particular grade levels using materials and information from Massachusetts and Tianjin.

We also visited the Museum of Mathematics in New York and interviewed visitors there on their reaction to aesthetic appeal and educational value of the mathematical models. We presented our museum trip experience to the Math Club at WPI.

We attended a high school science fair to figure out the effect of model-related activities on the basic education and their impact on students' mathematical creativity.

Chapter 2: Education Report

2.1 Massachusetts Education in United States

2.1.1 General education in the United States

Most of the countries start math education early. The K-12 education system adopted by the United States, Canada and possibly other countries starts math education from primary education in elementary school for 5 - 6 years, then secondary education for 2-3 years in middle school and 4 years in high school. In the K-12 education of 2011-2012, about 87% of school-age children in the US attended public schools, about 10% attended private schools[1] and roughly 3% were homeschooled[2]. Different from other countries, mathematics is separated by topics in high school in the US. Students in high school dedicate the first year and third year to algebra knowledge while the second year to geometry. This algebra-geometry-algebra model is followed by pre-calculus class that is provided to students who want to be further educated in universities. This class that combines high-level algebra, trigonometry, and other calculus topics is offered in grade 12 in the high school or in the first year in college.

2.2.2 General math education in Massachusetts

"The Common Core concentrates on a clear set of math skills and concepts. Students will learn concepts in a more organized way both during the school year and across grades. The standards encourage students to solve real-world problems."

(Common Core State Standards Initiative, *COMMON CORE STATE STANDARDS FOR MATHEMATICS* , March 2011)

In order to improve the mathematics achievement in the United States, researches of math education in high-performing countries have been conducted for decades. Greater focus and coherence need to be addressed in the mathematics education in the US. The Common Core State Standards come out as a result. The Standards are designed to be specific and organized for planning out the entire K-12 math education. Each grade level standards have Standards that

---

[1] U.S. Department of Education, National Center for Education Statistics, Private School Universe Survey (PSS), 1995–96 through 2011–12. April 2013. Retrieved from:
<http://nces.ed.gov/programs/digest/d13/tables/dt13_205.20.asp >

[2] J. Michael Smith,President, HSLDA. U.S. Department of Education: Homeschooling Continues to Grow! (September 3, 2013). Retrieved from: <https://www.hslda.org/docs/news/2013/201309030.asp >

define what students should know and be able to solve, Clusters that group the closely related standards, and Domains which are the topics such as Number systems and Functions, and it also contains multiple standards.

There are 46 states adopting the Common Core State Standards including Massachusetts who adopted the Standard at July 2013 and added State specific additions to the document. (Academic Benchmarks, *Common Core State Standards Adoption Map, 2016*)

Massachusetts has particular curriculum framework for mathematics subject from pre-kindergarten to grade 12, incorporating with the Common Core State Standards for Mathematics. In the curriculum framework, there are 8 Mathematical Practice standards:

> *"1. Make sense of problems and persevere in solving them.*
>
> *2. Reason abstractly and quantitatively.*
>
> *3. Construct viable arguments and critique the reasoning of others*
>
> *4. Model with mathematics.*
>
> *5. Use appropriate tools strategically*
>
> *6. Attend to precision.*
>
> *7. Look for and make use of the structure.*
>
> *8. Look for and express regularity in repeated reasoning.''*
>
> (Massachusetts Department of Elementary & Secondary Education, *MASSACHUSETTS CURRICULUM FRAMEWORK FOR MATHEMATICS*, March 2011)

These standards are the expectations for students after getting the K-12 math education.

To achieve the expectations, the content standards are organized in the same way as the State Common Core Standards from pre-kindergarten to grade 8. There are also unique Massachusetts Standards, most of which are detailed requirements to be reached at certain grade level. For the basic 10-year education, each grade has very clear topics to teach and these topics are across several different fields.

Table 1 Math Domains of PreK-8 learning Progress
(*MASSACHUSETTS CURRICULUM FRAMEWORK FOR MATHEMATICS*, March 2011)

Progression of Pre-K–8 Domains										
Domain	**Grade Level**									
	PK	K	1	2	3	4	5	6	7	8
Counting and Cardinality	MA									
Operations and Algebraic Thinking	MA									
Number and Operations in Base Ten										
Number and Operations – Fractions										
The Number System							MA			
Ratios and Proportional Relationships										
Expressions and Equations										
Functions										
Measurement and Data	MA									
Geometry	MA									
Statistics and Probability										

When it comes to Grade 8, students have to know about functions, geometry like two-dimensional figure, rotations, and translations in coordinates, also to understand the Pythagorean Theorem, to solve volumes of cylinders and cones, spheres problems and to do simple investigations on statistics and probability.

After 10 years foundation education, students start high school life. There are lists of conceptual categories students have to understand during high school years, including number and quantity, algebra, functions, modeling, geometry, statistics and probability. All students should study in order to be college and career ready. The conceptual categories do not indicate the sequence of high school courses. "Massachusetts educators requested additional guidance about how these 9–12 standards might be configured into model high school courses and represent a smooth transition from the grades pre-k–8 standards"(*MASSACHUSETTS CURRICULUM FRAMEWORK FOR MATHEMATICS*, March 2011). So there are two Model Pathways and related model courses presented in the high school math education framework. The traditional Pathway includes Model Algebra I, Model Geometry and Model Algebra II; the integrated Pathway includes Model Mathematics I, Model Mathematics II and Model Mathematics III. There are other advanced model courses like Model Precalculus and Model Advanced Quantitative Reasoning.

Besides the traditional classes in school, there is a Virtual High School providing high-quality courses online for about 6000 students in 200 middle schools and high schools in Massachusetts

every year. The courses vary from English, art to math, science and other interesting topics. (The Virtual High School, *The VHS Collaborative in Massachusetts*, 2016)

2.2.3 Math book used in Massachusetts high school

There are standard textbooks all the high school in Massachusetts have to use. The textbooks are Algebra 1, Geometry and Algebra 2. Published by different publishers, textbooks have different styles and contents. But the main knowledge points are all covered in the textbooks. The most common publishers are McGraw-Hill and Prentice Hall. Figure 1 are the textbook examples of these two publishers in which both are describing the Quadratic Functions.
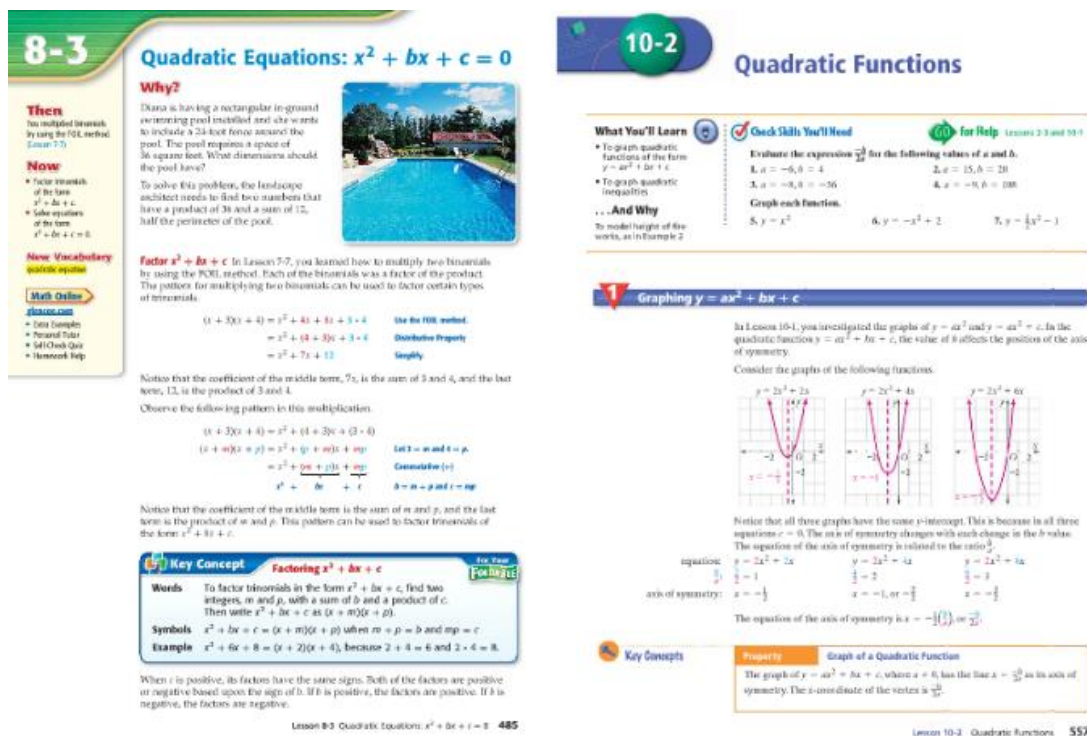
Figure 1 Math Textbook examples
(Left is *McGraw-Hill Algebra 1* p485, right is *Prentice Hall Algebra 1* p557)

Schools would choose textbooks that cover the same material but with a different topic organization. Teachers can teach topics not in order as long as the students can fully understand the topics and proficiently apply the concepts. Besides official textbooks, teachers can assign other materials to students. The form not limited to textbooks, but also contextual math problems, math reports on the popular journals and a variety of media. Pre-Calculus and Calculus are also provided for interested students who would like to take Advanced Placement test for college credit. Textbooks of Calculus vary from schools. Teachers decide the materials to teach.

2.2.4 Math assessment for high school students

Assessments require varying amount of times and are assigned in a variety of forms. Homework, Mathematics group projects, magazines and journals, speeches, and brainstorm activities are common ways for teachers to observe students' performance and their ability of solving math problems. Teachers may assign math projects like studying a typical theory or holding math activities such as solving intellectual problems to let students learn the materials. Students usually have quizzes in classes. Homework not only in paperwork form but can be done on the computer. Using applications, teachers think it is convenient to collect students' data and can get scientific statistic feedback from students.

Additionally, students are encouraged to use calculators and calculators are allowed during the tests.

2.2.5 Teaching tools and methods

Some of the teachers use computers as teaching tools. Making a PowerPoint and present to students how questions are solved. Other teachers prefer to outline the important math concepts and give examples. Teachers will take models and show students the math concepts behind the models. Also, teachers can show students videos and animations to explain the relationship of algebra and geometry. Students are free to ask questions in the class.

2.2.6 Math tests for MA high school students

MCAS

The Massachusetts Comprehensive Assessment System, also called MCAS, is a statewide standards-based assessment program. The State and federal law mandate all the students educated with Massachusetts public funds and who are enrolled in the tested grades to take MCAS test. (Massachusetts Department of Elementary & Secondary Education, *Massachusetts Comprehensive Assessment System,* Oct 2014)

MCAS contains four types of questions: multiple-choice questions, short-answer questions, open-response questions and writing prompts. Also, there are four performance levels: Advanced (260-280), Proficient (240-259), Needs Improvement (220-239) and Warning/Failing (200-219). In order to graduate from high school, student must "either earn a scaled score of at least 240 on the grade 10 MCAS English Language Arts and Mathematic tests or earn a scaled score between 220 and 238 on these tests and fulfill the requirements of an Educational Proficiency Plan".

In 2015, the most recent result of 2015 mathematics MCAS test came up. There about 79 percent of students reached the score of Needs Improvement or higher. Comparing with 75% in 2010, the math results were always above 75% in recent 5 years. It even reached the highest 80% at 2013. Focusing on the test result of 2011, about 77% student reached the score of Needs Improvement or higher. It was very close to the result in 2015. About 48% Grade 10 students had advanced math performance in 2011 and about 29% student had proficient performance. About 21% student needed improvement in the test. When it came to 2015, advanced performance Grade 10 students increased to 53%. Proficient performance Grade 10 students decreased to 25%. And students need improvement increased to 23%.

Year	Advance	Proficient	Need Improvement/Warning/Failing
2015	53%	25%	21%
2011	48%	29%	23%

Table 2 MA High School Student Math Performance Comparison of Year 2015 and 2011
(Data from Massachusetts Department of Elementary & Secondary Education MCAS Results 2015 and 2011)

Figure 2 is an example question asking the midpoint of a line in the coordinate system in 2011 for Grade 10 students.
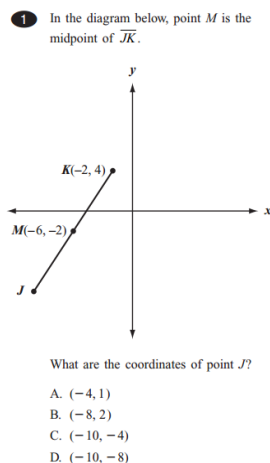
Figure 2 Example question in MCAS Year 2011
(Massachusetts Department of Elementary & Secondary Education, Grade 10 Mathematics Released Item Document)

SAT

SAT is the standardized college entrance test in the United States published by the College Board, a private, nonprofit organization. SAT has 3 parts: critical reading with 800 points, mathematics with 800 points and writing with 800 points to evaluate the general performance of students' knowledge. SAT can be taken all years around with multiple times which provides student more opportunities to get high scores. There are also subject tests such as math, chemistry and physics. Calculators can be used on the SAT math section. The content of math SAT test covers the number and operations, algebra and functions, statistics and probability, and geometry.

