THE VALUATION OF CREDIT
DEFAULT SWAPS

by

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                                            Mathematical Sciences

Table of Contents

ACKNOWLEDGMENTS

I dedicate this work to my parents and my brother- may their souls rest in peace-, and to all those who, directly or indirectly, have helped me and supported me all these years.

I would like to thank my advisor for all the patience and guidance, my PhD office mates, all my teachers, my classmates for all the support and advice.

INTRODUCTION

The credit derivatives market has known an incredible development since its advent in the 1990's. Today there is a plethora of credit derivatives going from the simplest ones, credit default swaps (CDS), to more complex ones such as synthetic single-tranche collateralized debt obligations. Valuing this rich panel of products involves modeling credit risk. For this purpose, two main approaches have been explored and proposed since 1976. The first approach is the Structural approach, first proposed by Merton in 1976, following the work of Black-Scholes for pricing stock options. This approach relies in the capital structure of a firm to model its probability of default. The other approach is called the Reduced-form approach or the hazard rate approach. It is pioneered by Duffie, Lando, Jarrow among others. The main thesis in this approach is that default should be modeled as a jump process.

The objective of this work is to value Asset-backed Credit default swaps using the hazard rate approach. The first section of the first chapter deals with the formal modeling of credit risk and the second section with managing credit risk. Then in chapter 2, section 1 is dedicated to corporate credit defaults swaps and section 2 to asset-backed CDS. Section 3 looks at the use of credit defaults swaps as a risk management tool. Section 4 then deals with the valuation of asset-backed CDS. The third chapter consists of the description of the methods used in this work for valuing asset-backed credit defaults swaps as well as the result of the implementation of a numerical example. We then close with a conclusion on the applicability of the hazard rate approach to price asset-backed credit default swaps.

CREDIT RISK

Credit Risk or Default Risk is defined as risk due to uncertainty in a counterparty's or obligor's ability to meet its obligations. Credit Risk comes in various forms because there are many types of counterparties and obligations. We can characterize Credit Risk by three elements: *credit exposure*, *default probability* and *recovery rate*.

Credit exposure refers to the magnitude of loss in the value of the outstanding obligation when default occurs. Default probability represents the likelihood that the counterparty will default on its obligation either over the life of the obligation or over some specified horizon, such as a year. In the event of default, recovery rate is the fraction of the exposure that may be recovered through bankruptcy proceedings or some other form of settlement.

Credit Risk came out as the key risk management challenge in the late 1980s and one way to hedge Credit Risk is through the use of *Credit Derivatives* and *Securitization* with loans and bonds as collateral assets.

In the effort to account for Credit Risk, it is important to be able to model it. Modeling Credit Risk is done through two major approaches, the *Structural Approach* and the *Reduced Form Approach*, also known as the *Intensity-based* or *Hazard Rate Approach*. Both approaches will be covered in this work. In this section, we will delve into the details of Credit Risk modeling and management.

1.1. MODELING CREDIT RISK

1.1.1. The structural approach

The structural framework is based on the idea of linking the credit quality of a company to its financial and economic conditions. Therefore defaults are resultant of the firm's capital structure. There are two main structural models, Merton's model and the Black-Cox or First Passage Model. In the following, we put ourselves in a continuous trading economy. We assume that a money market account and default free zero coupon bonds are traded in this economy. Non arbitrage is assumed in the market of these traded securities, ensuring the existence of a risk-neutral probability measure [5]. Time T, T>0, is the final date of the model.

Merton's Model

This approach was introduced in 1974 by Merton [12] and is considered as the first structural model. Merton applied the idea of Black-Scholes [3] for option pricing to modeling a firm's liability. Let $\left(\Omega,\left(\mathcal{F}_t\right)_{t\geq0},P\right)$ be a probability space and $\left(W_t^1\right)_{t\geq0}$ is a Wiener process $\mathcal{F}_t$ –adapted. In this model, the asset value $V=(V_t)_{t\geq0}$ of a firm is modeled as a geometric Brownian motion. In other words, we can write the change in the asset value dV_t as:

$$\begin{cases} dV_t = rV_t dt + \sigma_V V_t dW_t \\ V_0 = v \end{cases} \tag{1}$$

where r is the risk-free interest rate and $, \sigma_V$ is the volatility of V.

Moreover this model assumes that the firm's liability D can be modeled as an outstanding T-maturity zero-coupon bond. The company then defaults if at time of servicing the debt, the asset value is less that its outstanding liability, i.e. $V_T < D$. Therefore the firm's equity can be seen as a European call option with strike price D on the asset value and expiration time T. This is a direct result of the assumption that default can only occur at maturity. Thus we can write that

$$E_T = max\left(V_T - D,0\right) \tag{2}$$

Other assumptions include the absence of:

- r Transaction costs,
- r Bankruptcy costs,
- r Taxes,
- r Or problems of assets indivisibilities [7].

Since the asset value of the firm is not typically traded, market prices cannot be observed for it. Thus we cannot compute σ_V from market data. Even tough balance sheet information is available, due to the coarse frequency (quarterly or annually); this information might not be helpful in estimating σ_V on a more frequent basis as it is needed for hedging purposes. To go around this problem, we use the known relationship between equity E_t and asset value V_t through the fundamental theorem of accounting

$$V_T = D + E_T \tag{3}$$

The advantage of using this relationship is that equity is traded and market prices are readily available for it.

We also assume the following model for equity:

$$dE_t = rE_t dt + \sigma_E E_t dW_t \tag{4}$$

where σ_E is the volatility of E_t.

