Improvement of The Fresnel Thermal Solar Power System

Major Qualifying Project Completed in Partial Fulfillment of the
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ABSTRACT

Solar Energy is one of the cleanest and powerful forms of renewable energy. This project focuses on improving the efficiency of the Fresnel Thermal Solar Power System (FTSPS) by modifying its design and mechanism. The proposed design reduces the overshadowing by minimizing the size and gaps of the mirror strips, and utilizing the Alternating Mirror Arrangement (AMA) method. Using the proposed strip-rotation mechanism makes FTSPS more reliable, durable and energy-efficient while reducing the manufacturing and maintenance cost.

ACKNOWLEDGMENTS

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The project would not have been possible without the help of our advisor Professor Selcuk Guceri. Our advisor provided us guidance, assistance, and general help whenever needed for the completion of our project. Prof.Guceri provided us with his valuable time and guidance in helping our project develop from a prompt to a complete deliverable. The team also would like to thank Dr. Mehul Bhatia who provided us with constructive criticism on our project and guidance in helping our project meet all the requirements of an MQP. Moreover, we recognize the support of the WPI Mechanical Engineering for the supplies and equipment necessary for carrying out our fieldwork.

The team would also like to extend our gratitude to our peer and colleague Aidan Kennedy who helped us with the initial calculations of the project.

CHAPTER 1: INTRODUCTION

Solar Energy is one of the most clean and powerful forms of renewable energy. Energy is needed in every aspect of our lives and with humanity on the verge of depleting its fossil fuels, we have to look for new forms of sustainable energy. We can achieve this by using the sun's powerful rays to produce steam that runs our turbines. There are several types of Solar Thermal Power Plants that can convert Solar Energy to produce steam. These power plants are the Dish-Stirling system, Central Receiver Systems-Power Tower**,** Parabolic Trough Collector System, and the Fresnel Thermal Solar Power System (FTSPS).

While the Dish-Stirling system, Central Receiver Systems-Power Tower**,** Parabolic Trough Collector System, have their respective advantages, this project focuses on developing the FTSPS. The system has many advantages compared to other thermal solar systems such as low manufacturing cost and high durability; however, the main flaw in the system is not having high efficiency. The team used the current concept of FTSPS and improved on its mechanical design and actuation.

The final design reduces the manufacturing and maintenance cost, and improves the overall efficiency of the FTSPS. The final design was achieved by finding out the Zenith angle for Worcester, MA.With the Zenith Angle, the team found the strip angle for each strip and the team found that the rate of change among all strips is consistent. Therefore, the team decided that the gear ratio for the design should be one-to-one with a rack and pinion mechanism rotating each mirror. In order to increase efficiency and optimize the design, the team used the Alternating Mirror Arrangement (AMA), decreased the spaces between the mirror strips as much as possible, and designed an omega shaped heat collector that increases the chances of absorbing energy.

CHAPTER 2: BACKGROUND

2.1 Solar Power and Irradiation

For many centuries humanity has used steam power as its primary source of energy. One of the most popular uses of steam power is the locomotive engine, an excellent example of the use of steam power converted to mechanical power. Modern steam turbines operate on the same principle of energy conversion at a much higher energy conversion rate. High pressured steam produces rotational energy that is used to drive electricity generators. However, most modern and past methods use fossil fuels such as natural gas and coal (Avila-Main, 2011) to generate the steam required for energy uses. With humanity depleting its natural resources at a rapid rate, we have to look for new methods to generate steam. Moreover, the burning of fossil fuels for the production of steam produces harmful greenhouse emissions that lead to global warming and many more environmental concerns. Therefore a new way to generate steam is through solar radiation. The steam that is generated by solar radiation is identical to the steam obtained by burning fossil fuels without having any carbon dioxide or other greenhouse gas emissions. The conversion principle of solar heat to electrical and mechanical energy is similar to the techniques used in traditional fossil-fuel combustion systems. Concentrating solar thermal systems is best suited to attain higher efficiencies and high temperatures under high pressure while meeting the requirements of large-scale turbines that need large amounts of high-quality steam. The methodology of energy conversion through solar energy is shown below

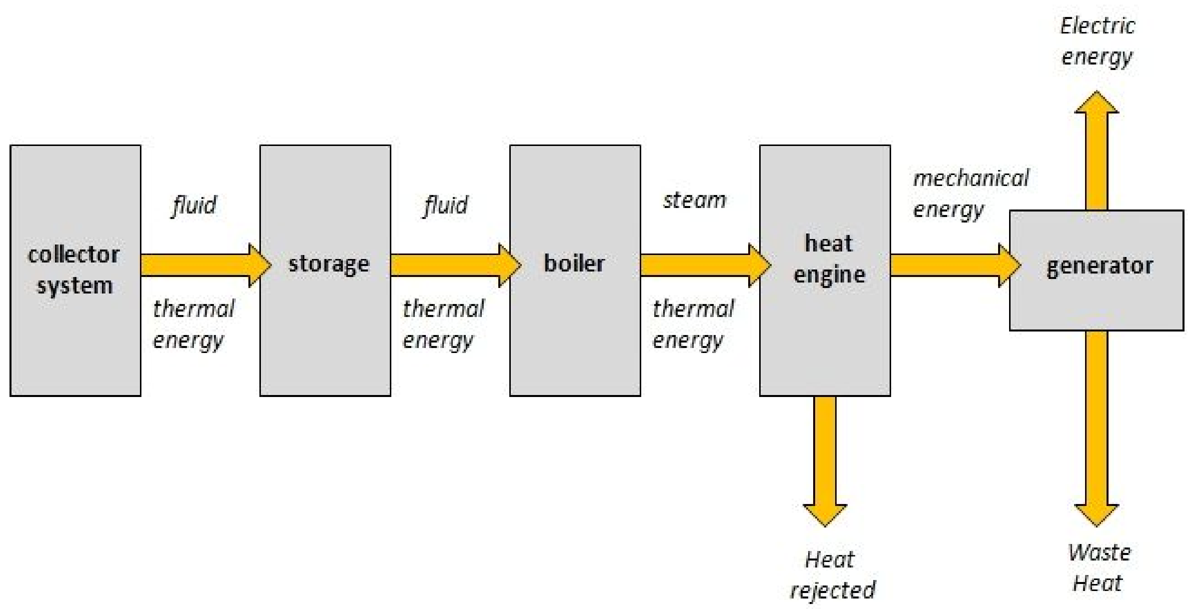


Figure 1 Generic solar thermal power system

The solar energy is received and converted to heat via the collector system. The heat is then transferred through the thermal fluid to storage and then to the boiler where steam generation takes place. The steam is transferred to the turbine in its heat engine. The steam is converted to mechanical energy, and some heat will be rejected. If the desired output is electric energy, the mechanical power is transferred to a generator which converts it to electric power. However, one of the key challenges in the system is the energy losses. The efficiency of solar collectors reduces with an increase in the operating temperature (Hu et al., 2010). Also, the heat engine efficiency increases with higher operating temperatures. The operators maximize the solar power output at the optimum conditions. No machine operates at a 100% efficiency. Plant plate temperatures do not provide efficient temperatures to run heat engines. Therefore, evacuated tubular collectors or concentrating collectors such as parabolic systems are desired. The paper will analyze solar thermal power systems, its financial requirements, and its future needs.