In 2015, about 50311 high school students in Massachusetts took SAT. In the math section, the average score was 521 points so that it was about 65% correctness rate. Math test average of 2011 was the same as the result of 2015 but with only 49279 students taken.

Figure 3 is an example question of 2011 SAT math test asking about calculation of coordinates:
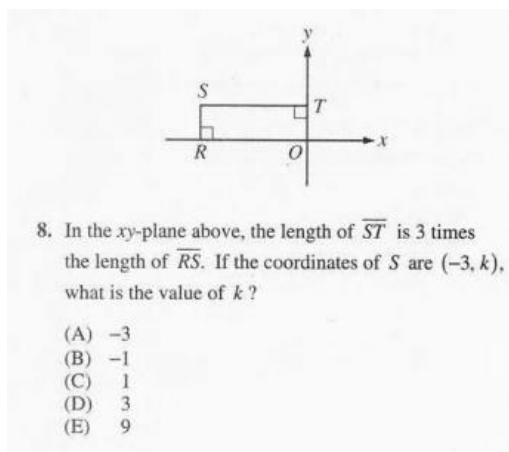
Figure 3 2011 SAT math section example
(The CollegeBoard, *The SAT*, MAY 2011)

AP

Advanced Placement is a program published by the College Board, offers college-level tests and curriculums for high school students. The content covers a wide range of topics such as history, languages, physics, and math. The final score is from 1-5 in which 5 means extremely qualified;

4 means well qualified; 3 indicates qualified; 2 is possibly qualified and 1 is no recommendation. Most colleges require 4 or 3 as the minimum score in the subjects to receive college credit.

In the year 2015, about 49032 high school students in MA took various AP subject tests. About 15519 students took math and computer science subjects that include calculus topics, statistics, and computer science. Approximately 7054 students took basic Calculus AB and about 6363 students took Statistics. In the year 2011, only about 10809 students took math related AP test and only about 5700 students took Calculus AB, about 3941 students took Statistics. (Data from Massachusetts Department of Elementary & Secondary Education, *Advanced Placement Performance Report, 2011 & 2015*)

The results of 2015 and 2011 advanced placement math tests for calculus show there were no big distinctions between the outcomes of these two years. Apparently, the number of students taking higher-level calculus courses increased with a higher percentage in getting qualified scores. However, students' exam grades drop in Statistics area from 66.1% to 60.7%.

Subject	2015	2011
Calculus AB	63.6%	63.1%
Calculus BC	86.1%	85.3%
Statistics	60.7%	66.1%

Table 3 MA High School Student AP Math Performance Comparison of Year 2015 and Year 2011 in Score 3-5
(Data from Massachusetts Department of Elementary & Secondary Education, Advanced Placement Performance Report)

Figure 4 is an example of 2011 AP Calculus AB:

For $0 \leq t \leq 6$, a particle is moving along the x-axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t) = 2\sin\left(e^{t/4}\right) + 1$. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{t/4}\cos\left(e^{t/4}\right)$ and $x(0) = 2$.

(a) Is the speed of the particle increasing or decreasing at time $t = 5.5$? Give a reason for your answer.

(b) Find the average velocity of the particle for the time period $0 \leq t \leq 6$.

(c) Find the total distance traveled by the particle from time $t = 0$ to $t = 6$.

(d) For $0 \leq t \leq 6$, the particle changes direction exactly once. Find the position of the particle at that time.

Figure 4 2011 AP Calculus AB example question
(The CollegeBoard, *AP Calculus AB 2011 Scoring Guidelines,* 2011)

2.2 Tianjin Education in People's Republic China

2.2.1 General education in China

Nine-year compulsory education is mandatory for every child in China, no matter the child lives in city or in countryside. The education includes 6 years of elementary school and 3 years of junior high school. Children are not required to go to kindergarten and teachers in kindergartens design the courses in the kindergarten. After 6 years old, children have to attend elementary school for formal education and start systematic knowledge learning. Formal math education starts right after going to elementary school. In 2011, about 99.8% children went to public elementary school[3], about 93% graduate elementary school students were involved in junior high school and about 84% junior high school students went to senior high[4]. After 12 years' basic education, students take math education for another 3 years in senior high school. Course syllabus follows National Class schedule.

Students will be divided into science focus or liberal art focus on the second year of senior high school. This division is the requirement of curriculum developing toward greater depth, which also determines the subjects a student will be tested on for college entrance examination. Moreover, students will select majors in university based on their choice of science or liberal arts.

There are 6 to 7 math topics discussed each semester in high school except the last semester. About half of the topics will cover geometry or spatial problems and another half of the topics will cover algebra and logic problems. The last semester in senior year will be used to review the previous materials to get students prepared for the final test so no new knowledge will be taught.

2.2.2 General math education in Tianjin

All the elementary schools, whether in big city such as Beijing and Shanghai or in countryside, design their courses following the Standard of Chinese Courses in the full-time Compulsory Schooling. Elementary math education is separated to 3 phases: 1-3 grades, 4-6 grades and 7-9 grades. For every continuous three years, math education will repeatedly cover the material of algebra, geometry, statistics and probability, with greater depth and wider topics using comprehensive materials and practical problems. Grade 1 to 3 focus on observing and recognizing basic shapes for geometry and mastering basic integer and decimal arithmetic for

---

[3] "Net Enrolment Ratio of School-age Children in Primary Schools." 小学学龄儿童净入学率_教育部门户网站 _MOE.GOV.CN. Ministry of Education of the People's Republic of China, n.d. Web. 08 Apr. 2016. < http://www.moe.edu.cn/publicfiles/business/htmlfiles/moe/s7567/201309/156875.html>.
[4] "中国教育概况." - 中华人民共和国教育部政府门户网站. 中华人民共和国教育部政府, n.d. Web. 08 Apr. 2016. <http://www.moe.gov.cn/jyb_sjzl/s5990/201503/t20150331_186797.html>.

algebra. For grade 4 to 6, algebra focuses on understanding the details of fraction, unit transformation and also applying algebra to practical problems. Geometry focuses on angles, specific shapes such as trapezoid, parallelogram, circle and fan-shape. Students also start learning basics for statistics and probability such as various statistical graphs. In Grade 7 to 9, student will learn rational numbers, functions for algebra, relation between points, lines, planes and angles, the proof of triangles, inter relation of quadrilateral, transformation of shapes and coordinates for geometry and some more advanced probability topics. (*普通高中课程标准实验教科书*. Beijing: People's Education, 2007. 电子课本. People's Education Press.)

The high school math topics in Grade 10 include notation of set, basic functions, trigonometric function, vectors, trigonometric identity, solid geometry and analytic plane geometry. Then comes to Grade 11 with advanced statistic and probability, logic, conic section, spatial vectors, solid geometry, derivative and deduction, introduction to complex number, proof of geometric theory, parameter equality and inequality. Grade 12 is the last year of high school, material continuous with the topics of Grade 11 but most of the schools will start a large scale review as early as possible to prepare students for the college entrance test. (*普通高中课程标准实验教科书*. Beijing: People's Education, 2007. 电子课本. People's Education Press.)

2.2.3 Math book used in Tianjin high school

There are five required books *Math Compulsory Module 1-5* that are revised by National Education Department and are mandatory for every high school math education nationwide. Each book contains 3-4 topics. Teachers choose topics to teach in the order they prefer but all the materials should be covered. This also allows teachers to introduce their own prepared materials and even use other books as main teaching sources as long as the topics are covered. The official textbooks will be used as review materials at the end of learning process in high school, usually in senior year, because the college entrance test is based on the official textbook. High school does not offer any pre-college math preparation. Basic knowledge of derivative is consider as high school material and will be tested on the college entrance test.

Figure 5 is an example of a page in the first math textbook introducing power functions for Grade 10 in Tianjin:
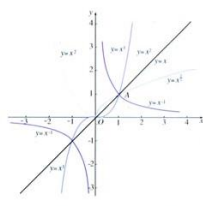
Figure 5 Example page of Math Compulsory Module 1
(*普通高中课程标准实验教科书.* **Beijing: People's Education, 2007.** 电子课本**. People's Education Press)**

2.2.4 Math assessments for high school students

The most common assessments are homework and tests. Homework has three main sources: the exercise books specified by Tianjin education department, the self-designed exercises books written by teachers in the school, which usually covers more materials than the textbooks, and the take-home examination paper. The first main form of homework appears usually in Grade 10 and Grade 11. Take-home examination papers are trivial for Grade 12 students who are preparing for the college entrance test. In Grade 10 and Grade 11, there are only mid-term exams and final exams. When come into Grade 12, students have to take exams every month. Beside regular monthly exams, there will be three simulation tests for all Tianjin high school students and other simulation opportunities like multiple schools taking exams together. All the tests and homework are in paper form. There is no activity or project for students during the school years.

Using calculator is not encouraged. Except large number calculation and particular topics like statistic and probability, small-scale calculations are all done by hand.

2.2.5 Teaching tools and methods

Using computer animation to teach is not so efficient for teaching math in high school teachers' minds. Almost all the teachers only use blackboard for regular teaching. Outlining the concepts first, then providing examples to explain or proof. Teachers write all the processes of how to solve a question on the board. Students follow teachers' thoughts and take notes. Teacher seldom takes models to class because most of explainable models can be drawn on the board. While drawing on the board, teachers explain the concepts beneath the models. Students usually do not ask questions in class because this will interrupt the thinking process and distract other students. But they are free to ask after class.

2.2.6 Math tests for Tianjin high school students:

High School Achievement Test

High school achievement test is a test written by Tianjin education Department and covers the basic materials in the official textbook with minimum requirements on subjects.

"When Student finish the learning in high school, if all High school achievement tests reach qualified grades, all subjects reach qualified grades, behavioral education scores reach qualified grades and physical education reach qualified grades, students are allowed to graduate."(Tianjin Education Department, 普通高中学籍管理规定, 1997)

Achievement test has four grade levels: A, B, C and D. In order to graduate, students have to reach the grade at least C. However, students can retake the test if receive a D. Math achievement test will be held at the beginning of senior year in high school.

Figure 6 is a sample question in 2011 Tianjin math achievement test:

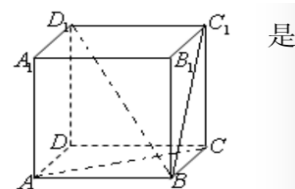（11）19.如图，在正方体 $ABCD-A_1B_1C_1D_1$ 中，下列结论正确的 是

A. $AD \perp BC_1$ B. $AC \perp BC_1$

C. $AD \perp BD_1$ D. $AC \perp BD_1$

Figure 6 Sample question in 2011 Tianjin math achievement test: "In the cube ABCD-A₁B₁C₁D₁, which of following statement is right?"
(Tianjin Education Department, *2011 年天津市普通高中学业水平(数学)*, 2011)

College Entrance Examine

College Entrance Examine is the most famous test in China. All high school students nationwide have to take this test in order to go to universities in China. There are various versions of entrance exams. Some provinces have their own college entrance test papers. There are also national test papers that have been used by multiple provinces. In recent years, increasing number of provinces decide to use national test papers or national test papers for some subjects. But Tianjin, like Beijing, Shanghai and other several provinces uses their own test papers for all the subjects, including math. Also, the math exam for science students and liberal art students are different with the math part much easier for liberal art students.

There are two parts in the exam. The first part contains multiple-choice questions. The second part contains blank-filling questions and free-response questions. There are no specific performance levels. The higher the score the higher chance a student has to get into good college.
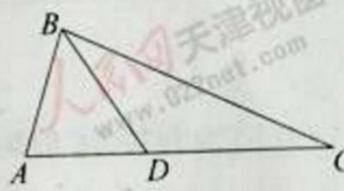
Students cannot use calculators in the tests.