With the assumption that the firm's equity is a call option on its asset value, we use Black-Sholes's option pricing theory to express equity as follows

$$E_t = f(t, V_t) \tag{5}$$

where

$$f(t, x) = xN(d_1) - De^{-r(T-t)}N(d_2) \tag{6}$$

$$d_1 = \frac{\ln(x/D) + (r + \frac{1}{2}\sigma_V^2)(T-t)}{\sigma_V \sqrt{T-t}}, \quad d_2 = d_1 - \sigma\sqrt{T-t}$$

Using Ito's lemma, we have:

$$dE_t = \left(\frac{\partial f(t,V_t)}{\partial t} + \frac{\partial f(t,V_t)}{\partial x} rV_t + \frac{1}{2} \frac{\partial^2 f(t,V_t)}{\partial x^2} \left(\sigma_V V_t \right)^2 \right) dt + \frac{\partial f(t,V_t)}{\partial x} \sigma_V V_t dW_t \tag{7}$$

Thus identifying (7) with (4) and using (6), we have

$$\sigma_E E_t = \frac{\partial f(t,V_t)}{\partial x} \sigma_V V_t = N(d_1)\sigma_V V_t$$

Thus we can now express σ_V.

$$\sigma_V = \frac{\sigma_E E_t}{N(d_d)V_t} \qquad (8)$$

Where $N(x) = \frac{1}{2\pi} \int_{-\infty}^{x} e^{-\frac{z^2}{2}} dz$ is the cumulative distribution function of the standard normal random variable.

We then use $\hat{E}_t$ the observed market price of equity, and $\hat{V}_t$ obtained through averaging a time series of asset values from the available balance sheets to estimate σ_V as

$$\hat{\sigma}_V \approx \frac{\sigma_E \hat{E}_t}{N(\hat{d}_1)\hat{V}_t} \qquad (9)$$

Where

$$\hat{d}_1 = \frac{\ln(\hat{V}_t / D) + (r + \frac{1}{2}\hat{\sigma}_V^2)(T-t)}{\hat{\sigma}_V \sqrt{T-t}} .$$

Then, we can solve numerically for $\hat{\sigma}_V$ \qquad (10)

The only other parameter needed in the approximation is the liability D. To estimate D, the face value of the zero-coupon T-maturity bond used to model the firm's liability, Sundaram [21] points out that "default tends to occur in practice when the market value of the firm's assets drops below a critical point that typically lies below the book value of all liabilities, but above the book value of

short-term liabilities." A good approximate for D is a value in between these to values.

To obtain the maturity of the zero coupon bond, "we can either choose it to represent the maturity structure of the debt, for example as the Macaulay duration of all the liabilities, or simply as a required time horizon (for example, in case we are pricing a credit derivative with some specific maturity)" [7]

While the assumptions make it easy to implement this model, they are clearly unrealistic. All improvements of this model are attempts to adopt more realistic assumptions. Other concerns are the assumptions of a constant and flat term structure, the use of a simple zero coupon bond to model the firm's debt and the predictability of default occurring only at maturity T, resulting in very low short-term spreads.

Once we have estimated σ_V, the parameters of the model in (1) are all specified.

Next there are at least two quantities of interest for modeling Credit Risk:

  r The default probabilities.

  r The time of default.

The next section covers these two concepts with more details.

The First Passage Model: Black-Cox Model

Rather than making the assumption that default occurs only at maturity of the zero coupon bond, this model assumes a threshold in the form of a stochastic process $D = (d_t)_{t \geq 0}$ [6]. The firm will default at time t once its asset value V_t hits

d_t. In this model, the time of default is given as the *first passage time* of the value

process $V = (V_t)_{t \geq 0}$ to either a deterministic or a random barrier [7].

Default can then occur at any point of time and the main issue becomes finding

the correct threshold process d_t. As in the Merton Model, we also consider the

following model for the asset value V_t

$$dV_t = rV_t dt + \sigma_V V_t dW_t$$

Where σ_V is estimated using (10).

The random variable *time of default* is then defined by

$$\tau(t) = \inf \{ s \geq t \mid V_s \leq d_s \} \tag{11}$$

This represents the first default after time t.

We can now write the default probabilities explicitly using the assumptions on V_t

$$P(\tau \leq T) = P\left(\min_{s \leq T} V_s < d_s \right) = P\left(\min_{s \leq T} V_0 e^{rs + \sigma_V W_s^1} < d_s \right) = P\left(\min_{s \leq T} e^{rs + \sigma_V W_s^1} < \frac{d_s}{V_0} \right)$$

$$= P\left(\min_{s \leq T} \left(rs + \sigma_V W_s^1 \right) < \ln \frac{d_s}{V_0} \right)$$

It can be shown that the random variable $G_t = \min_{s \leq t} \left(rs + \sigma_V W_s^1 \right)$ has an inverse

Gaussian[1] distribution. Thus, assuming that $D = (d_t)_{t \geq 0}$ is such that $d_t = d$

$\forall t \geq 0$,

$$P\left(\tau \leq T\right)=1-N\left(\frac{rT-\ln\left(d/V_0\right)}{\sigma_V\sqrt{T}}\right)+e^{\frac{2r\ln\left(d/V_0\right)}{\sigma_V^2}}N\left(\frac{\ln\left(d/V_0\right)+rT}{\sigma_V\sqrt{T}}\right) \tag{12}$$

The other improvements of the Black-Cox model from Merton's model relate to the inclusion of transaction costs, taxes, stochastic interest rates, debt subordination, jump in V_t among others. These improvements however introduce more analytical complexity into the problem. Certain authors point that it is more suitable to model the ratio of the asset value over the threshold $\frac{V_t}{d_t}$.

This would help to go around the problem of having to find an explicit expression for d_t.

1.1.2. The reduced-form approach

The main idea behind the reduced-form approach- also known as the hazard rate approach- is that the time of default of a firm can be modeled as a jump process, specifically as the first jump of a Poisson process. Market data are then used to determine the parameters associated with the default intensity. This approach was introduced by various authors among whom Jarrow and Turnbull [14], Madan & Unal [16], Duffie & Singleton [5] and Hughston & Turnbull [9].