Solar energy concentrated on the Earth's surface is commonly known as Solar Irradiance, which is the power per unit area (W/m^2) electromagnetic radiation from the sun. The derivation for an equation of irradiance on a given day is as follows:

$$\cos(\Theta) = \sin(\phi)\sin(\delta) + \cos(\delta)\cos(\phi)\cos(h)$$

Where Θ = *Zenith Angle,*

ϕ = *latitude,*

δ = *Solar declination,*

h = *relative longitude,*

Therefore Irradiance is

$$Q = \begin{cases} S_o \frac{R_o^2}{R_E^2} \cos(\Theta) & \cos(\Theta) > 0 \\ 0 & \cos(\Theta) \leq 0 \end{cases}$$

Where Θ = *Zenith Angle,*

S_0 = *Solar Constant,*

R_0 =*Mean Distance,*

R_E = *Separation of Earth from the Sun.*

The average of Q over a day is the average of Q over one rotation, or the hour angle progressing from $h = \pi$ to $h = -\pi$:

$$\overline{Q}^{\text{day}} = -\frac{1}{2\pi}\int_{\pi}^{-\pi} Q \, dh$$

Using integration we get that the formula for an equation of irradiance on a given day is:

$$Q = S_0 \left(1 + 0.034 \cos\left(2\pi \frac{n}{365.25}\right)\right)$$

2.2 Types of Thermal Solar Power Systems

There are many types of Thermal Solar Power Systems that convert Solar Energy into Steam to produce electricity. Each of them have their respective advantages and disadvantages that allow them to produce clean Solar Energy. Some of the Thermal Solar Power Systems are as follows.

2.2.1 Dish Stirling System

One of the most common types of systems, the dish Stirling system utilizes a parabolic dish of mirrors to focus and concentrate solar energy on a power collector that generates electricity. It is a CSP (Concentrating Solar Power) technology that produces smaller amounts of electricity than other known CSP technologies - in the range of 3-25 KWs - but it is still beneficial for modular use. Major parts of the Dish Stirling System are the following:

1. Solar Concentrator

   Dish gathers solar energy directly from the sun and reflects the beam of concentrated sunlight onto a thermal receiver that collects solar heat. The dish is placed on a structure that tracks the sun for maximum efficiency.

2. Power Conversion Unit

After receiving the concentrated sunlight a power conversion unit absorbs the concentrated beams of solar energy, converts the energy to heat, and transfers the heat to the engine/generator.

The engine takes the heat received from the unit and converts it into electrical energy.

2.2.2 Central Receiver Systems - Power Tower

One of the most popularly used Solar Systems is the Central Receiver Systems - Power Tower, a central system that sends focused sunlight onto a remote central receiver unlike concentrating systems that reflect light onto a focal line. It consists of many tracking mirrors in a field surrounding a main external receiver installed on a tower. The Central Receiver Systems - Power Tower can reach much higher levels of concentrated solar energy than most linear systems. This solar energy is then used as heat to produce steam and use that steam to move turbines to generate electricity.

Figure 2: Solucar PS10 solar power plant, Spain.

These power plants are very large scale and produce a lot of energy (Central Receiver Systems, 2020).

2.2.3 Parabolic Trough Collector Systems

The use of Parabolic Trough Collector Systems is widespread in utility-scale power thermal power plants. Parabolic Trough Collector Systems can concentrate direct sunlight energy to reach output fluid temperatures in the range up to 500 C (Concentrating Solar Power, 2020). It consists of a cylindrically curved parabolic mirror that reflects the sunlight onto a tabular receiver positioned in the focus line of the parabola. Which is then received through a tabular fluid that absorbs heat and transfers it via circulation to the boiler of another device to produce steam.

Figure 3: Kuraymat parabolic trough solar power plant

The power plant usually consists of large rows of mirrors that are mounted in parallel on either a north-south axis or an east-west axis.

2.2.4 Fresnel Solar Reflectors

Although the before mentioned Solar Thermal Power have their respective merits. This project focuses on the Fresnel Solar Reflector. Named after their similarity to a fresnel these mirrors are capable of concentrating solar energy 30 times its normal intensity (Dey, C.J., 2004). Using long, thin segments of mirrors the Fresnel Solar Reflector focuses solar energy onto a specific collector which is located at a point that can be reached by all of the fresnel solar reflectors. The energy collected is transferred to generate steam, which is then used to produce electricity.

Figure 4: A Solar Fresnel Mirror apparatus in Arizona.

2.3 Design of a Fresnel Solar Reflector

Located at the base, the Fresnel Solar Reflector makes use of the Fresnel lens effect that allows concentrating mirror systems with a huge aperture and a short focal length and simultaneously reduces the volume of material required for the reflector. Therefore it greatly reduces the cost as sagged-glass parabolic reflectors are really expensive.

Located at the focal line of mirrors, the absorber runs parallel to and above the reflector segment to transport radiation into the working thermal system. This part is one of most important aspects of the design. Developing an efficient heat collector will result in an increase in the overall efficiency.