The total points for math exam are 150. Science students in 2015 got average 95.84 and liberal art students got average 88.34. 2014 math average of science students was 92.96 and math average of liberal arts student was 77.93. (Sina education, 天津 2015 高考各科平均成绩及各分数段统计, June 2015)

Figure 7 is an example of 2011 Tianjin College entrance math test:

Figure 7 Sample question in 2011 Tianjin College entrance math test: "In triangle ABC, D is on edge AC, AB =AD, 2AB = sqr(3)BD, BC = 2BD, so sinC equals which value?"
(Tianjin Education Department, *2011 年普通高等学校招生全国统一考试 (天津卷)*, 2011)

2.3 PISA TEST

PISA (Programme for International Student Assessment) is a global evaluation test for 15-year-old school pupils' performance in reading, science and math. The PISA can be used as an education outcome evaluation for students around 10th grade in the US. From 2003 to 2012, about 64 countries and economies participated PISA's every 3 years' tests.

Although with excellent teaching resources and sufficient education funding compared with the rest countries in the world, the United States showed a mean performance that was not significantly different from the average of the test results in PISA. In the most recent two assessment results from 2009 to 2012, United State dropped from rank 17 to 36 as more countries involving. The countries with the neighboring math scores like Sweden, Slovak Republic, Spain and Hungary in 2009 were still similar to the test score of the United States in 2012. More countries attending into PISA allows a country to see its true pace of math, reading and science education comparing with the pace of the rest of countries in the world. (Peterson, Paul E., Ludger Woessmann, Eric A. Hanushek, and Carlos X. Lastra-Anadón. *Globally Challenged: Are U. S. Students Ready to Compete?* (n.d.): n. pag. Aug. 2011)

From the latest data analysis, US students acted mediocrely. Other countries like Norway, Portugal, Spain, Sweden, and Hungary had mean scores that were not significantly different from the score of US, which were far below other good development countries like Korea, Japan, Finland, Canada, Switzerland and Netherlands. Although performance levels among the countries ranked 23rd to 31st were not greatly different from the performance of United States, 22 countries did remarkably outperform the United States in the share of students reaching the proficient level in math in 2012. The United States also spent over 115000 dollars per student in 2012 but the same result level country like the Slovak Republic only spent about 53000 dollars per student, about half of the US's spending. Therefore, excessive money investing does not translate into better performance in math. (Peterson, Paul E., Ludger Woessmann, Eric A. Hanushek, and Carlos X. Lastra-Anadón. *Globally Challenged: Are U. S. Students Ready to Compete?* (n.d.): n. pag. Aug. 2011)

Less than one-third US students performed at the proficient level whereas a majority of students reached the proficient level in six top scored countries including Shanghai and Hong Kong. Shanghai, the top-performer, had a 75% in math proficiency average rate comparing with 32% average rate of the United States. About 25.8% US students shared the low achievement in mathematics, whereas 3.8% Shanghai students were at this level. In the US, only 8.8% students shared the top performance while about 55.4% student in Shanghai obtained the top performance. Many researchers thought Shanghai is a thriving district with much better educational resources than the rest Chinese areas, and it should be posed to compare with Massachusetts and Minnesota that are similar and also top education states in the US. However, Shanghai highly

outperformed the Massachusetts students who had 51 percent proficiency in math and the Minnesota students who had 43 percent proficiency. The mathematics scores of Shanghai "indicate a performance that is the equivalent of over two years of formal schooling ahead of those observed in Massachusetts." (OECD, PISA result from PISA 2012, 2012).

However, if comparing the result of students from Massachusetts with top performers' results in Canada, Japan, Netherlands, and Netherlands, the percent of proficient was similar (OECD, *PISA 2009 Results: Executive Summary*, 2010. & OECD, *PISA 2012 Results in Focus*, 2014).

"From PISA test, students in the United States have particular weaknesses in performing mathematical tasks with higher cognitive demands, such as taking real-world situations, translating them into mathematical terms, and interpreting mathematical aspects in real-world problems. And Students in the U.S. are largely satisfied with their school and view teacher-student relations positively. But they do not report strong motivation towards learning mathematics: only 50% students agreed that they are interested in learning mathematics."(Peterson, Paul E., Ludger Woessmann, Eric A. Hanushek, and Carlos X. Lastra-Anadón. *Globally Challenged: Are U. S. Students Ready to Compete?* (n.d.): n. pag. Aug. 2011)

Chapter 3: Activities for Math Education Research

3.1 National Math Museum (Momath) Trip

To find out whether exhibits, arts and activities relating to math topics contribute to the improvement of math education, we took a trip to the National Museum of Mathematics. We took pictures for the exhibits in the museum and also interviewed some tourists with the questions we prepared.

3.1.1 Introduction to National Museum of Mathematics (MoMath)

Located at 11 East 26th Street in Manhattan, New York, National Museum of Mathematics contains innovative exhibits that will engage people from 5 to 105 years old, aiming to improve math perception and understanding of the public. The exhibits have a special emphasis on the materials for 4th through 8th grade[5]. The museum has a variety of sponsors from technology companies to hedge fund investment foundation, including Google, John & Laura Overdeck, Simons Foundation and so on. There are also a great number of mathematicians contributed to this museum. They design many exhibitions and also offer the opportunities for the public to participate into math activities. A picture for sponsors is shown in Figure 8:
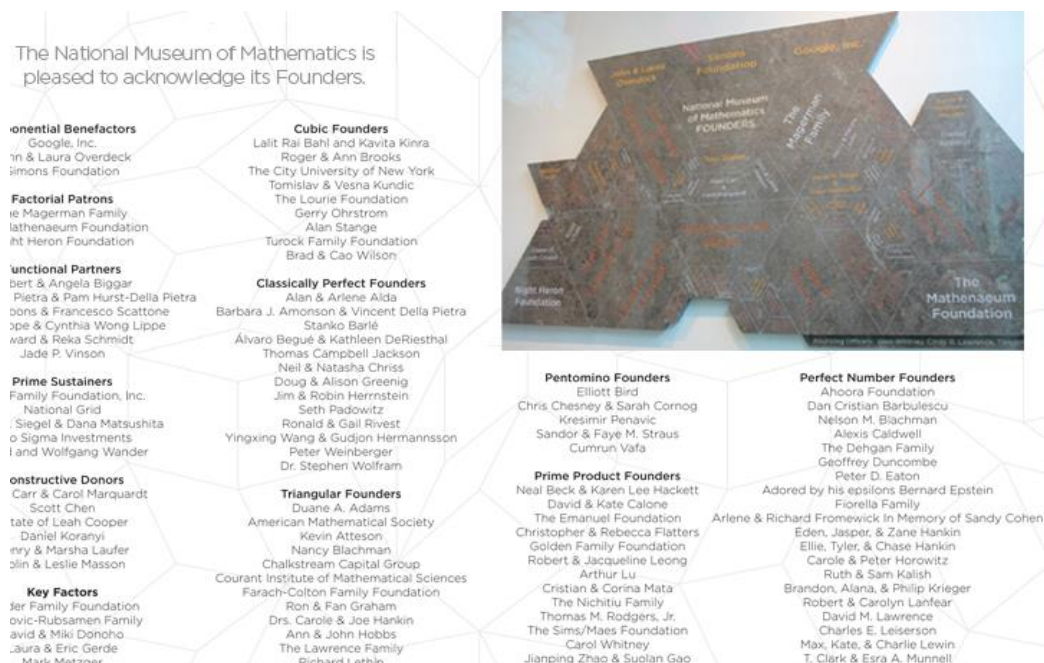
Figure 8 Sponsors and contributors for Momath
("National Museum of Mathematics." 2009. 12 Apr. 2016)

---

[5] "National Museum of Mathematics." 2009. 12 Apr. 2016
 <http://momath.org/>

The National Museum of Mathematics contains two floors. On the first floor, there are exhibits including Hyper Hyperboloid, Coaster Rollers, Square-Wheeled Trike, Tracks of Galileo, and Motionscape. A map of the first floor is presented in the Figure 9:
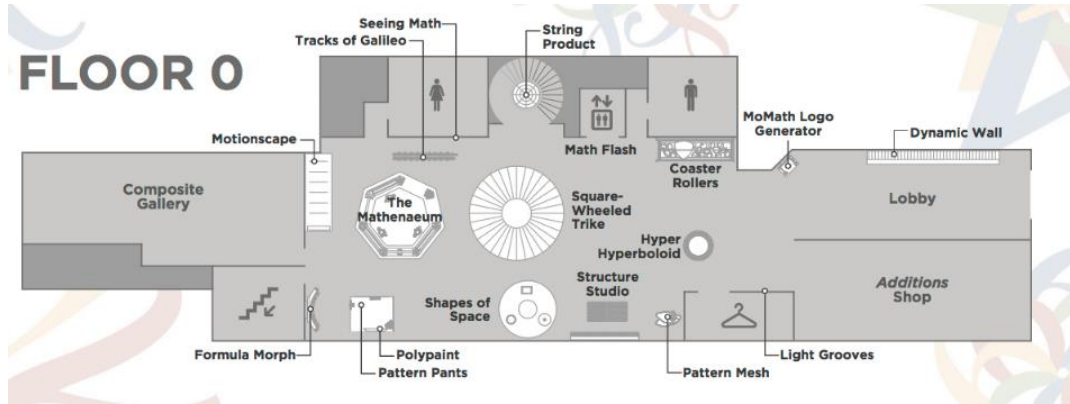
Figure 9 First floor in Momath
("Exhibit Guide – National Museum of Mathematics." 2013. 12 Apr. 2016)

The String Product exhibit is built between the two floors. The second floor, shown in Figure 10, contains Math Square, Tessellation Station, Edge FX and Human Tree.
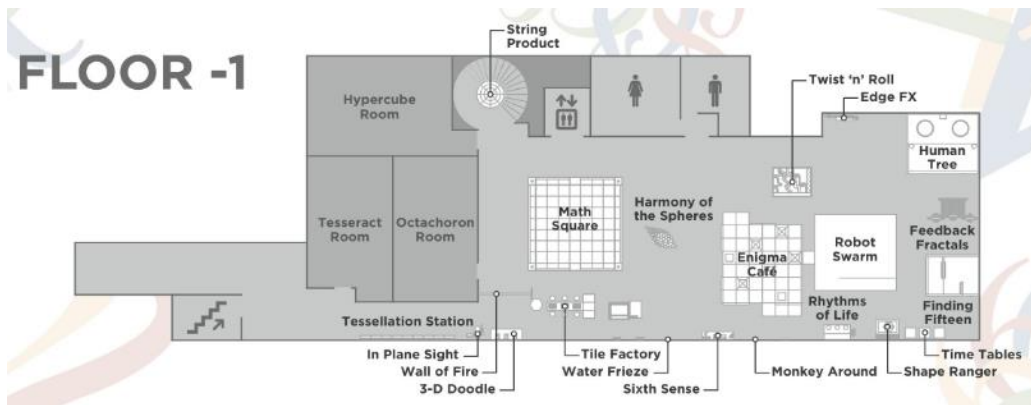
Figure 10 Second floor in Momath
("Exhibit Guide – National Museum of Mathematics." 2013. 12 Apr. 2016)

3.1.2 Major Exhibitions in the MoMath

We took photos of some major exhibits in the trip. Exhibitions cover a wide range of topics from algebra, geometry, space, modeling, designing, logics to theory discovering and problem solving. The first one is Hyper Hyperboloid, which is located right in front of the front door.

Figure 11 Hyper Hyperboloid Exhibition

A person can sit on the chair inside the cylinder and spin the chair. The platform under the chair will spin as the person spins. It allows the person inside to construct a quadratic equation through a visible contour around him by turning the chair. It also demonstrates that a curved surface can be created out of straight lines. We think it is fun to see how a touchable hyperboloid model can be made in a real activity and the activity let people understand the creation process of hyperboloid.

The exhibit next to the Hyper Hyperboloid is Coaster Rollers.

Figure 12 Coaster Roller

Visitors can sit on the plastic sled and they can have a ride supported by the objects spread in the container. The objects in the container have different shapes, and the MoMath called these objects "acorns". It is astonishing that without round shape objects supporting the move, like rolling using wheels, the visitors in the sled can still have a smooth ride when the "acorns" rolling. We thought about this phenomenon for a long time, then, the staff told us the reason that people do not have bumpy ride over the rollers. It is because the center of these "acorns" are constantly at the same height while rolling so that the distance between the ground and the sled will always be the same.

After Coaster Rollers, there is Square-Wheeled Trike. It is located behind the Hyper Hyperboloid and is probably the biggest exhibit on the first floor.