The intensity-based model

Unlike the structural models, the hazard rate model does not use the capital structure of the firm to find the time of default but it rather takes it as the first jump in a Poisson process. To better understand this approach, we explore the

"time to default" process. As in section 1.1.1, we let $\left(\Omega,(\mathfrak{F}_t)_{t\geq0},\mathbb{P}\right)$ be a probability space and let τ denote the time to default or the waiting time for the happening of default. Let $S(t)$ be the probability of survival after time t. In other words,

$$S(t) = P(\tau > t) \tag{13}$$

Thus

$$F(t) = 1 - S(t) = P(\tau \leq t) \tag{14}$$

is the probability of default before or at time t.

Since we assume no default at time 0, we have $S(0) = 1$.

The question then is to find an appropriate distribution that can be used to model the default probabilities.

For that purpose, the notion of *hazard function* λ_t is introduced [5, 14].

This function is defined as:

$$\lambda_t = \lim_{h \to 0} \frac{P\left[(t < \tau < t+h)/\tau > t\right]}{h} \tag{15}$$

Then by definition of conditional probability

$$P\left[(t < \tau < t+h)/\tau > t\right] = \frac{F(t+h) - F(t)}{S(t)} = \frac{-\left[S(t+h) - S(t)\right]}{S(t)} \tag{16}$$

Therefore since $S(0) = 1$ it follows that:

$$\lambda_t = -\frac{S'(t)}{S(t)} \quad \text{and hence} \quad S(t) = e^{-\int_0^t \lambda_s ds} . \tag{17}$$

Observe that if we assume λ_t to be constant and equal to λ, we have

$$S(t) = e^{-\lambda t} \tag{18}$$

Therefore, under these assumptions, τ has an exponential distribution with parameter $\lambda > 0$.

Knowing this, it seems plausible [2] to assume that there is a sequence $(\tau)_{i \geq 0}$ of independent random variables, exponentially distributed with parameter $\lambda > 0$ such that: τ_1 is the time to the 1st default, τ_2 is the time between the 1st and 2nd default,..., τ_n is the time between the $(n-1)^{th}$ and n^{th} defaults and so on.

The sequence $(\tau_i)_{i \geq 0}$ represents an infinite sequence of random variables on the probability space. The variable $\mathbf{M}_n = \tau_1 + \tau_2 + ... + \tau_n$ characterizes the time to the nth default with $\mathbf{M}_0 = 0$. With the assumptions that two defaults cannot happen simultaneously and that only a finite number of defaults can happen in each interval time, $(\mathbf{M}_n)_{n \geq 0}$ is an increasing sequence that converges to infinity. Thus

$$0 = \mathbf{M}_0(\omega) < \mathbf{M}_1(\omega) < \mathbf{M}_2(\omega) < ... \text{ and } \sup_n \mathbf{M}_n(\omega) = \infty.$$

We then look at the process N_t defined as the number of defaults in the time interval $[0,t]$. In other words,

$$N_t = \max[n : \mathbf{M}_n \leq t] \tag{20}$$

Note that because of the nature of the sequence $(\mathbf{M}_n)_{n \geq 0}$, $N_t = 0$ if $t < \mathbf{M}_1 = \tau_1$.

In particular

$$N_0 = 0.$$

We deduce that the number of defaults in the time interval $(s,t]$ is the increment $N_t - N_s$.

From (12), it can easily be shown that

$$[N_t \geq n] = [M_n \leq t]$$

and deduce that

$$[N_t = n] = [M_n \leq t < M_{n+1}] \tag{21}$$

It can be shown [2] that N_t is a homogenous Poisson process with intensity λ, i.e.:

$$P[N_t = n] = \frac{1}{n!}(tl)^n e^{-(tl)} \quad n = 0,1,... \tag{22}$$

and the increments are independent and have a Poisson distribution:

$$P[N_t - N_s = n] = \frac{1}{n!}(t-s)^n \lambda^n e^{-(t-s)\lambda} \quad n = 0,1,... \quad 0 \leq s \leq t \tag{23}$$

This model can be generalized by letting λ_t be a stochastic process. For example, Duffie & al. [5] in their work, modeled λ_t as a diffusion process of the form

$$d\lambda_t = \mu\lambda_t dt + \sigma\lambda_t dW_t \tag{24}$$

where W_t is a Brownian motion, μ and σ are respectively the mean and volatility of λ_t. The Poisson process is then called a Cox process.

1.2. MANAGING CREDIT RISK

Let's introduce some common terms that will be used in the remainder of this thesis.

Credit Derivative is a derivative instrument designed to transfer credit risk from one party to another. A *Credit Event* is an event such as a debt default or bankruptcy that will affect the payoff on a credit derivative A *Reference Asset* is an asset, such as a corporate or sovereign debt instrument that underlies a credit derivative. A *Reference Entity* is the issuer of a Reference Asset. A *Notional Amount* is the amount of the Reference Asset par value to which a contract applies. The *Par value* of a bond is usually the amount the issuing company promises to pay at the maturity date of the bond. In the most general sense, a *spread* is the difference between two similar measures. In the credit derivatives market, spread is computed using bid and offer quotes from dealers. A *basis point (bps)* is a unit equal to 1/100th of 1%. In this paper, it is used as a unit for the spread.