2.4 Thermal Solar Power Systems Comparison Table

The Parabolic Trough Collector Systems, FTSPS, Central Receiver Systems -Power Tower and the Dish Stirling System have their respective advantages and disadvantages, output, optical loss and thermal loss defined below:

Type of Solar Thermal System	Wattage (GWh)	Advantages	Disadvantages	Optical Loss	Thermal Loss
Parabolic	20.339	Most efficient use of concentrated power	Oil Bases' thermal infrastructure restricts temperature to 400 C.	41.372 %	19.858 %
Fresnel Thermal Solar Power System (FTSPS)	16.117	1) Small Volume 2) Light Weight 3) Mass Production 4) Less Maintenance Cost	1) Imperfection causes rays to be improperly focused at the receiver. 2) Light can be lost due to incidence on the draft feet	54.139 %	6.994 %
Heliostat	16.657	1) High temperature 2) Hybrid with other thermal systems is possible. 3) Better option to use if the surface isn't flat	1) The investment cost is high, 2) Tests needed are of great scale.	33.774 %	16.064 %
Dish	18.86	1) High conversion efficiency. 2) Follow the sun in all directions. .	1) Can't store energy since it is a direct operation 2) Cost of series production in verification stages.	~5.43 %	20.150%

Table 1: Comparison of different thermal power plants in terms of their respective factors

Sources: (Kedar N., 2020)

CHAPTER 3: METHODOLOGY

After researching and analyzing the different types of Solar Systems, the team decided to use the Fresnel Thermal Solar Power System and come up with a design that best utilizes the advantages of the system while simultaneously minimizing the disadvantages. This decision was made because of the low cost of maintenance and low cost of manufacturing of the FTSPS. Our goal for this project is to develop and optimize the FTSPS and increase the efficiency. The team achieved this goal by:

3.1 Calculating the Sun Angles

There are two sun angle calculations for a given day, Zenith and Azimuth Angle.

The Zenith can be calculated using the following formula:

$$\cos(\Theta) = \sin(\phi)\sin(\delta) + \cos(\delta)\cos(\phi)\cos(h)$$

Where Θ = *Zenith Angle,*

ϕ = *Latitude,*

δ = *Solar Declination,*

h = *Relative Longitude*

The Azimuth can be calculated using the following formula

$$\cos(\phi_s) = \sin(\delta)\cos(\phi) - \cos(\delta)\sin(\phi)\cos(h)/\sin(\Theta_S)$$

Where ϕ_s = *Azimuth Angle,*

Θ_S = *Zenith Angle,*

δ = *Solar Declination,*

ϕ = *Latitude,*

h = Relative Longitude

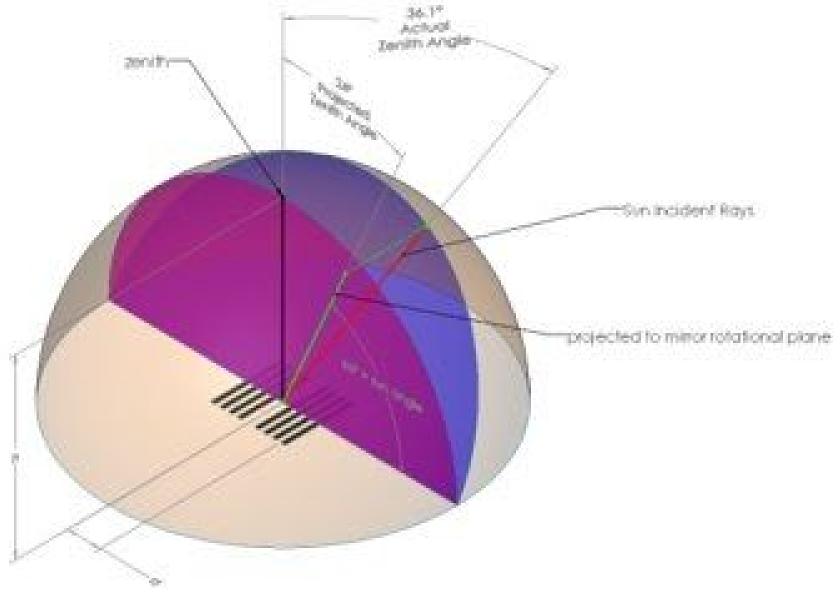


Figure 5: Zenith and Sun Angle

The team calculated the Zenith and Azimuth angle with the help of a website (SunCalc sun position, 2020). After getting the angles, the team created a Matlab script that calculates the angles and finds the corresponding strip angle for each lense in the fresnel solar thermal system. Further on the team simulate the motion to verify the calculations.

3.2 Setting The Dimensions of the Device

In order to calculate and suggest the dimensions of the system, the team used the following equation below:

$$y = \frac{x^2}{C}$$

This equation solves for the double height of the focal point in a parabolic curve. Where, $C = 2f$ and f is the focal point height. The team thought of the fresnel system as a simplified parabolic curve, therefore the team decided to choose a C value that is high, which provides a wider width for the system. After meticulous trial and error the team found that the value of *C* was 40.

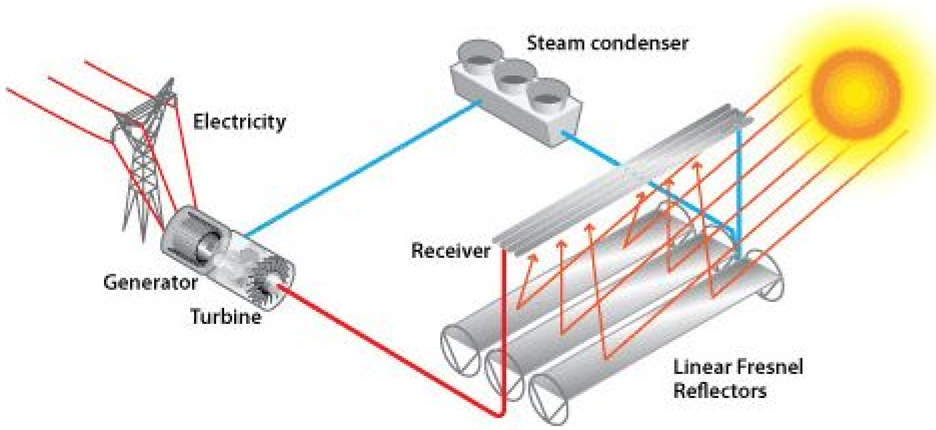


Figure 6: Design of a fresnel solar reflector

3.3 Calculating the Mechanism and Drive Chain

After calculating the sun angle for each strip, the team found that the difference in the change of the angle for each strip was close to 0 since they are all in close proximity to each other following the sun's orbit. The team experimented with different mechanisms for the drive chain exploring options with having a motor for each strip to follow the sun, a chain-and-sprocket mechanism that has one motor to control all strips, and a rack-and-pinion. The team compared the advantages and disadvantages of each mechanism, and decided on the most fit option.

3.4 Minimizing the Space of the System and Overshadowing

To minimize the space of the Fresnel Solar Thermal System, the team did various simulations in SolidWorks, making sure that the reflectors do not overshadow one another in the new confined space. The simulations were done using the SolidWorks model that the team developed with trying different placements of the reflectors and finding the optimal position for the Fresnel Solar Power Plant.