Figure 13 Square-Wheeled Trike

On the sunflower shaped surface, there are multiple circles carved to indicate different wheel orbits. Two strange bicycles, each with 3 different sizes square-wheels, can be ridden on the uneven-looked surface. The rider won't feel bumpy at all and it is just like riding a normal bicycle on a flat surface. This looks pretty interesting and visitors will have same question with the Coater Roller. From the previous exhibit, we could guess with the combination of the uneven surface, the wheel center height remains the same all the time. But this time, it seems more complicate. The 3 square-wheels follow 3 different circles and never fall off. How to calculate the exact size of the square wheel to fit curve of the surface and how to maintain the center of the square at the same height as it rolls? This interesting question leaves people to think about and making people think is exactly the goal of the Math museum.

Next to Square-Wheeled Trike is Track of Galileo.
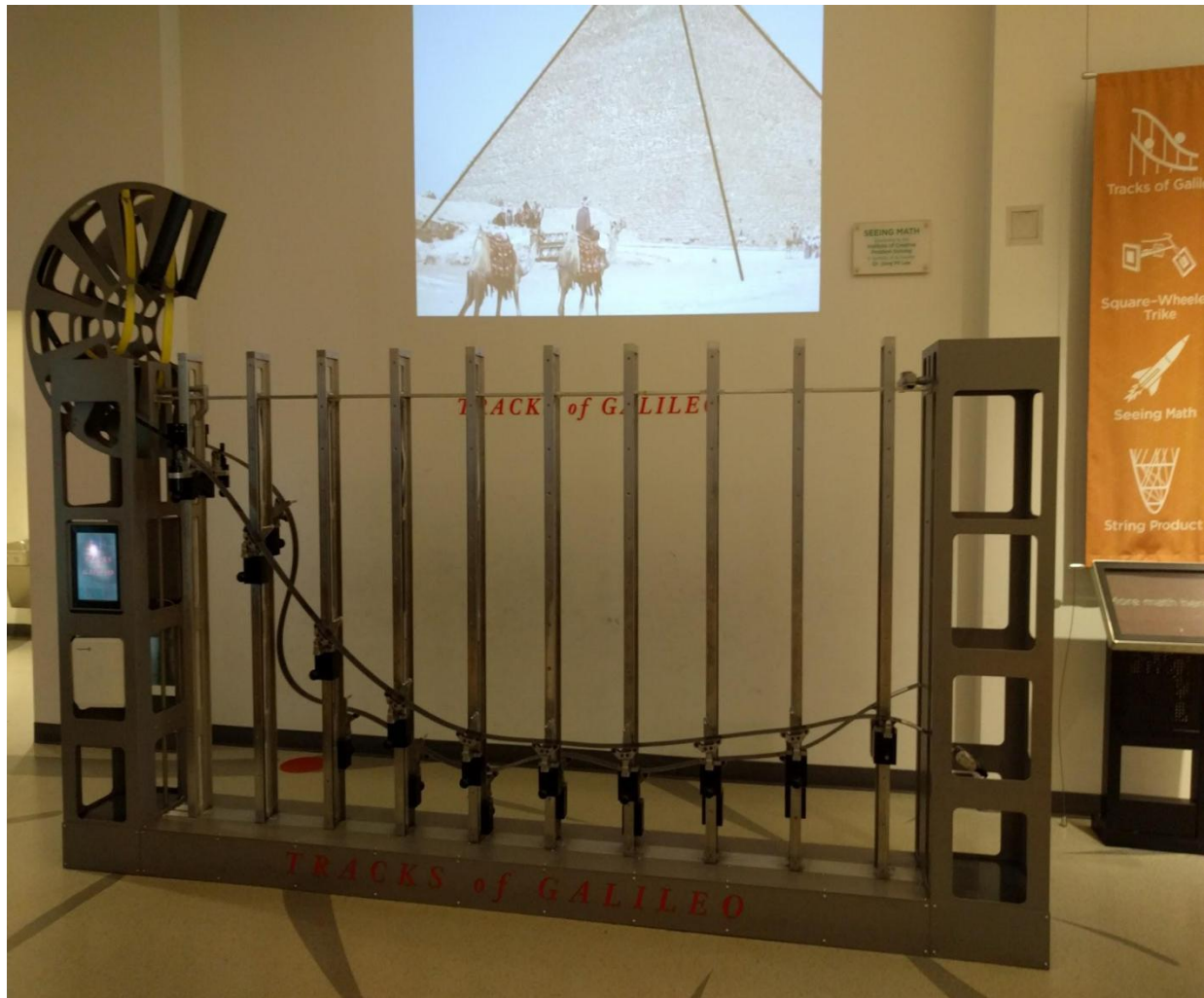
Figure 14 Track of Galileo

The high and low ends of the two tracks are fixed and visitors can adjust the middle parts of the two tracks freely. Two carts are located at the high end of both tracks. Visitors can release the two carts at the same time and the time required for carts on both tracks to reach the bottom will be shown on the screen. The exhibit challenge the visitors to create a track that allows cart to arrive destination with the shortest time. The first idea of most people is to create a straight line because a straight line between two points is the shortest. However, the answer is if the shape of the track is a cycloid, the cart will use the least time to finish the ride. Why a cycloid track promise the shortest time? Another question is posted in people's minds. The exhibition here really inspire people to think.

Then, there is the Motionscape.

Figure 15 MotionScape
("Calculus Is Fun! Math Exhibit Demos Principles of Motion." 2014. 11 Apr. 2016)

The Motionscape has three modes: position, velocity and acceleration. A big screen in front of the track shows the instruction, player's current motion and score. During the play, the player looks at the screen and tries to keep a circle in certain range shown on the corresponding to the mode. The computer also calculates the other two parameters of player. For example, in position mode, the computer give the position range, the player have to catch up to keep the circle in the range. The system calculates the velocity and acceleration at the same time. Players can see the motion locus on the coordinate in the game. On velocity mode, the circle location is determined by the speed of the player and position and acceleration will be calculated. On acceleration mode, it is determined by the acceleration of the player. This exhibit successfully demonstrates the concept of location, speed and acceleration to visitors. Furthermore, it shows people the relation between three parameters which involved the knowledge of calculus.

There is also Structure Studio on the first floor, where visitors can use flat chips in different shapes to create three dimensional structures.

Figure 16 Three dimensional Structures by plastic pieces

Figure 17 Three dimensional Structures by papers

The exhibit that spans the two floors is the String Product.
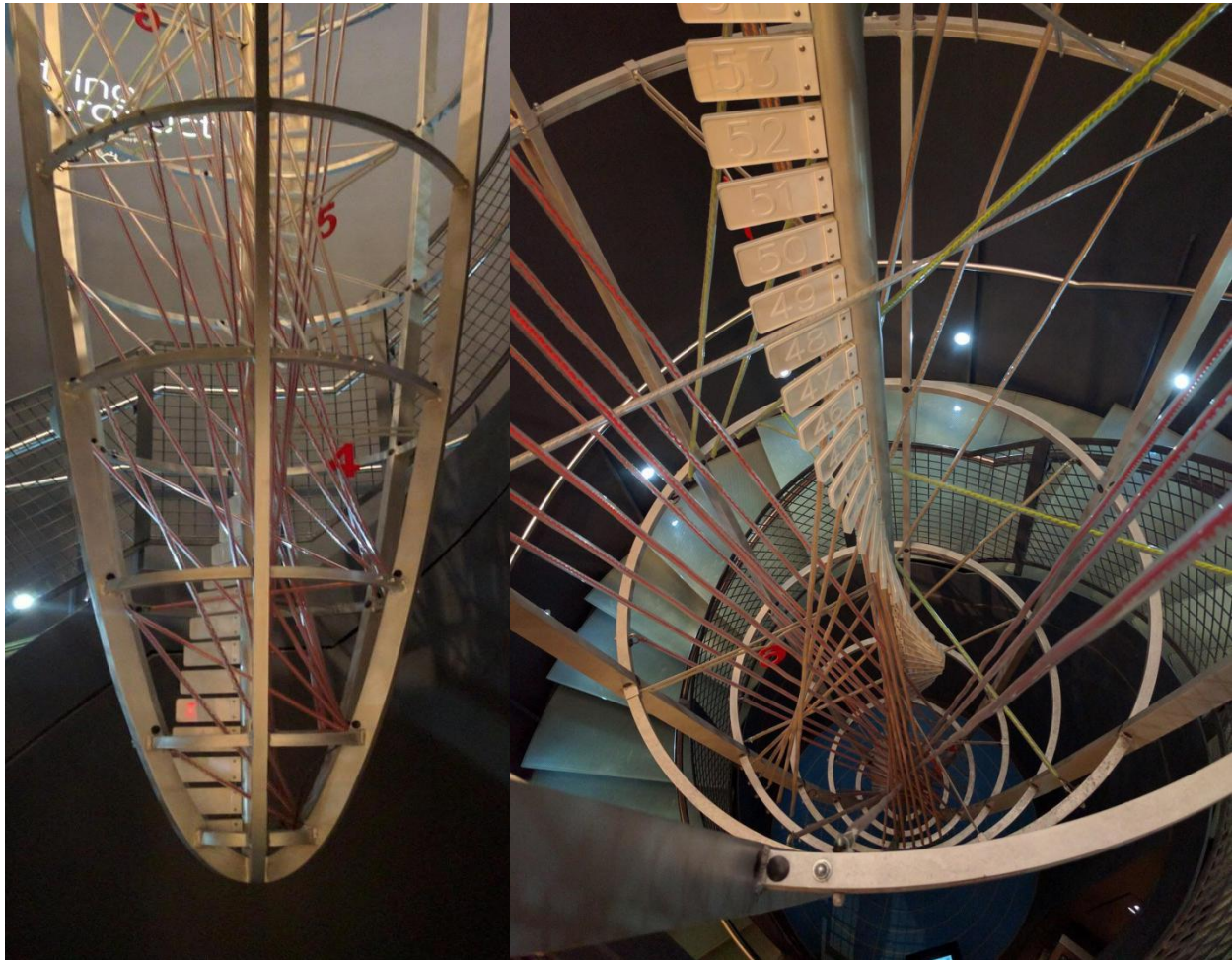
Figure 18 String Product

String Product is a very large exhibit. It is a three dimensional parabola representing equation of $y=x^2$. Inside of it, there are lines from one side of parabola to the other side. The circles embedded in the parabola represent the square of integers ranging from 1 to 10. For example, product of 1 and 1 is on the lowest circle; product of 2 and 2 is on the second lowest circle and so on. This exhibit contains the knowledge of parabola. There are buttons from 1 to 9 located at the bottom of this exhibit. After pressing two numbers, the line connecting the square of each number will light up and the intersection between the line and the vertical axis is exactly the product of the two numbers pressed. Visitors can find the subtle relations between numbers, equation and model in this String Product.

On the second floor, the first exhibit we saw was the Math Square:
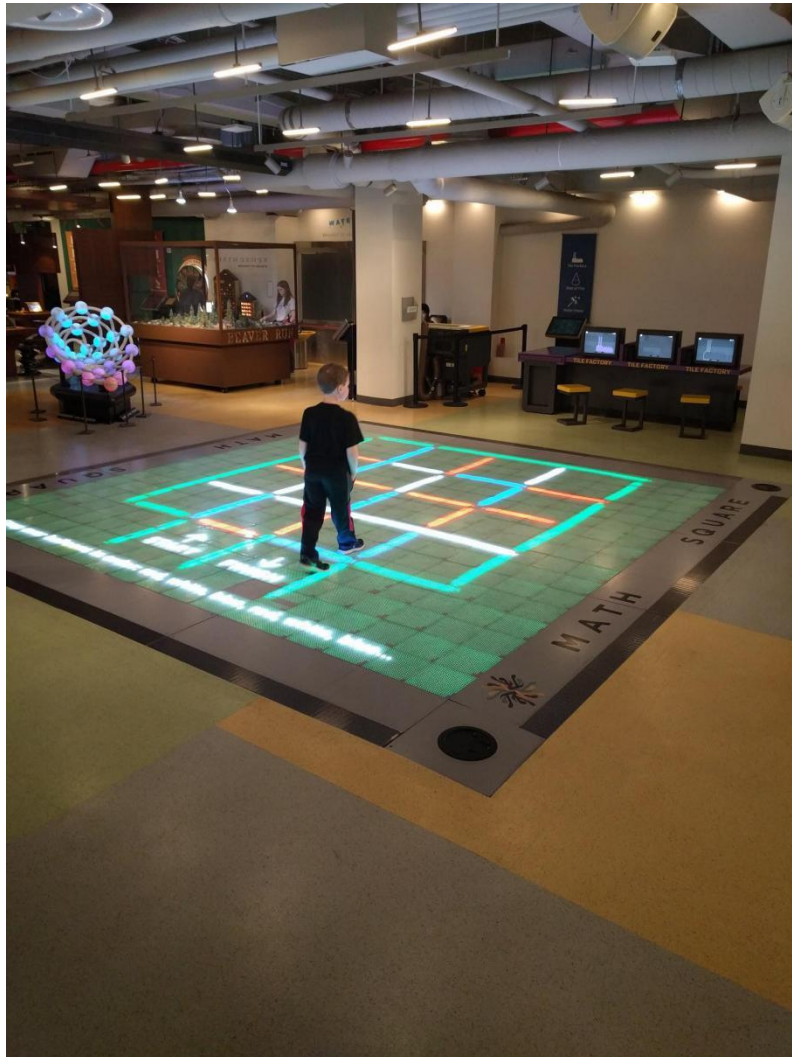
Figure 19 Math Square

The Math Square is a giant computer monitor that can display different puzzles. The screen can sense the location of a person and the direction that person is moving. Figure 19 above shows one of the puzzles, a maze. In this maze, the player starts from the entrance, and can only go through colors in a specific order instructed by the screen and try to get to the exit. The Math Square also provides several other puzzles focusing on logic and algebra, such as make polygon with multiple people, squaring the squares, color constraint satisfaction problem and Sokoban. If visitors play the game and think carefully about the math behind each puzzle, they will find that the answers are extremely profound and it is really an exercise for the brain.