1.2.1. Credit Derivatives

Credit Derivatives represent one of the most important innovations of the financial industry in the last 15 years [1]. They allow isolating and trading the Reference Entity's credit risk through a partial or total transfer. They come in various types and flavors, the most common being:

- r *Collateralized Debt Obligations* (CDO)

- r *Credit Default Swaps* (CDS)

Other types of credit derivatives include Total Return Swaps and Credit-Linked Notes. While the latter are still frequently used and described in the literature [10], this project will focus on CDS and CDO. The figure below illustrates the Credit Derivatives market as of 2003.
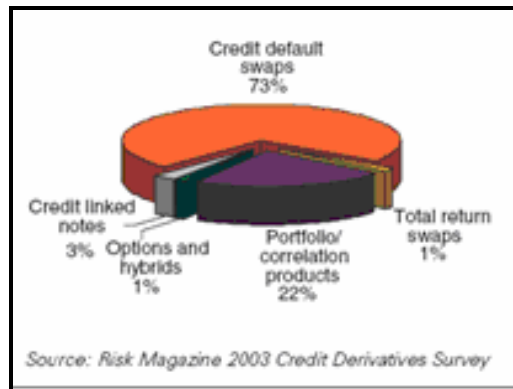

**Figure 1Credit Derivative market breakdown
by instrument type**

Collateralized Debt Obligations (CDO)

CDO ([10]) are investment-grade securities backed by a pool of bonds, loans and other assets. CDO are also referred to as *portfolio correlation products*. They represent a way of packaging Credit Risk. Four classes called *tranches* are created from a portfolio of corporate bonds or bank loans or asset-backed securities. The first tranche owns 5% of the principal of the portfolio and bears the first 5% default losses. 10% of the principal belongs to the second tranche which also takes the next 10% default losses. The third tranche has 10% of the principal and takes in the next 10% default losses. Finally the fourth tranche owns the remaining 75% of the principal and absorbs the residual losses.

The figure below illustrates a CDO.
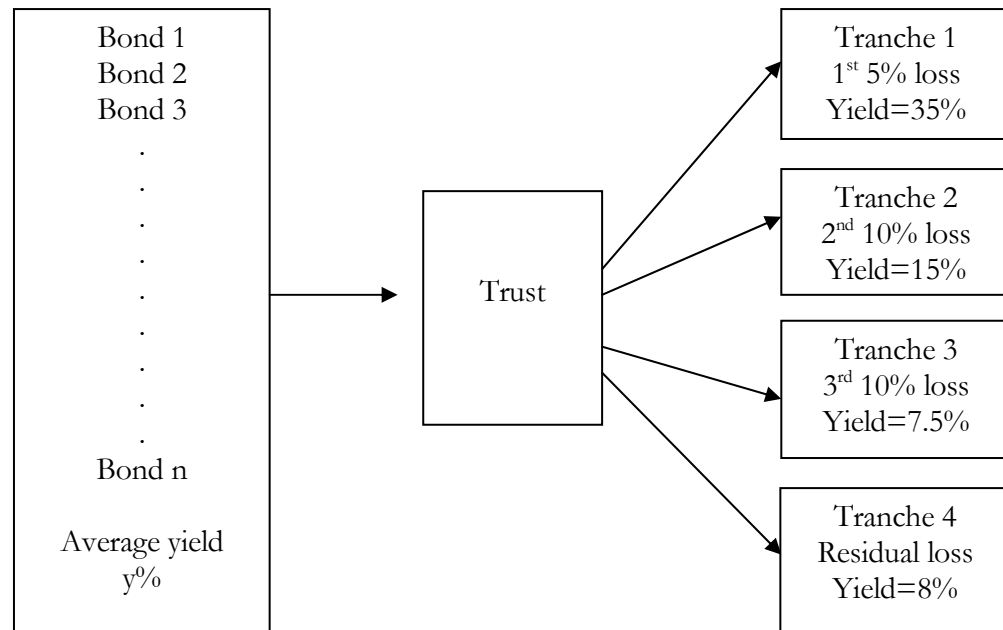
Tranche 4 is usually called the super senior tranche, tranche 3, the senior tranche, tranche 2, the mezzanine and tranche 1, the subordinate tranche or "toxic waste". The structure of the CDO is supported by a rating given by the ratings agencies such as Moody's, S&P, and Fitch. Although these ratings might vary slightly among these agencies, the highest rates, equivalent to almost default-free are the AAA or Aaa, and the lowest, also called "junk" are rated C, Ca or D.

Tranche 4 is usually rated AAA by S&P and Aaa by Moody's because almost no default risk is associated with that tranche. Despite the existence of default risk in Tranche 3, it is still lower than that of the entire underlying portfolio. Unlike tranche 3, tranche 2 is probably more risky than the portfolio. Tranche 1 can be very risky. A 5% loss in the portfolio will translate in a 100% loss in that tranche.

That is the reason why this tranche is usually retained by the creator of the CDO, given the high risk associated with it. CDO allow creating high-quality debt with average quality. An important issue is the correlation between bonds in the portfolio, given that the risk to which the mezzanine, senior and super senior tranches are exposed depend on that. Recently, *copula* models, which are statistical models, are being used to incorporate the correlation between the elements of a CDO.

Credit Default Swaps (CDS)

CDS ([1, 19]) are the simplest type of credit derivatives and act as a form of insurance. If the reference asset is one bond of a single firm, they are called *single-name* CDS. If the Reference Asst is a portfolio of bonds, then we talk about *multi-name* CDS. There are also CDS backed by an index such as the Dow Jones index. In that case, they are called CDX.

In a CDS contract, one party (the protection buyer or Credit Risk seller) is protected from a Reference Entity default through the payment of a regular Premium to the other party (the protection seller or Credit Risk buyer), entitling the former a payment of any non recoverable amount in the event of the Reference Entity default. This process is illustrated below:

Premium Payments

Protection Buyer — Credit Event — Contingent Payment — Protection Seller

Reference Entity
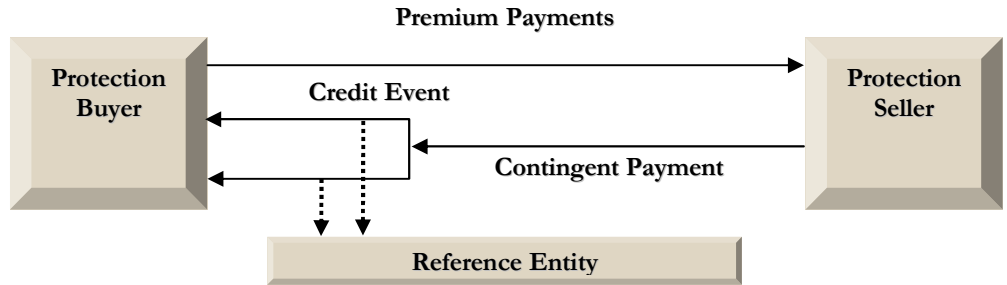
The calculation of the value of a CDS is then done using the risk-neutral valuation method: The expectation of the discounted value of the contingent payment to the protection buyer subtracted from the sum of the payments to the protection seller. In other words, if we denote the value of the CDS by V_{CDS} we have:

$$V_{CDS} = E\left[\sum_{i=1}^{n} ANot(i)1_{\{\tau_i > t_i\}} B(t_i)\Delta t_i - \sum_{i=1}^{n} (1-R)Not(i)(1_{\{\tau_i > t_{i-1}\}} - 1_{\{\tau_i \le t_i\}})B(t_i) \right]$$

Where

 r n is the number of payments by the protection buyer to the protection seller if no default were to occur until maturity,

 r A is the premium that the buyer pays to the seller at each payment in case of no default,

 r Not() is the value of the notional at each payment date,

 r $(t_i)_{1 \le i \le n}$ are the time elapsed (in years) between the premium payment dates and the start date of the contract,

r $(\tau_i)_{1 \le i \le n}$ are as defined in Section 1.1.2,

r $B(t_i)$ is the discounting factor at t_i,

r R is the recovery rate of the Reference Asset in case of default,

r And $E[\ ]$ is the expectation under the probability Π.

Therefore under the assumptions that default events, recovery rates and interest rates are mutually independent we obtain:

$$V_{CDS} = \sum_{i=1}^{n} ANot(i)S(t_i)B(t_i)\Delta t_i - \sum_{i=1}^{n}(1-R)Not(i)(S(t_{i-1})-S(t_i))B(t_i) \tag{25}$$

At origination, under the risk neutral valuation framework, we should have

$$V_{CDS} = 0$$

More details about CDS will be provided in Chapter 2.

1.2.2. Synthetic Securitization

The idea behind synthetic securitization is among others, to allow credit protection, capital relief and exposure to an asset without having the obligation to retain ownership of the asset. For example a bank can transfer the Credit Risk associated with a portfolio of BBB-rated (medium credit quality) corporate loans to a bankruptcy remote special purpose financing vehicle without actually transferring the underlying loans. The special purpose vehicle (SPV) can then issue securities whose interest and principal payments are provided by cash flows coming from the loans. The SPV then creates a portfolio of single-name CDS on each the security type and buys protection synthetically for the CDO, therefore replicating the CDO synthetically. In this way, the bank is able to

avoid sensitive client relationship issues arising from loan transfer notification requirements, loan assignment provisions, and loan participation restrictions. It is also able to maintain client confidentiality.

Almost inexistent in 2001, the synthetic products market has grown quickly as illustrated by the figure below (reference).
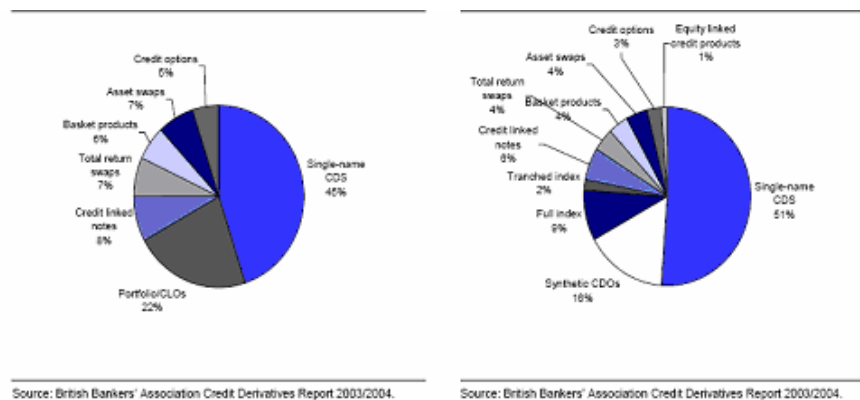

Source: British Bankers' Association Credit Derivatives Report 2003/2004.

Figure 2 Breakdown of the CDS market in 2001 and………………2003

There are many types of synthetic credit derivatives such as Synthetic CDO (SCDO), Single Tranche CDO (STCDO).

Synthetic CDO (SCDO)

Synthetic CDO are artificial CDO that are backed by a pool of credit derivatives such as CDS, forwards, and options. SCDO achieve exposure to the pool of assets underlying these derivatives by synthetically selling CDS. In such a CDS, the SCDO receives a periodic payment from a counterparty that seeks protection against the default of a referenced asset. A special purpose vehicle is used to structure a SCDO and it can issue floating or fixed rate obligations tranched in a

variety of ways with respect to seniority and payment. SCDO obligations can have special features customized or tailor-made to investor requirements.

Single-Tranche Synthetic CDO (STSCDO)

A single-tranche CDO is one where the sponsor sells only one tranche from the capital structure of a synthetic CDO. It is also known as mezzanine-only CDO, instant CDO (iCDO), custom tailored CDO tranches and bespoke (meaning custom tailored) CDO tranches, STCDO deals are based on synthetic CDO technology. A bank arranger will create a customized tranche for an investor. The investor chooses the initial portfolio – usually a portfolio of diversified corporate reference credits. The portfolio may be static, or the investor may "lightly manage" the portfolio, usually at no extra cost. A tranche is defined by its attachment point and its detachment point. These two elements characterize its position in the CDO structure as illustrated in the figure below.
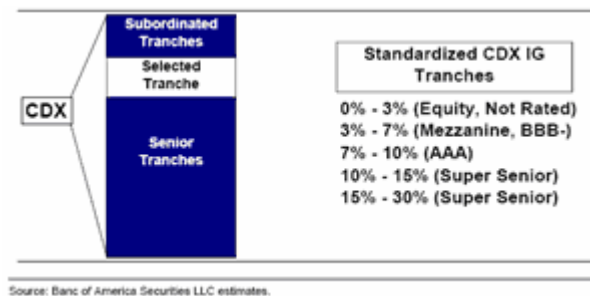

Source: Banc of America Securities LLC estimates.