3.5 Designing a Heat Collector

In order to increase the efficiency of our system, a design that ensures maximum conversion of Solar Energy to thermal energy is essential. The team studied multiple ideas on the shape of the thermal collector that allowed as much reflected solar rays as possible to be absorbed. This design had to be suitable and efficient with the thermo-fluid considerations and sizing of pipe.

CHAPTER 4: FINDINGS

The FTSPS is an efficient Solar Thermal Power system and the team found the following results and observations while improving on the design of FTSPS. The results and observations are as follows:

4.1 Sun Angle Calculations

After calculating the zenith angle through the SunCalc sun position- und sun phases calculator website, the team compared the website results to MATLAB results which were produced with a built-in function called solarPosition. To produce the MATLAB results the team assumed the longitude and latitude to be of Worcester, MA.

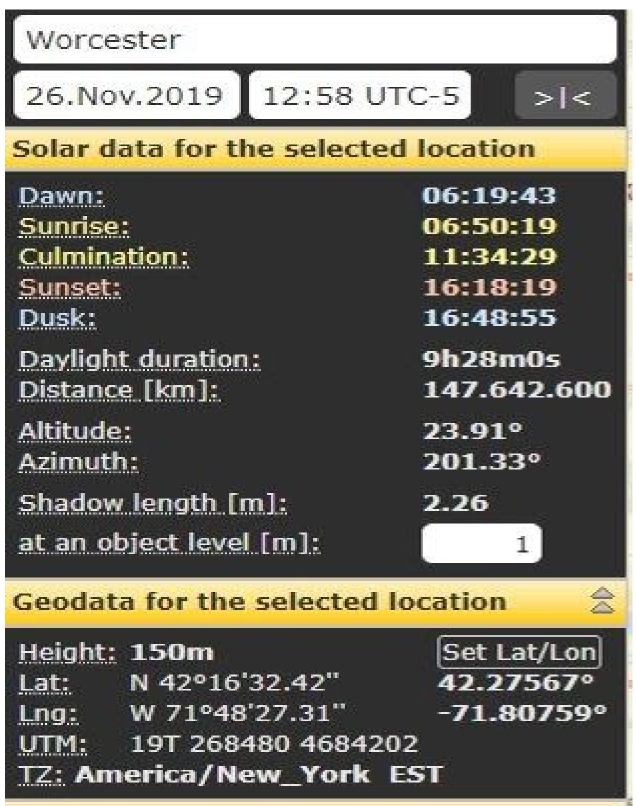


Figure 7: Website Calculations

Furthermore, after finding the sun angles the team calculated the strip angle for each strip in order to reflect the sun rays to the focal point through a MATLAB script (Appendix A). The figures below show the formula of the geometrical proof of the formula used in MATLAB.

$$\Psi = mirror\ angle, measured\ from + x$$
$$\zeta = zenith\ (measured\ from + z)$$
$$\alpha = azimuth\ (cw\ from\ north / + y)$$
$$\theta_1 = sun\ angle\ used\ in\ mirror\ angle\ calculations$$
$$\theta_2 = angle\ from\ mirror\ to\ collector$$

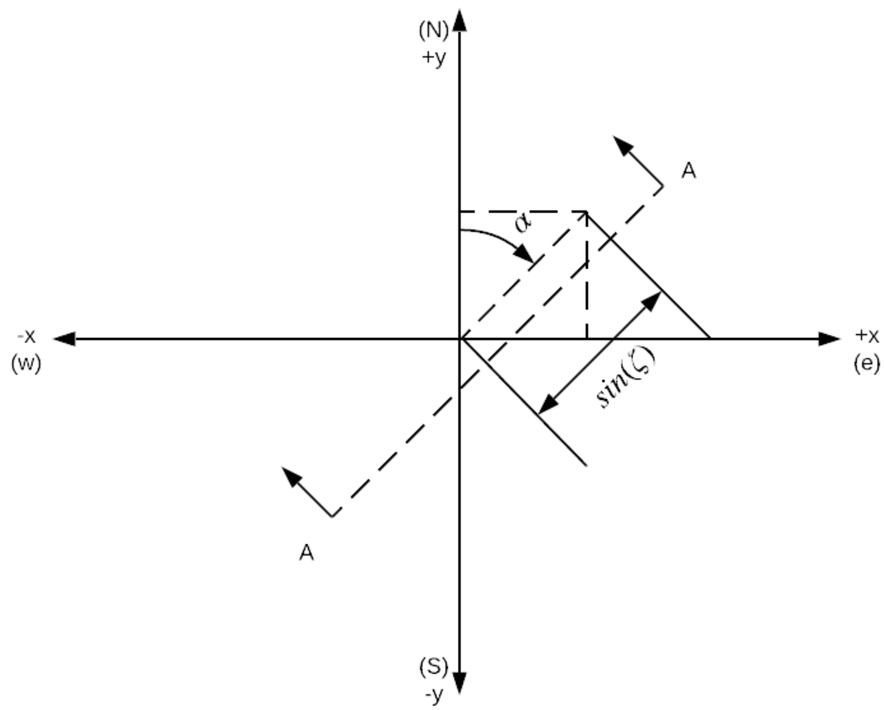


Figure 8: Geometrical proof for strip angle formula (Step 1)

$\Psi = mirror\ angle, measured\ from + x$

$\zeta = zenith\ (measured\ from + z)$

$\alpha = azimuth\ (cw\ from\ north\ / + y)$

$\theta_1 = sun\ angle\ used\ in\ mirror\ angle\ calculations$

$\theta_2 = angle\ from\ mirror\ to\ collector$

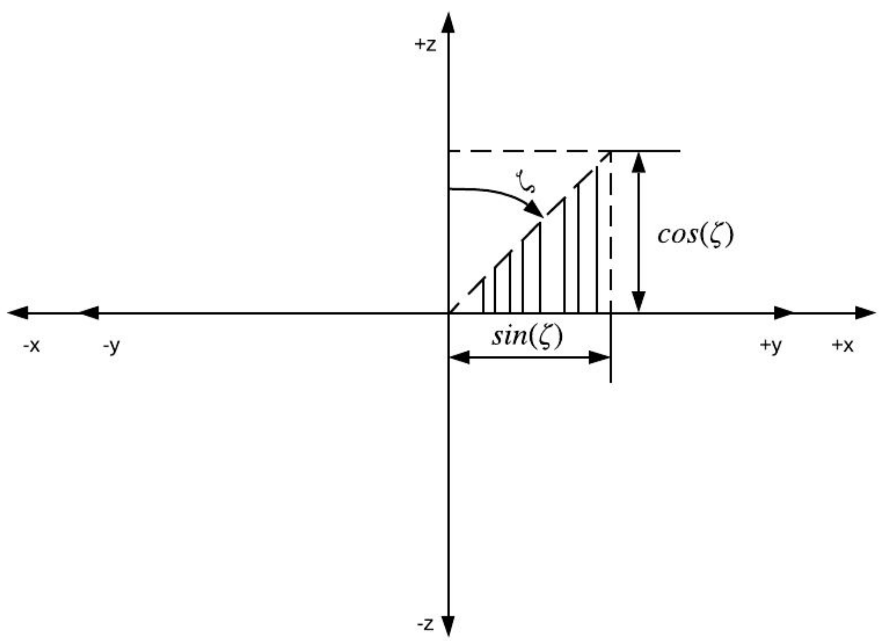


Figure 9: Geometrical proof for strip angle formula (Step 2)