Next to the Math Square is the Tessellation Station. There are a lot of different shapes provided at the Tessellation Station. Through the lenses of both geometry and art, visitors can use shapes to create different tessellations and explore the symmetries of polygons.

Figure 20 Tessellation

There is also a fun exhibition called Monkey Around:

Figure 21 Monkey Around

If the visitor move the handle under the exhibit to the right, it obviously appears 11 red monkeys and 12 blue monkeys on the left picture. But if the handle is moved to the left, the second tier in the middle rotates clockwise and after it stops, there are 12 red monkeys and 11 blue monkeys on the right picture. Why does the number of monkey change?

There is also the Edge FX on the second floor.

Figure 22 Edge FX

There is a lever next to this exhibit. As small balls keep falling down, if the lever is at the center, the shape formulated by the accumulated balls at the bottom slots will look like a bell curve. If the lever position is changed to create a bias, this bias will finally be shown by the shape formed by accumulated balls in a long run. This exhibit demonstrates the idea of probability and statics to visitors. A similar exhibit is also placed in Boston Science Museum.

The exhibit next to Edge FX is called Human Tree.

Figure 23 Human Tree

This exhibit uses a camera to detect the movement of the player and replace the left and right arms of the player to the body of the player. The Human Tree shows the concept of recursion and dynamic fractal tree.

There is also a model for Pythagoras Theorem and mechanical cipher machine on the same floor.
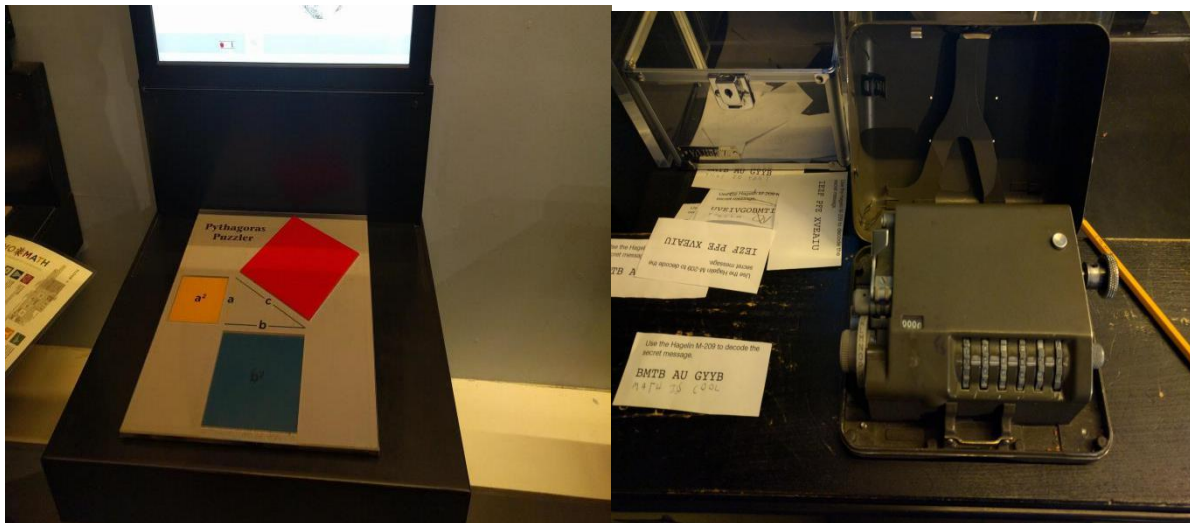
Figure 24 Pythagoras Triangle and Hagelin M-209

There are other interesting models and exhibitions that are not shown in this section. But details can be found in the museum or online.

3.1.3 Math Museum Interviews

In order to find out how visitors of different ages and identities think about the math museum, we interviewed both visitors and staffs with the questions we prepared (Shown in the reference).

On the day we went to the museum, there were four school trips with two of them led by school teachers and the rest led by student parents. Students in three groups are from 4th to 6th grades and students in the other group are from 9th to 12th grade.

From the answers given to us by the staffs, there are about 250 to 1000 people ranging from kindergarten children to adult visiting the museum on weekdays. There will be more people on weekends and when there are school trips. The school trips are arranged by schools and there will be a reasonable fee charged by museum. According to the stuffs, more children come to the museum than adults do. All the grades nationwide come to this museum to take a look at the interesting exhibitions.

The museum holds the "Math Speech" on the first Wednesday every month. Mathematicians come to the Math Speech and usually talk about some high level topics such as quantum mechanics and how to multiply two 3-digit numbers in mind. There is also "Family Friday", which is held on the first Friday every month and involve topics related closely to children. Activities including music and math magic will also be held occasionally. The museum also has classrooms where teachers can give lectures during school trips. Additionally, there are professional staffs at the museum who can teach additional knowledge such as cryptography and tessellation. Some of the exhibits are designed by mathematicians and the others are designed by some of the sponsors. Exhibits such as "String Product" are designed specifically for course materials. The most popular exhibits, said by some staffs, are Square-Wheeled Trike and Coaster Roller because visitors can interactive with exhibitions.

We also interviewed some visitors in order to know how they feel about the museum.

The first visitor we interviewed was a 4th grade teacher at a school in Washington DC. She and some other teachers are the leaders for this school trip and this is the first time they came to this museum. They prepared a survey, shown in Figure 24, for students to fill out during the trip. The exhibit she liked most was the Square-Wheeled Trike.
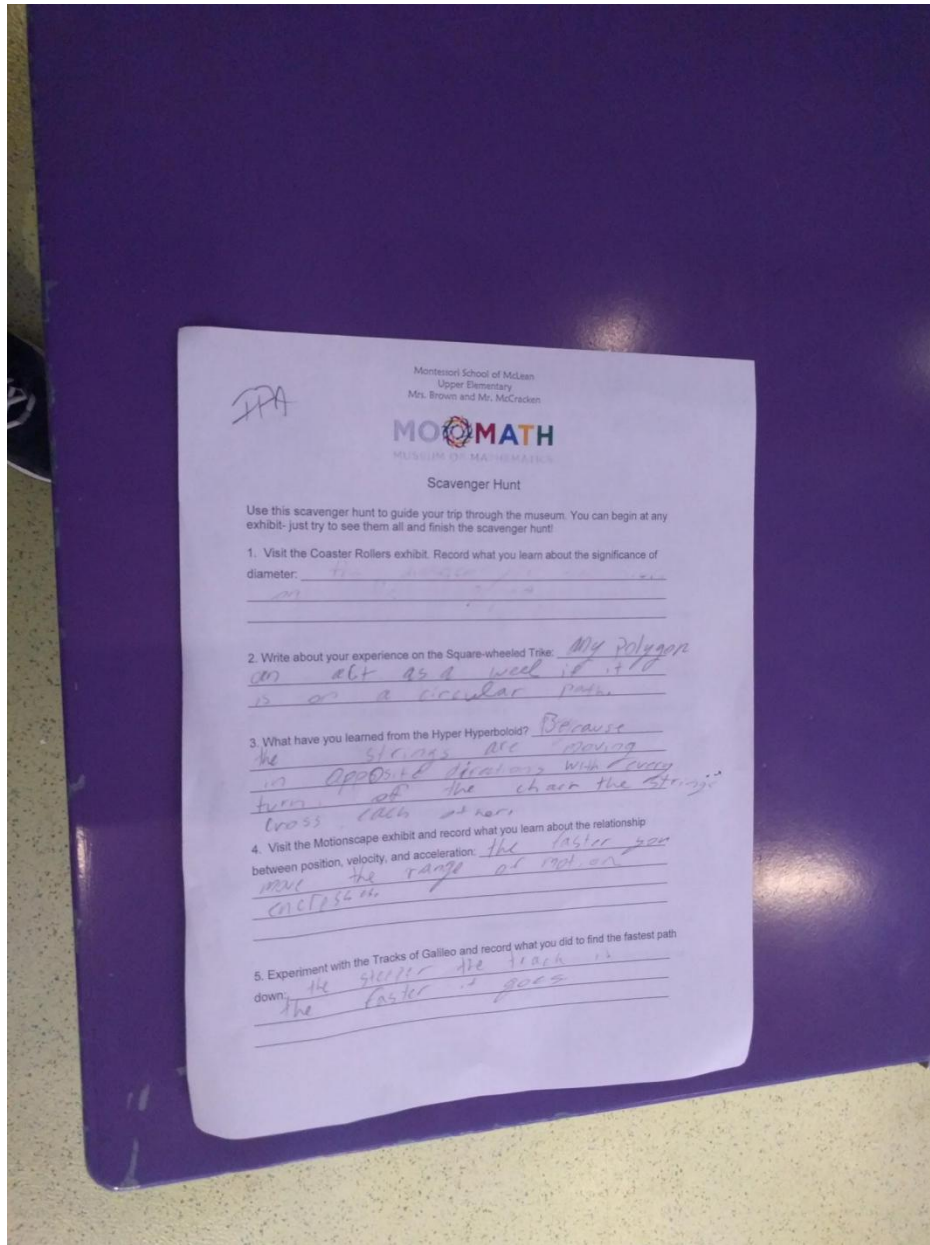
Figure 25 Questionnaire distributed by the school for Grade 5 museum trip

The second visitor was a team leader for a trip organized by a technical high school with students ranging from 9th grade to 12th grade. This school-organized trip to the math museum on a regular base and it was the second time he had been here. He had been a math teacher before and he thought that some geometry-related exhibits are connected to course material. Also, in his opinion, students couldn't fully understand the mathematical knowledge behind some of the exhibits since they hadn't started learning calculus. Finally, he liked the Math Square most.

Then, we interviewed a 6th grade child. He was from a school in Virginia and it was his first time here. He was taking this trip because he thought the knowledge here was related to his

school project. In his opinion, the exhibits were related to the materials, such as geometry and some science, he learned in class. The exhibit he liked most on the first floor was the MotionScape since it taught the concept of position, velocity and acceleration and on the second floor was the Math Square since it was related to logic.

The next person was a 5th grade child. The trip was organized by her school and was led by students' parents. This was her second time being here since her school organized the trip on a regular base. She thought the exhibits with geometry concept were related to course materials. Also, she thought the museum was interesting because she could interact with all the exhibits. The exhibit she liked most was the Squared-Wheeled Trike.

Another visitor is a math professor. He came here as a parent representative for a school trip and this is his first time here. He was not sure whether the exhibits were related to course materials or not. The exhibits he liked most were Square-Wheeled Trike, Coaster Roller, MotionScape and Track of Galileo. In his opinion, the museum was smaller than he expected and some of the machines were broken.

Overall, the public gave positive feedback to the museum materials. Children demonstrated their interests in the objects in museum. Most of them are delightful when playing with exhibitions. Adults accompanied with children learn math as well. The professions of adults could provide a substantial support to the explanation of the exhibitions.

3.2 Math Club Representation

To get advice from mathematics major college students on the influence of the National Museum of Mathematics on math education, we presented our trip to the math club in Worcester Polytechnic Institute. The WPI Mathematics Club is a student organization whose purpose is to foster interest, understanding and enjoyment of mathematics by the WPI community. The math club holds an activity every week, usually Wednesday afternoon, in the Math Lounge in Stratton Hall and 10 people on average will show up at the math club.[6]

We made a slideshow with the photos we took during the trip and explanations corresponding to the photos. Then, we went to the math club during its second meeting in March D term 2016 to present our trip. There were six math major students in that meeting. During our presentation, they were able to figure out most of the math knowledge behind the exhibits. They also showed great interest in some of the exhibits such as square-wheeled trike and string product.