Figure 3Tranches on CDX or more generally
Single Tranche CDO

CREDIT DEFAULT SWAPS

A CDS is a form of OTC credit derivative security, which can be regarded as default insurance on reference assets such as bonds or loans. CDS are one of the most innovative financial instruments in the last decade and are expanding very rapidly and successfully. According to the November 2005 report of the Bank of International Settlements (BIS) on the Global OTC Derivatives Market at end-June 2005 "Notional Amounts outstanding of credit default swaps rose by 60% during the first half of 2005 to $10.2 trillion, weathering the sell-off in credit markets triggered by downgrades in the US auto industry in March.3 Growth was particularly strong in multi-name contracts, whose Notional Amount more than doubled to $2.9 trillion, single-name CDS increased by 43% to $7.3 trillion. The vast majority of contracts have maturities between one and five years." [22]

CDS is a sophisticated form of a traditional financial guarantee, with the difference that it needs not be limited to compensation upon an actual default but might even cover events such as downgrading, apprehended default etc.

CDS covers only the Credit Risk inherent in the asset, while risks on account of other factors such as interest rate movements remain with the originator.

CDS are used as a way of hedging a specific exposure to a specific asset or as a mean to gain exposure to a specific asset.

Two parties are involved in a CDS, protection buyers or risk sellers and protection sellers or risk buyers. Protection buyers express their negative view of the performance of the specific asset by seeking insurance for a default event. Protection sellers on the other hand gain exposure to the specific asset.

The protection buyer, hereafter referred to as the buyer agrees to pay a Premium to the protection seller until a credit event occurs or maturity is attained. The protection seller, hereafter referred to as the seller, agrees to pay the contingent value to the buyer in case a default event occurs.

At the time of definition, a CDS contract comes with various specifications. Apart from the Premium, the Maturity Date and the Notional Amount, the other ingredients to a CDS contract:

- Specification of the credit events

- Type of settlement (physical or cash)

- Payment frequency

- Business day convention

- Discounting factor

- Effective date

According to an article that appeared on Bloomberg News on November 18th 2005, "GM was among the five companies most frequently included in credit-derivatives contracts in 2004, along with Ford Motor Co., France Telecom SA, DaimlerChrysler AG and Deutsche Telekom AG, Fitch said. Investors bought

more contracts protecting payments from Korea, Italy and Russia than any other governments."

We observe a wide range of participants in the credit-derivatives market. In the same article cited above, it is said that "The survey of 120 banks and financial institutions showed that banks are typically net buyers of debt insurance because they can use default swaps to reduce the risk of corporate loans. Banks used credit derivatives to transfer a record $427 billion of credit risk from their balance sheets to other counterparties in 2004, up from $260 billion a year earlier, Fitch said." According to a survey from the BBA, "Banks, security houses, and hedge funds dominate the protection-buyers market, with banks representing about 50 percent of the demand. On the protection-sellers side, banks and insurance companies dominate [23].

We can distinguish two types of CDS, corporate CDS and Asset-Backed CDS.

2.1. CORPORATE CDS

The Reference Asset, also know as Reference Obligation, in this case is a corporate bond and the Reference Entity is a firm. One of the distinctive features of corporate CDS is that the contract terminates at the first credit event or when the bond matures, whichever comes first.

The buyer agrees to pay a Premium to the seller until the underlying Reference Entity defaults or the bond matures, whichever comes first. The CDS Premium is

typically quoted in basis points per $100 of the Notional Amount of the reference asset. In return the seller makes the engagement to buy the defaulted bond at par value from the buyer.

In the case of a corporate CDS, the guidelines for credit events set by ISDA (the International Swaps and Dealers Association) are:

- r Failure to Pay,

- r Loss event,

- r Bankruptcy,

- r And downgrading.

The type of settlement can be:

- r Physical, in which case the seller buys the reference asset at par

- r Cash, entitling the buyer the reimbursement of the defaulted amount.

In [10] we can see the following example: suppose two parties enter into a five-year CDS on March 1, 2002. Assume that the notional principal is $100 million and the buyer agrees to pay 90 bps annually for protection against default by the reference entity. If the Reference Entity does not default (i.e. there is no default event), the buyer receives no payoff and pays $900,000 on March 1 of the years 2003, 2004, 2005, 2006 and 2007. If there is a credit event, a substantial payoff is likely.

 Suppose the buyer notifies the seller of a credit event on September 1, 2005 (halfway through the 4th year). If the contract specifies physical settlement, the

buyer presents "$100 million divided by par value" of the reference obligation to the seller and receives $100 million. If the contract requires cash settlement, the calculation agent would poll dealers to determine the mid-market price of the reference obligation, a pre-designated number of days after the credit event. If the value of the reference obligation proved to be $35 per $100 of par value, the cash payoff would be$65 million.

In the two cases, physical and cash settlement, the buyer would be required to pay the seller the amount of the annual payment accrued during March 1, 2005 and September 1, 2005 (approximately $450,000), but no further payments would be required.