$$\Psi = mirror\ angle, measured\ from + x$$

$$\zeta = zenith\ (measured\ from + z)$$

$$\alpha = azimuth\ (cw\ from\ north / + y)$$

$$\theta_1 = sun\ angle\ used\ in\ mirror\ angle\ calculations$$

$$\theta_2 = angle\ from\ mirror\ to\ collector$$

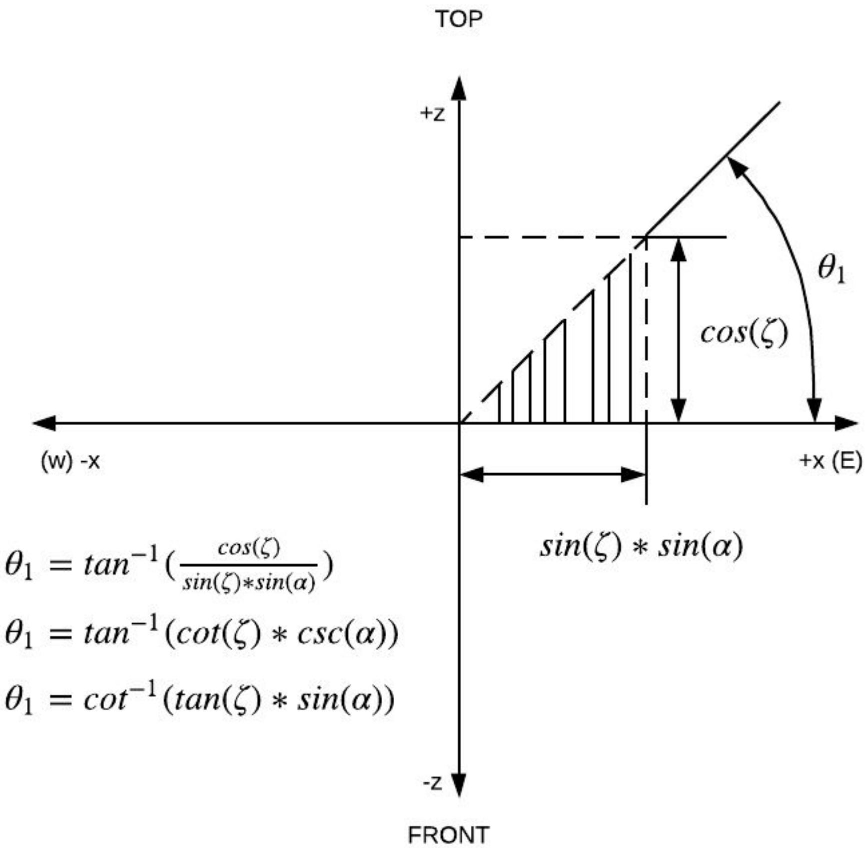


Figure 10: Geometrical proof for strip angle formula (Step 3)

$\Psi = mirror\ angle, measured\ from + x$

$\zeta = zenith\ (measured\ from + z)$

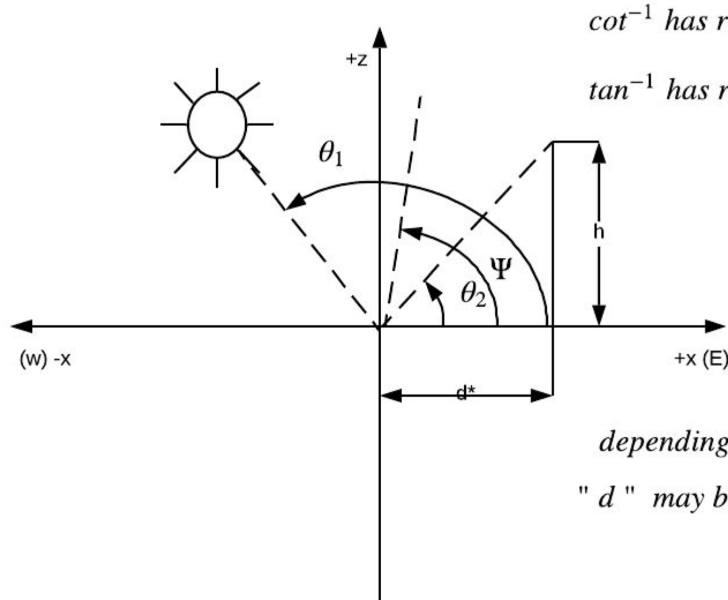
$\alpha = azimuth\ (cw\ from\ north\ / + y)$

$\theta_1 = sun\ angle\ used\ in\ mirror\ angle\ calculations$

$\theta_2 = angle\ from\ mirror\ to\ collector$

$cot^{-1}\ has\ range\ \theta\ to\ \pi$

$tan^{-1}\ has\ range\ \frac{-\pi}{2}\ to\ \frac{\pi}{2}$



$depending\ on\ the\ coordinates$

$"\ d\ "\ may\ be\ replaced\ by\ "\ -d\ "$

$$\theta_2 = cot^{-1}(\frac{d}{h})$$

$$\Psi = \frac{\theta_1 + \theta_2}{2}$$

$$\Psi = \frac{1}{2} * (cot^{-1}(\frac{d}{h}) + cot^{-1}(tan(\zeta) * sin(\alpha)))$$

Figure 11: Geometrical proof for strip angle formula (Step 4)

In the above proof, we show that both Zenith and Azimuth are necessary in order to locate the sun. The team also proved that the horizontal distance of the strip to the perpendicular line that intersects through the focal point, as well as the height of the focal point, is necessary to calculate the strip angle.

To confirm the results the team simulated the motion in SolidWorks shown in Figure 14 in Section 4.4.