After the presentation, we asked them whether they will be interested in taking a trip to the math museum. They said that the exhibits in the museum are very interesting and a lot of math knowledge lies behind the exhibits. Also, because almost all the exhibits are interactive, they really want to try by themselves and thought it would be a good idea to take a trip to the math museum.

We also asked them whether they think the museum would be helpful for students at school, whether the museum trip will increase student's understanding of the course material. Most of them thought that since some of the exhibits are related to the course work and a great part of the exhibitions are full of fun, a trip to the math museum will help students to gain deeper understanding to the materials they learned.

One of them, on the other hand, had a different opinion. He thought that middle school and high school students don't have enough knowledge to fully understand the math material behind most of the exhibits. Students may think the activities are intrinsically attractive, but never seriously think about the methods to solve the problem or the theories that explain the phenomenon behind. So the trip won't help them a lot in understanding the course material. However, since the exhibits are not like other ordinary museums that cannot touch, or cannot even take pictures, a trip to such a fun museum may stimulate the interests for students to learn the relating math materials and inspire them to exercise their minds in solving difficult puzzles. The exhibits also show to the public how math materials they learned in class can be used in real life or presented as a form of art. After they know what they learned can be utilized in numerous ways, students will more willingly and happily learn math.

---

[6] "Mathematical Sciences: Student Groups - WPI." 2012. 15 Apr. 2016
<https://www.wpi.edu/academics/math/student-groups.html>

3.3 Worcester High School Science Fair

To find out whether high school students will use their mathematical knowledge and skills practically in solving real-world problems, we went to the "61st Annual Worcester Regional Science & Engineering Fair". We wanted to find out how many projects were directly related to mathematics, such as math theorem proving and math modeling, and also find out how many projects had mathematical concepts involved.

The Worcester science fair was held at Worcester Polytechnic Institute at March 11, 2016. According to the fair introduction on the official website[7], a lot of students presenting had won awards at science fairs in their high schools. There were 121 projects in total. We went to the science fair after judging session finished.

Based on our observation, all project was related to mathematics directly or indirectly. All projects used some statistics knowledge to find patterns in their data and plot some graphs for analysis. However, some of them such as data distribution, algorithm analysis and physicals experience are more math related. Five projects, "Artificial Intelligence Using Bayesian Networks and MEBN", "Music Math: Does Music Follow a Zipfian Distribution?", "Novel Genome-Specific Data Compression Algorithms", "The Physics Behind Hockey" and "The Science Behind Kicking a Football" required a great quantity of math knowledge. Most of the other projects were related to biology and chemistry. There were also some projects concentrated on environmental science and social science.

---

[7] "High School Fair Deadlines - Worcester Regional Science ..." 2011. 1 May. 2016
<http://www.wrsef.org/high-school-fair/Default.aspx>

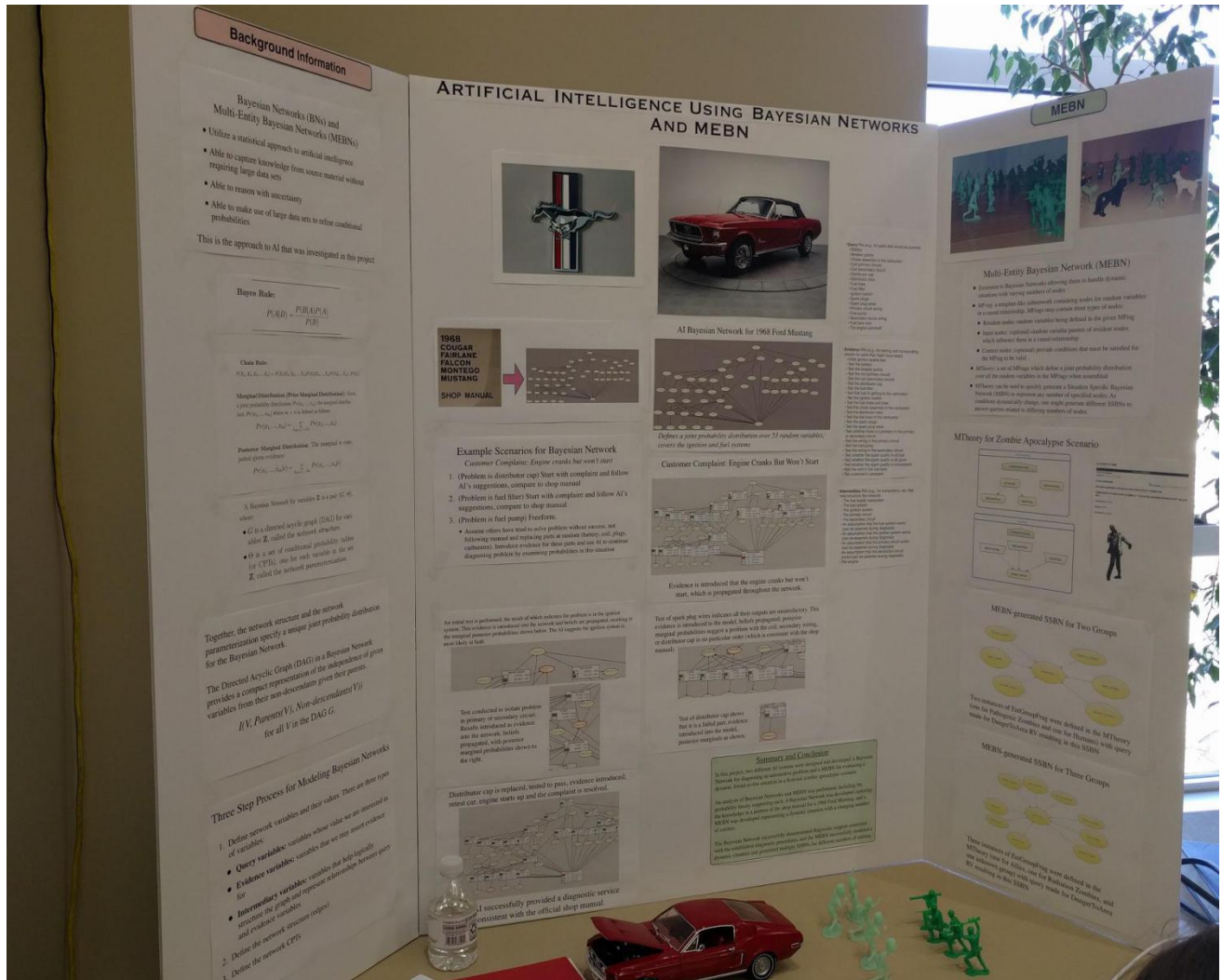Figure 26 is the poster for Artificial Intelligence Using Bayesian Networks and MEBN:

Figure 26 Artificial Intelligence Using Bayesian Networks and MEBN

In order to investigate the failure of starting a car, the student listed out all the possible causes for a car starting failure. With each cause and condition, the student assigned a reasonable probability that was found online. Based on the simple one-layer Bayesian networks, the student increased hypothesis conditions and expand the model to make a multi-entries Bayesian Networks (MEBN). The basic probability knowledge is taught in the high school class, but constructing a model of Bayesian network and using it in a realistic situation is beyond the materials talked about in the school. To proficiently use Bayes rules for this project, the student has to be very familiar with probability and statistics.

Figure 27 is the poster for Music Math: Does Music Follow a Zipfian Distribution:
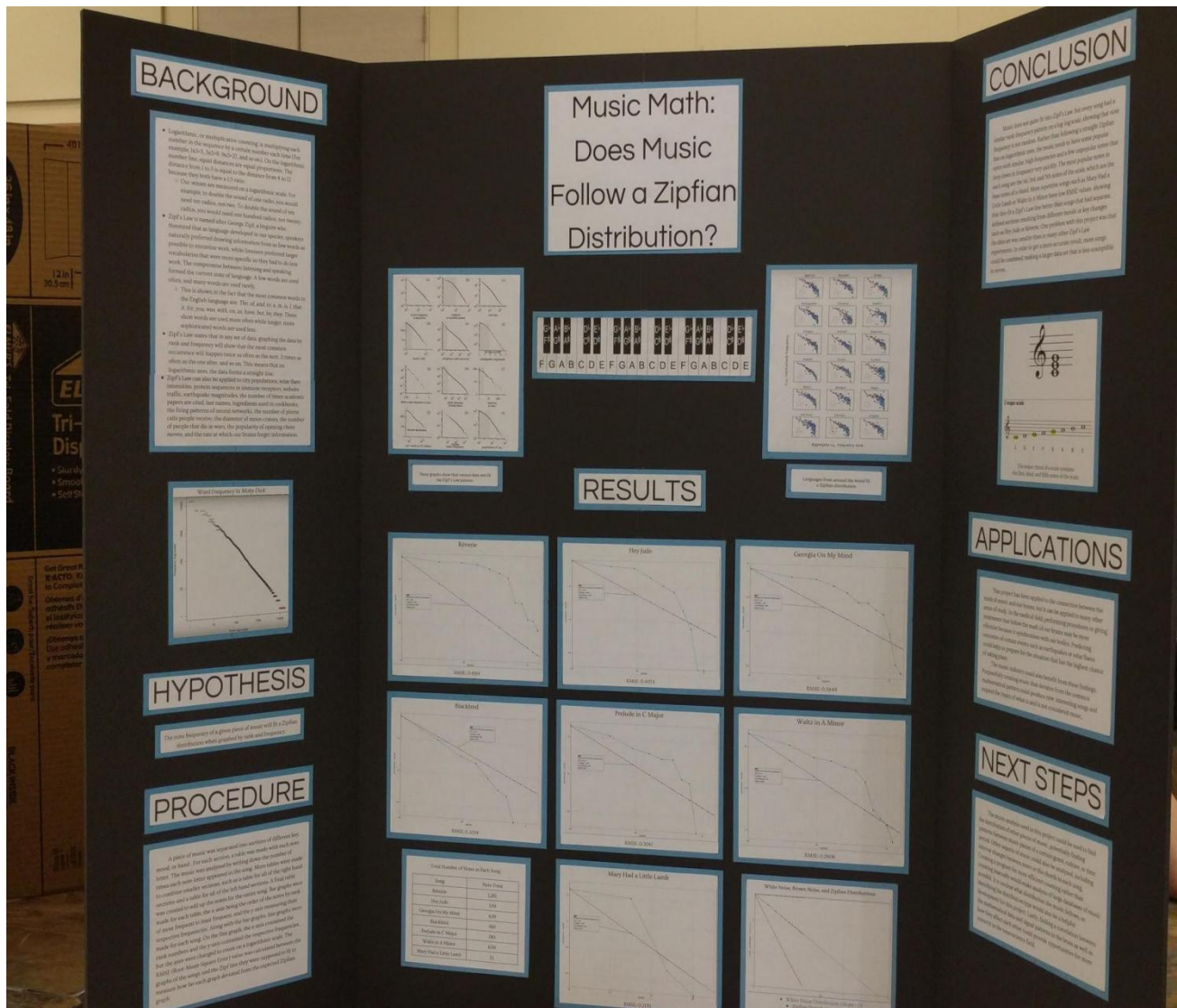

Figure 27 Music Math: Does Music Follow a Zipfian Distribution

The student of this project wanted to see if a piece of music follows the Zipfian distribution. Zipf's distribution[8] is under the topic of mathematical statistics. Zipf's law is being used in the data analysis of many social sciences and physics studies. The common use of Zipf's distribution is in language research to give information of ranking frequency of the words. The student demonstrated the hypothesis he made and modeled 9 songs with each note in the song analyzed. Although the conclusion of this research proved music doesn't follow the Zipfian distribution, the student found other interesting results such as notes relation in the scales.

---

[8] Zanette, Damián, and Marcelo Montemurro. "Dynamics of text generation with realistic Zipf's distribution." *Journal of quantitative Linguistics* 12.1 (2005): 29-40.