Payı
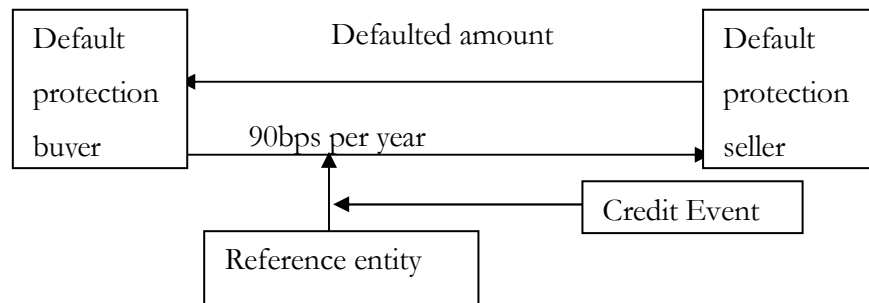
2.2. ASSET-BACKED CDS (ABCDS)

Asset-backed CDS are written on Asset-backed securities. Parties involved are the same as in a corporate CDS. Contract elements such as the Premium, the Maturity Date, the business day convention, discounting factor are also the same. The main difference resides in the following:

The ABCDS is a PAYGO contract; defaulted amounts are paid and reimbursed as they happen, therefore the Notional Amount is not constant and might increase or decrease. The contract is not terminated at the first credit event but when the Outstanding Notional Balance becomes null or at Maturity Date, whichever comes first. Additional provisions are the payment by the buyer to the seller of any reimbursed defaulted amount. Also credit events include Principal Writedown (reduction), Failure to pay while Bankruptcy is no longer considered.

2.3. CDS AS RISK MANAGEMENT TOOL

CDS be used by commercial banks as well as portfolios managers to diversify their credit portfolios, reduce their exposure to credit risk and trade credit risk. Let's look at a specific example in which CDS are used as a way of effectively hedging a specific exposure to a specific asset or as a mean to gain exposure to a specific asset. The table below shows quotes for CDS as a market maker might provide them in January 2001.

		Maturity			
Company	Rating	3 years	5 years	7 years	10 years
M. Lynch	Aa3/AA-	21/41	40/55	41/83	56/96
Enron	Baa1/BBB+	105/125	115/135	117/158	182/233
Nissan	Ba1/BB+	115/145	125/155	200/230	244/274

For the last 4 columns, the pair of numbers represents bid and offer quotes for CDS with maturity 3, 5, 7 and 10 years. Suppose that a bank had several hundreds million dollars of loans outstanding to Enron in January 2001 and was concerned about its exposure. How can it use CDS to hedge its exposure to a default by Enron? A possible hedging strategy would be to buy a $100 million 5-year CDS on Enron from the market maker in table 1 for 135bps or $1.35 million/year. This would shift to the market maker part of the bank's Enron credit exposure. Instead of shifting the credit exposure to the market maker, the bank can also choose to exchange part of the exposure for an exposure to a company in a totally different industry, say Nissan. The bank could sell a 5-year $100 million CDS on Nissan for $1.25million/year at the same time as buying a CDS on Enron. The net cost of this strategy would be 10bps or $100,000.

As it happens, Enron defaulted within 12 months of January 2001. Either strategy would have worked out well!

2.4. ASSET-BACKED CDS VALUATION

In this section, we discuss the pricing of CDS in the reduced form framework.

In pricing CDS, there are two cases to consider: the primary market or at origination and the secondary market or after origination. In the primary market,

we are pricing a CDS that has not been initiated yet while in the secondary

market, we are pricing an existing CDS. Both cases will be covered in this section.

Let's consider a CDS contract with a constant notional $Not(t)$. We assume that

in the event of default, the seller will only pay the non- recoverable part

$(1-R)Not(t)$ of the notional. We also assume, for simplicity, the recovery rate R

to be constant. Let T be the length of the contract in years.

A CDS has two cash flow legs:

  r   The fixed leg or the Premium payments that the buyer pays to the seller

  r   And the floating leg or protection payments that the seller pays to the

      buyer in the case of a credit event.

 For simplicity we assume no counterparty credit risk. Then the pricing strategy is

as follows:

  r   In the primary market, risk neutral valuation is used to equate the present

      value of these two legs and determine the fair price.

  r   In the secondary market, current CDS market prices are used to determine

      the implied default probability and compute the value of the CDS contract.

  Each case will be studied in details.

2.4.1.   Pricing CDS in the primary market

Let $(m_k)_{k\geq 0}$ be defined as a sequence of a set of dates representing the possible

maturity dates of a CDS contract. m_k is usually chosen such that the length of

the contract follows the usual bond length agreement. In other words,

$$m_1 = \frac{1}{2} \, year$$

$$m_2 = 1 \, year$$

$$m_3 = 2 \, years$$

Also let $(A_k)_{k \geq 1}$ be a sequence representing the CDS market premiums corresponding to the sequence $(m_k)_{k \geq 1}$. We know that each of these premiums will depend on how the market perceives the credit risk associated with the Reference Entity, i.e., the probability of default of the Reference Entity. A_k is paid periodically, for example every quarter, until the end of the contract or until a certain credit event occurs for a corporate CDS or the Notional has been entirely paid in the case of ABCDS or the contract reaches maturity.

Consider a CDS contract in which the Premium A is paid every quarter, at a set of dates to which we associate a sequence $(t_i)_{1 \leq i \leq n}$, that represent the time elapsed between the payment dates and the start date of the contract.

Thus $t_1 < t_2 ... < t_n = T$.

We set $t_0 = 0$.

Given this definition, there will be some t_i such that $t_i = m_k$ for $m_k \leq T$.

Let $F(t_i)$ be the probability that the Reference Entity will default by time t_i (see equation (14)) and $S(t_i) = 1 - F(t_i)$, the probability that the Reference Entity will survive t_i.

One of the challenges for pricing ABCDS is that unlike with corporate CDS where a credit event signifies the end of the CDS contract, a credit event merely implies a contingent default payment if the notional is not exhausted. Another salient feature is that contingent payments are dependent on the type of credit event. Thus to express the default leg, one needs to take into account the various type of credit events and their related time of default, as well as the likely correlation among them.

In this work, to simplify the model, we assume only one type of credit event, a principal writedown, implying a reduction in the notional.