4.2 SolidWorks Modeling and Final Mechanism

With the strip calculations and the SolidWorks simulation, the team found that the direction and the rate of change among all strips are consistent. Therefore, the team decided that the gear ratio should be one-to-one with the assumption being that the initial position for the strips is correctly placed with the relevant solar position - which can be calculated through the MATLAB code in Appendix A.

Since the gear ratio is one-to-one the team decided to compare and analyze different mechanisms. With meticulous research and SolidWorks simulations, the decided the criteria of the mechanism to be as follows:

i) The Mechanism should have motion-sensitive stability in order to make sure that the motion can be very small due to the slow rate of change of the sun position,

ii) The Mechanism should be cost-efficient and durable to extreme weather conditions.

The team decided to use the rack and pinion mechanism since it satisfies the above conditions. Figure 12 shows the rack and pinion mechanism that the team used. This mechanism can hold gears and makes its motion consistent and steady.

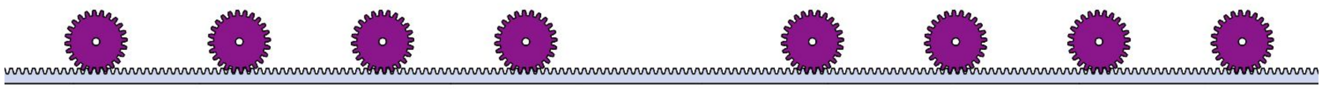


Figure 12: Rack and Pinion Mechanism

4.3 Design Decisions

In order to increase efficiency and optimize the design, the team made several crucial decisions:

a) Decreasing overshadowing by using the Alternating Mirror Arrangement (AMA). Figure 10 shows a preliminary concept of the idea.
b) Increasing the efficiency of the reflected sun rays by decreasing the spaces between the mirror strips as much as possible.
c) Designing an Omega-like shaped solar collector on top of the focal point pipe (Figure 16) in order to increase the efficiency of collecting the reflected sun rays.

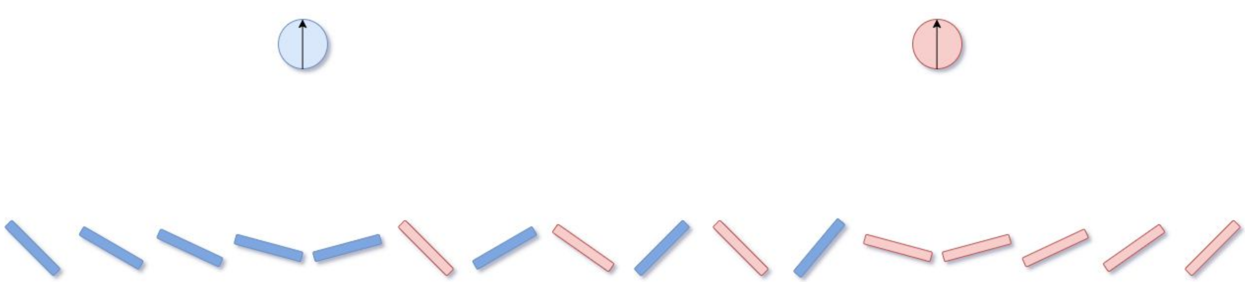


Figure 13: Concept of Alternating Mirror Arrangement (AMA).

The AMA is applied only to the shared mirror strips in between the two focal points. Whereas, the mirror strips on the sides of the system still reflect the sun rays to the closest focal point. Figure 10 shows that the blue strips reflect the sun rays to the blue solar collector and the red strips reflect the sun rays to the red solar collector. In this figure, the sun is supposed to be at noon as the arrows demonstrate.

4.4 Final Design

This section shows multiple of the results. Figure 14,15,16, and 17 show the preliminary assembly of the model before applying the AMA method. The line on the focal point shown in the figures points to the sun position. In the SolidWorks Model, one can rotate the focal point imitating the motion of the sun. As a result, the strips would move accordingly. Figure 14, 15 show sunrise position, and Figure 16,17 show sunset position.