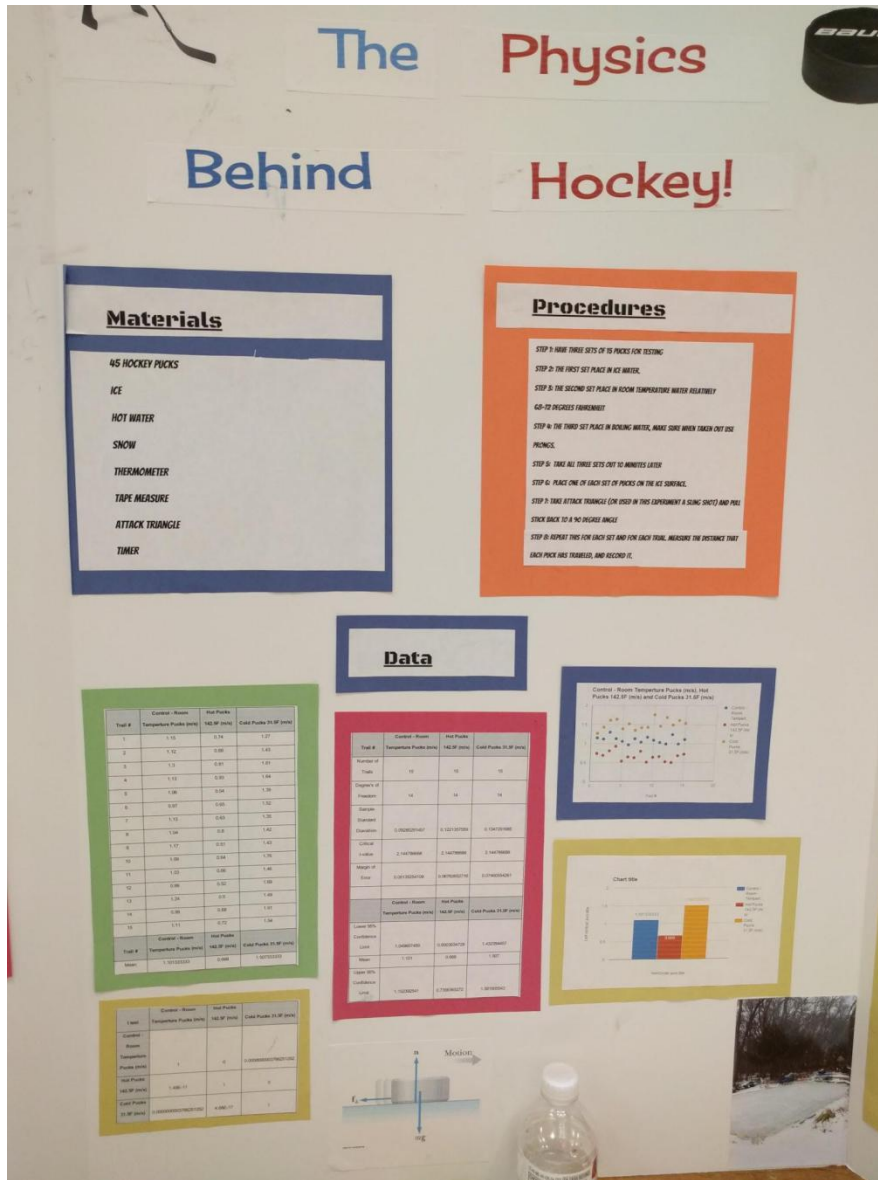Figure 28 is the poster of The physics Behind Hockey:

Figure 28 The physics Behind Hockey

This project was more like a physics experiment than a mathematical project. The students made a hypothesis that the coldest pucks would move the furthest on the ice compared to the warm and room temperature puck, and designed the controlled experiments. They gave treatments to the experiment units and blocked nuisance factors. Using knowledge of statistics, the student scientifically proved that his assumption was correct. While statistic design is generally talked about in the high school classroom, aspects such as experiment factor, factor level, confound factor and blockages are not. The student conducting this experiment used the mind of science and method of math he learned in the class.

Figure 29 is a poster of The Science Behind Kicking a Football:
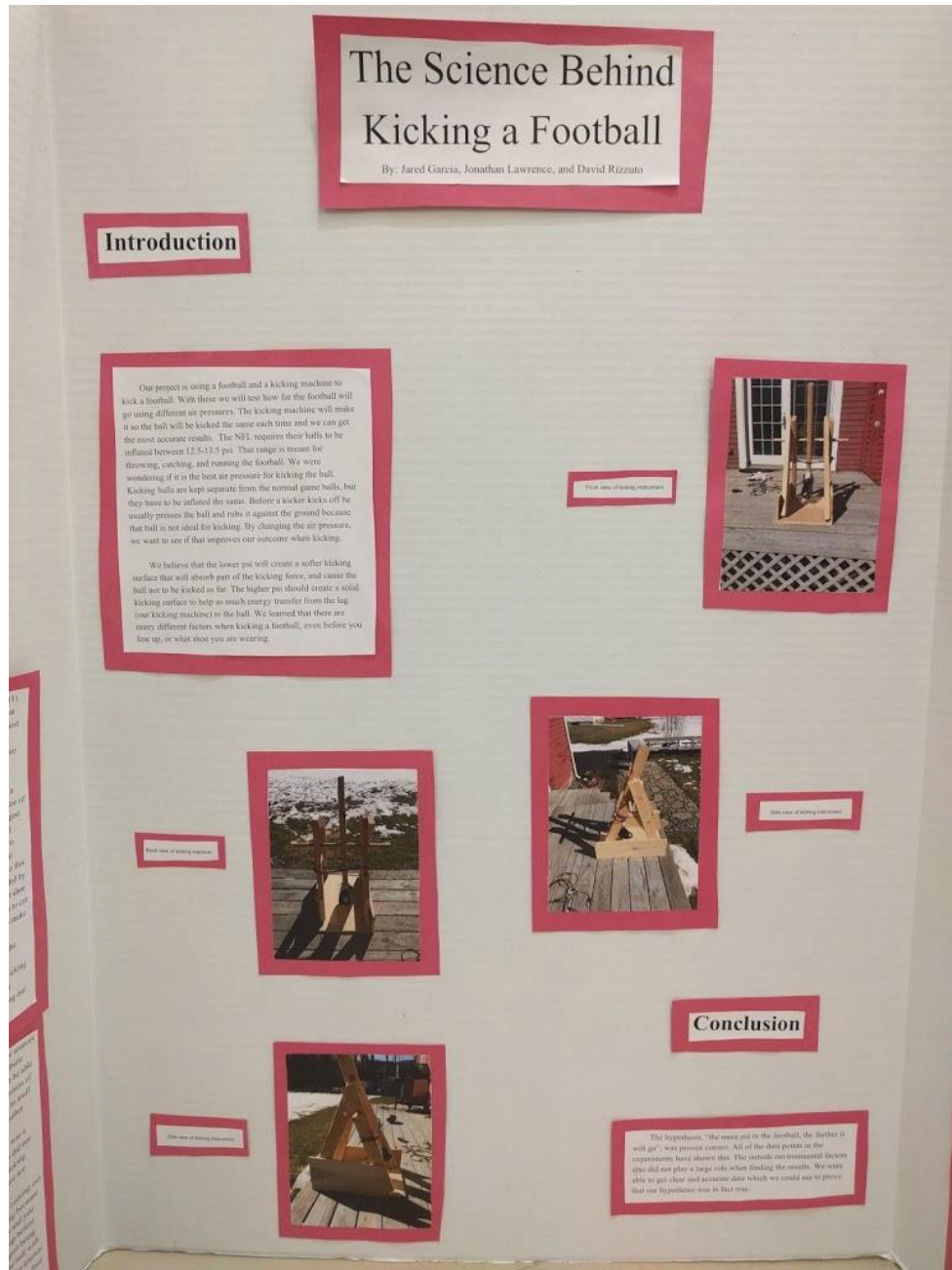
Figure 29 The Science Behind Kicking a Football

Similar to "The Physics Behind Hockey", this project was also an experiment that sampled from the population and conducted a t-test data analysis. Students did not learn t-test in a high school class; it is the material that will be covered in college statistic. The student also used bar graph data analysis in this project, which is primarily discussed in a high school class.

While the rest of the projects did tightly relate to mathematical topics, a majority of the remaining projects used statistics and data analysis.

Figure 30 shows two projects using tables and graphs to shown data relations:
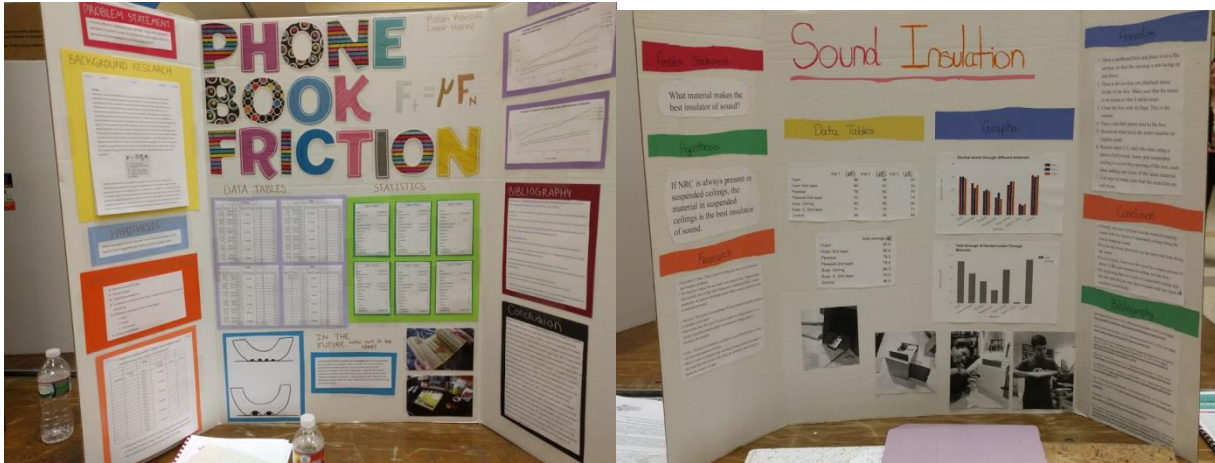
Figure 30 Other projects: Phone Book Friction (Left) and Sound Insulation (Right)

Chapter 4: Creation, Education and Activities

All humans have creativity, and it cannot be taught; it can be fostered; it can be unblocked; it is where science and art meet.

------Carlos Arozmena[9]

Carlos Arozamena, a professor of Industrial Design the Universidad Autonoma Metropolitana[10] in Mexico City, uses art in his teaching to foster creativity and follows the teaching philosophy of Eric Maisel.

In Fearless Creating[11], Dr. Eric Maisel outlines the stages of the creative process, from nurturing the wish to create, choosing next subject, starting work, working, completing work, to showing and selling work, identifies the anxieties and challenges associated with each stage, and presents tactics and strategies for meeting those challenges.

For an industrial designer creativity could be viewed as a black box in a pipeline. But rather than to research what is inside the black box, Carlos Arozamena uses Mailsel's approach which he summarizes as follows:

The phases of the creative process are
1. The wish to create.
2. The decision of which project to work on.
3. The decision to work
4. Working
5. Completion
6. Communication and presentation of the completed work.

Maisel addresses artists, but Arozamena applies the same ideas successfully in his teaching. If students have the wish to learn, the learning process is more efficient, so it is the professor's challenge to nurture the wish to learn in the students. It is important for students to decide to work and to consistently work until the work is actually finished. Upon completion the work should be presented and presentable.

The feedback from presenting the work to others (the feedback of the professor to the homework handed in on time) will create new ideas and the cycle can start again. Any creative person or any student will usually be working on different projects in different phases and phases will mix. Feedback will lead to revisions and refinement of the work and another completion.

---

[9] "Carlos Arozamena: Mirada y Pensamiento." 2008. 4 May. 2016
<http://carlosarozamena.blogspot.com/>

[10] "Universidad Autónoma Metropolitana Unidad Xochimilco - UAM." 2005. 4 May. 2016
<http://www.xoc.uam.mx/>

[11] Maisel, Eric. *Fearless Creating: A Step-by-step Guide to Starting and Completing Your Work of Art*. New York: Putnam, 1995. Print.

From Chapter 2 Education Report, we can see there are essentially no significant differences in the high school math curriculum of MA and Tianjin. However, Chinese student perform at double proficiency level as compared with MA students on the standardized tests (see section 2.3 PISA test).

We specifically examined the textbooks, the forms of homework, teaching methods and tests results in both MA and Tianjin. We found out the defects in each education systems.

MA textbooks with favorable type settings, massive colorful pictures, and thoughtful explanation of the concepts are very intriguing. Tianjin's textbooks use fewer pictures. Most of them are in black and white. Chinese students show no interest in reading a textbook. If the teacher does not talk about the book, students are not willing to even open the book. While the students are intrinsically captivated by the pretty figures in MA's math textbooks, they actually may be distracted by the beautiful and irrelevant photos. Also, most of time students are passionate about comely patterns, but fail to think critically about the math behind them if teachers do not mention them, which requires time and patience. On the other hand, the boring pictures in Tianjin's textbooks are quite content-related. Students could extract more useful information from the figures because with fewer pictures, teachers have more time to go through the math principles behind each one.