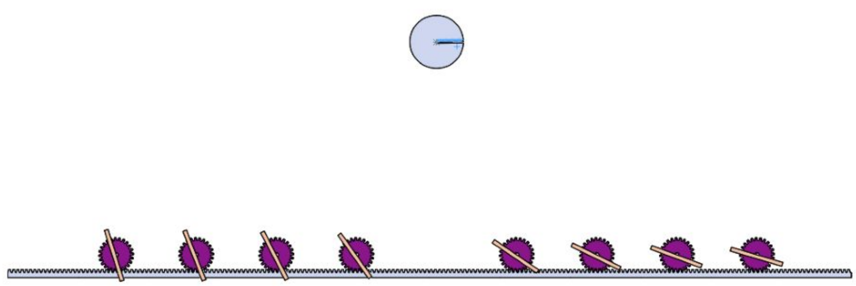


Figure 14: SolidWorks Design for the strips (Front View) Sunrise Position

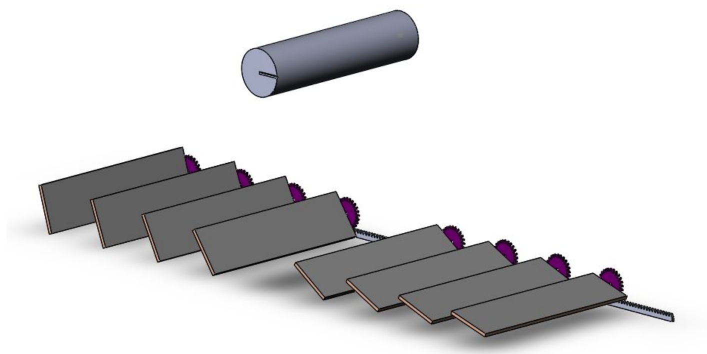


Figure 15: SolidWorks Design for the strips (Triometric View) Sunrise Position

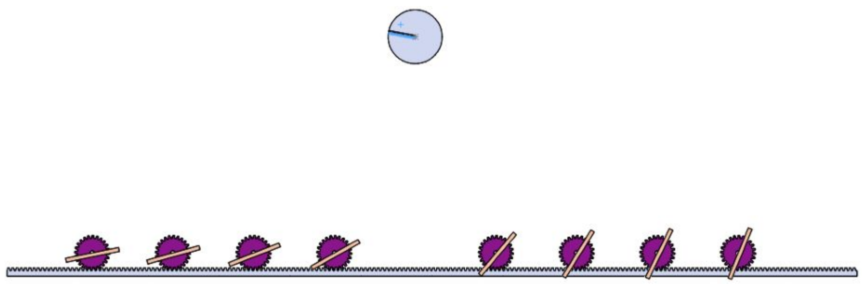


Figure 16: SolidWorks Design for the strips (Front View) Sunset Position

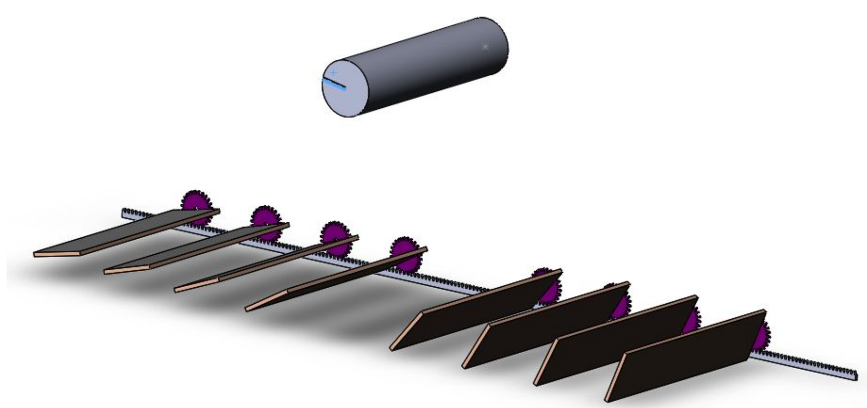


Figure 17: SolidWorks Design for the strips (Triometric View) Sunset Position

Figure 18 shows the mirror strip holder that the team used in the prototype, this part was 3D printed. Therefore, the upper part of the mirror holder was elastic enough to fix the mirror while rotating. The back of the part serves as a shaft holder which is connected to the gears driven by the rack and pinion.

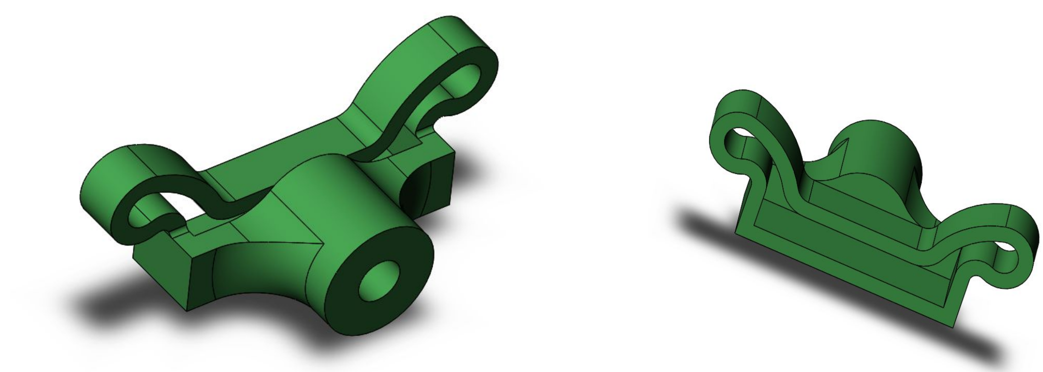


Figure 18: Prototype Mirror Strip Holder (Back Isometric View on the Left and Front Isometric View on the Right)

4.5 Heat Collector Design

The team designed the heat collector to be omega-shaped to, as seen in Figure 19 below, collect most of the reflected rays from the sun. This was achieved by keeping the opening gap double the width of each mirror strip to insure collecting all the solar radiation while keeping the pipe diameter less than the width of the mirror strip. This design decision and dimensions increased the overall efficiency of the system.

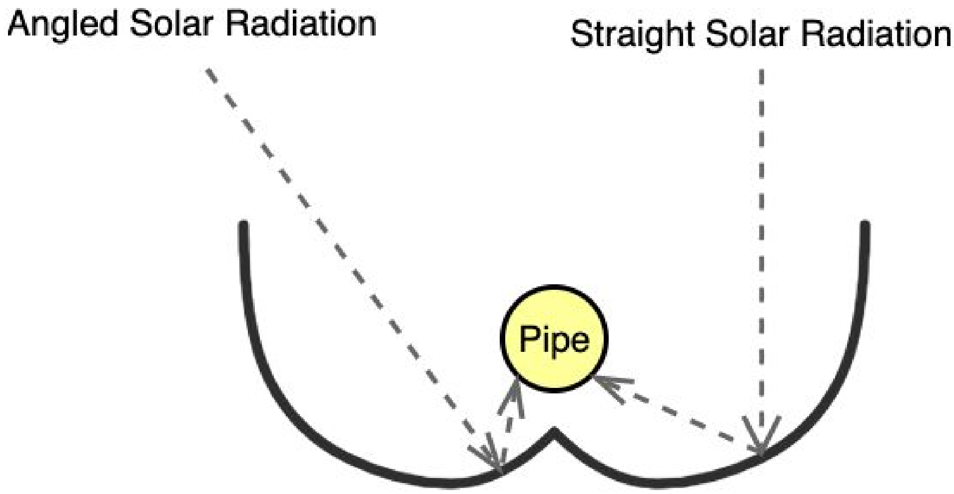


Figure 19: Heat Collector Design.

CHAPTER 5: CONCLUSION

The team has achieved the desired goals and requirements of the project that were reducing the manufacturing and maintenance cost and improving the overall efficiency of the FTSPS. The team started by solving the problem theoretically first and then moved to the design aspects.

The team utilized simple manufacturing processes to keep production costs low and to achieve resistance to the elements. The team came up with a design that uses cost-efficient materials and in return reduces the production and maintenance cost. In addition, the design is wind and dust resistant..

The team's design decreases overshadowing and reduces the wasted energy due to having bigger strips and larger spacing. Furthemore, the mechanism that rotates the strips is reliable, durable and energy efficient.

Unfortunately due to the pandemic of COVID-19, the team was not able to build and test the final real-life prototype of the project. However, the team provided scientific and the engineering concepts to back-up our results SolidWorks Simulations, as well as mathematical proofs. All in all, the team completed the requirements and fulfilled the goals of the project.

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APPENDIX A: MATLAB CODE

```
function [angles,projection] = solarPosition(datetime,latitude,longitude, ...
                              time_zone,rotation,dst)
%SOLARPOSITION Calculate solar position using most basic algorithm
%   This is the most basic algorithm. It is documented in Seinfeld &
%   Pandis, Duffie & Beckman and Wikipedia.
%
%                                    [ANGLES,PROJECTION]                         =
SOLARPOSITION(DATE,TIME,LATITUDE,LONGITUDE,TIME_ZONE)
% returns ZENITH & AZIMUTH for all DATE & TIME pairs at LATITUDE, LONGITUDE.
% ANGLES = [ZENITH,AZIMUTH] and PROJECTION = [PHI_X, PHI_Y]
% PHI_X is projection on x-z plane & PHI_Y is projection on y-z plane.
% DATETIME can be string, vector [YEAR, MONTH, DAY, HOURS, MINUTES,
SECONDS],
%   cellstring or matrix N x [YEAR, MONTH, DAY, HOURS, MINUTES, SECONDS] for N
%   times.
% LATITUDE [degrees] and LONGITUDE [degrees] are the coordinates of the site.
% TIME_ZONE [hours] of the site.
% ROTATION [degrees] clockwise rotation of system relative to north.
% DST [logical] flag for daylight savings time, typ. from March to November
%   in the northern hemisphere.
%
```

```
%% datetime
if iscellstr(datetime) || ~isvector(datetime)
    datetime = datenum(datetime); % [days] dates & times
else
    datetime = datetime(:); % convert datenums to row
end
date = floor(datetime); % [days]
[year,~,~] = datevec(date);
time = datetime - date; % [days]
%% constants
toRadians = @(x)x*pi/180; % convert degrees to radians
toDegrees = @(x)x*180/pi; % convert radians to degrees
%% Equation of time
d_n = mod(date-datenum(year,1,1)+1,365); % day number
B = 2*pi*(d_n-81)/365; % ET parameter
ET = 9.87*sin(2*B)-7.53*cos(B)-1.5*sin(B); % [minutes] equation of time
% approximate solar time
solarTime = ((time*24-double(dst))*60+4*(longitude-time_zone*15)+ET)/60/24;
latitude_rad = toRadians(latitude); % [radians] latitude
rotation_rad = toRadians(rotation); % [radians] field rotation
t_h = (solarTime*24-12)*15; % [degrees] hour angle
```

```
t_h_rad = toRadians(t_h); % [radians]

delta = -23.45 * cos(2*pi*(d_n+10)/365); % [degrees] declination

delta_rad = toRadians(delta); % [radians]

theta_rad = acos(sin(latitude_rad)*sin(delta_rad)+ ...

    cos(latitude_rad)*cos(delta_rad).*cos(t_h_rad)); % [radians] zenith

theta = toDegrees(theta_rad); % [degrees] zenith

elevation = 90 - theta; % elevation

day = elevation>0; % day or night?

cos_phi = (cos(theta_rad)*sin(latitude_rad)- ...

    sin(delta_rad))./(sin(theta_rad)*cos(latitude_rad)); % cosine(azimuth)

% azimuth [0, 180], absolute value measured from due south, so east = west = 90,

% south = 0, north = 180

phi_south = acos(min(1,max(-1,cos_phi)));

% azimuth [0, 360], measured clockwise from due north, so east = 90,

% south = 180, and west = 270 degrees

phi_rad = NaN(size(phi_south)); % night azimuth is NaN

% shift from ATAN to ATAN2, IE: use domain from 0 to 360 degrees instead of

% from -180 to 180

phi_rad(day) = pi + sign(t_h(day)).*phi_south(day); % Shift domain to 0-360 deg

% projection of sun angle on x-z plane, measured from z-direction (up)

phi_x = toDegrees(atan2(sin(phi_rad-rotation_rad).*sin(theta_rad), ...

    cos(theta_rad))); % [degrees]
```

```
% projection of sun angle on y-z plane, measured from z-direction (up)
phi_y = toDegrees(atan2(cos(phi_rad-rotation_rad).*sin(theta_rad), ...
    cos(theta_rad))); % [degrees]
phi = toDegrees(phi_rad); % [degrees] azimuth
angles = [theta, phi]; % [degrees] zenith, azimuth
projection = [phi_x,phi_y]; % [degrees] x-z plane, y-z plane
end



function theta1=stripangle(zenith, azimuith, Hight, distance)


    %x= atan2(distance,Hight);
    %y= atan2(sin(zenith)*sin(azimuith),cos(zenith));
  x=acotd(distance/Hight);
      y=acotd(tan(zenith)*sin(azimuith)); %


  if x>0
     %
    theta1=0.5*(abs(x)+y);
  elseif x<0


      theta1=0.5*(abs(x)+y);
```

```
  theta1= 180-theta1;
  end
  end

%%%%FINAL Code
datetimes= {'11/26/2019 11:35 AM'};%'11/26/2019 12:05 PM';'11/26/2019 01:05 PM';'11/26/2019 2:01 PM'};%;'11/26/2019 12:05 PM';'11/26/2019 01:05 PM';'11/26/2019 2:01 PM'};
lat=42.27567;
long=-71.80759;
TZ=-5;
rot=0;
DST=false;

ERRORZenith="3 Min and 23 second";

ERRORaz="1 Min and 39 second";

a=solarPosition(datetimes,lat,long,TZ,rot,DST);
zen= a(1)*pi/180;
az= a(2)*pi/180;
```

```
disp(datetimes);
for i=0:19
    d=2*(i)-19.5;
    if d>0
        d=d+1;
    end

   t1=stripangle(zen,az,20,d);
   %pause(0.5)
   disp(d)
   %pause(0.1)
disp(t1)
end
```

APPENDIX B: MATLAB Output

'11/26/2019 11:35 AM'

Strip Position
-19.5000

Strip Angle
112.2221

Strip Position
-17.5000

Strip Angle
110.6777

Strip Position
-15.5000

Strip Angle
108.9726

Strip Position
-13.5000

Strip Angle
107.0944

Strip Position
-11.5000

Strip Angle
105.0342

Strip Position
-9.5000

Strip Angle
102.7886

Strip Position
-7.5000

Strip Angle

100.3627

Strip Position
-5.5000

Strip Angle
97.7729

Strip Position
-3.5000

Strip Angle
95.0478

Strip Position
-1.5000

Strip Angle
92.2293

Strip Position
1.5000

Strip Angle
87.7707

Strip Position
3.5000

Strip Angle
84.9522

Strip Position
5.5000

Strip Angle
82.2271

Strip Position
7.5000

Strip Angle
79.6373

Strip Position

9.5000

Strip Angle
77.2114

Strip Position
11.5000

Strip Angle
74.9658

Strip Position
13.5000

Strip Angle
72.9056

Strip Position
15.5000

Strip Angle
71.0274

Strip Position
17.5000

Strip Angle
69.3223

Strip Position
19.5000

Strip Angle
67.7779