The test questions reflect the standard proficiency students should reach after understanding and practice. Though they cover the same topics, it is obvious that the MCAS and SAT in MA is simpler than High School Achievement Test and College Entrance Test in Tianjin. Tianjin's tests ask more sophisticated questions with more detailed concepts and greater depth. The MCAS and SAT have a large proportion of multiple-choice questions and about 1-2 open-response questions, which provides students with a higher chance of guessing and less motivation for serious learning. Using a computer-based test would have even worse feedback. Students are unable to show their thinking process to teachers on the screen and it is clumsier using software to write answers rather than fluently writing by hand. In Tianjin, with a paper-based test mechanism and open-response questions, teachers can see how students think and give partial credit. The feedback from the exams is accurate towards students' weak points. With fewer multiple-choice questions and more open-response questions, students have to master knowledge fast and optimally use what they learned to solve problems.

Considering the teaching methods in MA and Tianjin, great diversity leads to two distinct results of learning. In MA, various forms of researching and brainstorming the math ideas help students in class to see a beautiful math world with infinite possibilities and thus, curiosity beacons the way of discovering and learning. Students wish to learn and desire to have mental practice. In Tianjin's education, teachers care more about the grades and pay more attention to elevating scores. Peer pressure is the cardinal reason for high scores. Even if students have disinterest in math, they are coerced to learn to compete for good schools. The math-learning environment in

MA with more freedom and discretion elicits students' interest in math compared with the intensive math curriculums in Tianjin.

Both math education systems have flaws, but are complementary in different aspects. MA does extraordinarily well on captivating students to learn math, which is the first step in the creation process. However, it does not emphasize steps of 2-4 that require dedication to work. Students could be distracted by multiple ideas and not be devoted to complete any one of them because they shy away from the arduous solving process. Also, learning requires patience and time, a massive quantity of exercise is demanded. The interest in learning and creating of students with incomplete math foundation quickly fades away because they cannot move on to a higher level if their base knowledge is weak or missing. Excessive work choices with few practice and facile talks on the topics are futile to gain a substantial mastering, and in the long run, superficial learning manners harm the mental growth of students. Suggestions such as increasing the difficulties of exams and homework and practicing more on the topics could be made for teachers in MA.

Contradictorily, Tianjin does poorly on nurturing the wish of creation. The effect of pressure that comes to be the motivation of learning is ephemeral. Without this motivation and the wish to learn, student could easily stop learning math. This is also why many Chinese students dissipated themselves in the university for relaxation. On the other hand, Tianjin does devoted work and constant reviews supremely well. Undoubtedly, students earn profound understanding on the basic skills in high school. Tianjin students have solid basic skills when they come to the university. Nevertheless, as an inevitable part in the creation process, the importance of the wish to create should be pointed out in Tianjin math education.

From out trip to math museum, we figured out the public do have a sense that mathematics is important and the awareness of mathematics should be arise. But the devolving mathematical researching for dissecting the artifacts and models is far beyond the frivolous touching and impressions. Exercises and repetitions are necessary to learn basic skills so that the achievement for quantitative change to qualitative change can be made. This is progress from pure math knowledge accumulation to innovative problem breakthrough. Therefore, a large quantity of practice should be commensurate with emphasis of ideas' incubation in the math education.

To summarize, the cultivation of math creativity is based on both the appropriate wonder inducement and the lasting buttress of robust knowledge buildup, neither of which should be downplayed or overemphasized.

Reference

MA math education Reference

U.S. Department of Education, National Center for Education Statistics, Private School Universe Survey (PSS), 1995–96 through 2011–12. April 2013. <http://nces.ed.gov/programs/digest/d13/tables/dt13_205.20.asp>

J. Michael Smith,President, HSLDA. U.S. Department of Education: Homeschooling Continues to Grow! (September 3, 2013). Retrieved <https://www.hslda.org/docs/news/2013/201309030.asp>

"Mathematics Standards." | Common Core State Standards Initiative. Common Core State Standards Initiative, n.d. Web. 08 Apr. 2016.

<http://www.corestandards.org/Math/>

Massachusetts Department of Elementary & Secondary Education. "MASSACHUSETTS CURRICULUM FRAMEWORK FOR MATHEMATICS." (n.d.): n. pag. Mar. 2011. Web. <http://www.doe.mass.edu/frameworks/math/0311.pdf>

Massachusetts Department of Elementary & Secondary Education, *Massachusetts Comprehensive Assessment System*,Oct 2014 <http://www.doe.mass.edu/mcas/>

Massachusetts Department of Elementary & Secondary Education, *Massachusetts Comprehensive Assessment System 2011 results*, 2011 <http://www.doe.mass.edu/mcas/results.html?yr=2011>

Massachusetts Department of Elementary & Secondary Education, *Massachusetts Comprehensive Assessment System 2015 results*, 2015 <http://www.doe.mass.edu/mcas/results.html?yr=2015>

Massachusetts Department of Elementary & Secondary Education, 2014-15 SAT Performance Report, 2015 <http://profiles.doe.mass.edu/sat/sat_perf_dist.aspx?orgcode=00000000&orgtypecode=0&>

Massachusetts Department of Elementary & Secondary Education, 2010-11 SAT Performance Report, 2011 <http://profiles.doe.mass.edu/sat/sat_perf_dist.aspx?orgcode=00000000&orgtypecode=0&&fycode=2011>

Massachusetts Department of Elementary & Secondary Education, Grade 10 Mathematics Released Item Document, 2011
<http://www.doe.mass.edu/mcas/testitems.html?yr=11>

The CollegeBoard, The SAT, MAY 2011
<http://www.cracksat.net/sat-downloads/142.html>

Massachusetts Department of Elementary & Secondary Education, Advanced Placement Performance Report
<http://profiles.doe.mass.edu/adv_placement/ap_perf_dist.aspx?orgcode=00000000&orgtypecode=0&>

The CollegeBoard, AP Calculus AB 2011 Scoring Guidelines, 2011
<http://apcentral.collegeboard.com/apc/public/repository/ap11_calculus_ab_scoring_guidelines.pdf>

Carter, John A. Glencoe *McGraw-Hill Algebra 1*. Columbus, OH: Glencoe/McGraw-Hill, 2010. Print.

Kennedy, Dan, Randall I. Charles, Basia Hall, Bonnie McNemar, Alma Ramirez, Allan Bellman, and Laurie E. Bass. *Prentice Hall Mathematics*. Boston, MA: Pearson, Prentice Hall, 2007. Print.

Tianjin math education Reference

"Net Enrolment Ratio of School-age Children in Primary Schools." 小学学龄儿童净入学率_教育部门户网站_MOE.GOV.CN. Ministry of Education of the People's Republic of China, n.d. Web. 08 Apr. 2016.
<http://www.moe.edu.cn/publicfiles/business/htmlfiles/moe/s7567/201309/156875.html>.

"中国教育概况." - 中华人民共和国教育部政府门户网站. 中华人民共和国教育部政府, n.d. Web. 08 Apr. 2016.
<http://www.moe.gov.cn/jyb_sjzl/s5990/201503/t20150331_186797.html>.
普通高中课程标准实验教科书. Beijing: People's Education, 2007. 电子课本. People's Education Press. Web. 08 Apr. 2016.
<http://www.pep.com.cn/gzsx/jszx_1/czsxtbjxzy/xkbsyjc/gzsxxkbbx1_2/gzsxbx1kb_1/>

Sina education, 天津 2015 高考各科平均成绩及各分数段统计, June, 2015.
< http://edu.sina.com.cn/gaokao/2015-06-23/0039474175.shtml>

Tianjin Education Department, 2011 年普通高等学校招生全国统一考试(天津卷), 2011
<http://gaokao.eol.cn/2011gkst_11308/20110601/t20110601_625292.shtml>

Tianjin Education Department, *2011 年天津市普通高中学业水平(数学)*, 2011
<http://wenku.baidu.com/link?url=yGh5q2IlzuOWbmhpYPH4HADKh7vwv-1iv0wMD8AUJVOFA1h4ZUT0ml0X_dOmoTNAQRR1tw7khKfIEz2Mfb0A75MVWmrebnovaxMhQE6Yf8u>

Tianjin Education Department, *普通高中学籍管理规定*, 1997
<http://wenku.baidu.com/link?url=fXzrvD3FPCZjRAD-oMhtM1fL4PC6xo4Py17owcrVJEjLudY-R0NVJnTvFoweJK9oav059WWKpbVTCJNtnsHFYOIcIOYMMASylFSzgmG_9cm>

普通高中课程标准实验教科书. Beijing: People's Education, 2007. 电子课本.
People's Education Press
<http://www.pep.com.cn/gzsx/jszx_1/czsxtbjxzy/xkbsyjc/gzsxxkbbx1_2/gzsxbx1kb_1/>

PISA Test Reference

OECD. "PISA Result from PISA 2012." (n.d.): n. pag. Web.
<https://www.oecd.org/pisa/keyfindings/PISA-2012-results-US.pdf>

 Keith Devlin, *The Millennium Problems: The Seven Greatest Unsolved Mathematical Puzzles of Our Time* New York: Perseus Books Group (2002)

Peterson, Paul E., Ludger Woessmann, Eric A. Hanushek, and Carlos X. Lastra-Anadón. *Globally Challenged: Are U. S. Students Ready to Compete?* (n.d.): n. pag. Aug. 2011. Web.
<http://www.hks.harvard.edu/pepg/PDF/Papers/PEPG11-03_GloballyChallenged.pdf>.

OECD (2010), *PISA 2009 Results: Executive Summary*. (n.d.): n. pag. Web.
<https://www.oecd.org/pisa/pisaproducts/46619703.pdf>

OECD. *PISA 2012 Results in Focus*. (n.d.): n. pag. 2014. Web.
<https://www.oecd.org/pisa/keyfindings/pisa-2012-results-overview.pdf>

MoMath Reference

"Exhibit Guide – National Museum of Mathematics." 2013. 12 Apr. 2016
<http://momath.org/about/exhibit-guide/>

"Calculus Is Fun! Math Exhibit Demos Principles of Motion." 2014. 12 Apr. 2016
<http://www.livescience.com/44948-math-exhibit-demos-principles-of-motion.html>

"National Museum of Mathematics." 2009. 12 Apr. 2016
<http://momath.org/>

High School Science Fair Reference

"High School Fair Deadlines - Worcester Regional Science ..." 2011. 1 May. 2016
<http://www.wrsef.org/high-school-fair/Default.aspx>

Zanette, Damián, and Marcelo Montemurro. "Dynamics of text generation with realistic Zipf's distribution." *Journal of quantitative Linguistics* 12.1 (2005): 29-40.

Creation, Education and Activities Reference

"Carlos Arozamena: Mirada y Pensamiento." 2008. 4 May. 2016
<http://carlosarozamena.blogspot.com/>

"Universidad Autónoma Metropolitana Unidad Xochimilco - UAM." 2005. 4 May. 2016
<http://www.xoc.uam.mx/>

Maisel, Eric. *Fearless Creating: A Step-by-step Guide to Starting and Completing Your Work of Art*. New York: Putnam, 1995. Print.

Appendix

Questions Prepared for math museum trip

For staffs:
- How many visitors come on weekdays and weekends?
- What is the ratio of children to adults?
- Are there any visits organized by schools? If there are, what's the most common grade?
- Are there any exhibits for math materials taught at school? If there are, what are the corresponding grade levels?
- Does it cost a lot for school groups to visit?
- Is there any collaboration between the museum and local schools?
- What kinds of events does the museum hold? Why hold these kinds of events?
- Which part of museum is the most attractive place?
- Have any mathematicians come here to give speeches? Where were they from?
- Do any math teachers give classes here? Like teaching math modeling on computers?
- Who do you target with your events? (Who do the speeches and games target?)
- How is the feedback from the public?

For visitors:
- Which exhibit do you like the most? Is it related to any math knowledge/course you are familiar with or is it only because it looks interesting/beautiful?
- Do you think a trip to the math museum will help children to understand math more deeply or just inspire them to learn math?
- How many times have you visited the math museum?
- Do you ever come here to listen to the mathematicians speak?
- What inspired you to be here? Would you bring children here? Why?
- What things have you learned here?
- Do you think the museum is a good way to enhance public understanding and perception of mathematics?
- Does your occupation relate to math?