The present value of the expected cash flows to be paid by the buyer is:

$$PV_{FixedLeg}(A,\lambda,n) = \sum_{i=1}^{n} ANot(t_i)S(t_i)\Delta t_i B(t_i) \tag{26}$$

where $B(.)$ is the risk-free discount factor

On the other side of the contract, the present value of the expected cash flows to be paid by the protection seller is given by:

$$PV_{FloatingLeg}(l,n) = \sum_{i=1}^{n} (1-R)\,Not(t_i)\big(S(t_{i-1}) - S(t_i)\big)B(t_i) \tag{27}$$

Under a non-arbitrage assumption, (26) should be equal to (27) at the initiation of the CDS contract. Thus the fair Premium is given by:

$$A = \frac{\sum\limits_{i=1}^{n} (1-R)\,Not(t_i)\big(S(t_{i-1})-S(t_i)\big)B(t_i)}{\sum\limits_{i=1}^{n} Not(t_i)S(t_i)\Delta t_i B(t_i)} \tag{28}$$

At this point, if we know the survival probabilities $S(t)$ then we can compute the Premium A. Therefore our effort in this work will be using the market premiums $\big(A_k\big)_{k=1..K_T}$ where K_T is the index for the sequence $\big(m_k\big)_{k\geq1}$ such that $m_{K_T} = T$, to find the implied survival probabilities. These latter will be used in the above expression to compute A.

The algorithm for finding the implied hazard rate for each maturity will be described later.

2.4.2. Valuing an existing CDS

Suppose we have a T-maturity CDS contract that has been already initiated in the past at time t_{-k} and suppose that we are now at time t_0. We now want to determine the value of this contract. To do this, we first need to find out the default probabilities implied by the current market prices for smaller maturities CDS contract on the same reference asset. These probabilities will be implied from equation (28) using the market prices and used to compute each leg in (26) and (27) using the CDS price. The value of the CDS will be the difference between the fixed and floating leg.

$$V_{CDS}(A,\lambda,n) = PV_{FixedLeg}(\lambda,n) - PV_{FloatingLeg}(\lambda,n)$$

$$V_{CDS}(A,\lambda,n) = \sum_{i=1}^{n} ANS(t_i)\Delta t_i B(t_i) - \sum_{i=1}^{n} (1-R)\,N\big(S(t_{i-1}) - S(t_i)\big)B(t_i) \tag{29}$$

Given that in practice spreads are correlated with maturity, it is advisable to use the term structure of the hazard rate in (27) and (28). We incorporate that factor by using the spread of the various CDS contracts available in the market to determine the corresponding hazard rate and use bootstrapping to find the T-hazard curve. This process will be described in more details in the next chapter.

Data and Methods

In this work, we essentially used the risk-neutral valuation method within the reduced form framework to value ABCDS in the secondary market. To account for the dependence between spread and maturity, the bootstrapping method is used find the term structure of the hazard rate.

C++ code was implemented to price ABCDS using the aforementioned methods. The data source for this work is the INTEX database. The INTEX cash flow engine was used to generate cash to use to test the methods and also for comparing the effect of applying default at the tranche level and the collaterals level.

3.1. THE HAZARD RATE METHOD USING BOOTSTRAPPING

Recall that from the hazard rate defined in section 1.1.2.1.

$$\lambda_t = \lim_{h \to 0} \frac{P\left[(t < \tau < t+h)/\tau > t\right]}{h},$$

We used this formula to derive an expression for the survival function in terms of the hazard rate $S(t) = e^{-\int_0^t \lambda_s ds}$

If we assume the hazard rate is constant, we have from equation (16):

$$S(t) = e^{-\lambda t}$$

For short maturities, this assumption makes sense. However for long term maturities, given the known dependence between spreads and CDS maturities, we use bootstrap to obtain more reasonable default intensities.

Let $(t_i)_{1 \le i \le n}$ be the sequence of elapsed time between payment dates and the contract start date as in section 2.4.1.

We denote by λ_k the hazard rate that applies between m_k and m_{k+1}, where $m_k \in (m_k)_{k=1..K_T}$ and K_T is the index for the sequence $(m_k)_{k \ge 1}$ such that

$$m_{K_T} = T$$

Also let n_k be the number of payments for the m_k-maturity CDS.

Then $n_k = m_k \dfrac{12}{freq}$, where $freq$ is the payment frequency that can take value in $1, 3, 6, 12$ meaning monthly, quarterly, semi-annually or annually, respectively.

The idea of bootstrapping consists of finding the hazard rate λ_k that will make the value of the CDS equal to 0, assuming that the previous λ_j, for $j = 1..k-1$, are used for their corresponding period of time.

To illustrate the method, we look at a practical example. Suppose we have a 10-year maturity ABCDS, with quarterly payments. Then the method is implemented as follows:

Set

$$m_1 = 0.5, m_2 = 1, m_3 = 2, m_4 = 3, m_5 = 4, m_6 = 5, m_7 = 7, m_8 = 10$$

For j=1

Compute $n_1 = m_1 \dfrac{12}{3} = 2$

Set $V_{CDS}\left(A_1, \left(\lambda_j\right)_{1 \le j \le 1}, n_1\right) = \sum_{i=1}^{2} A_1 N S(t_i) \Delta t_i B(t_i) - \sum_{i=1}^{2} (1-R) N \left(S(t_{i-1}) - S(t_i)\right) B(t_i)$

$$V_{CDS}\left(A_1, \left(\lambda_j\right)_{1 \le j \le 1}, n_1\right) = \sum_{i=1}^{2} A_1 N e^{-\lambda t_i} \Delta t_i B(t_i) - \sum_{i=1}^{2} (1-R) N \left(e^{-\lambda t_{i-1}} - e^{-\lambda t_i}\right) B(t_i)$$

Solve numerically for λ_1 by setting $V_{CDS}\left(A_1, \left(\lambda_j\right)_{1 \le j \le 1}, n_1\right) = 0$.

For j=2

Set $n_2 = m_2 \dfrac{12}{3} = 1 \times \dfrac{12}{3} = 4$

Set

$$V_{CDS}\left(A_2, \left(\lambda_j\right)_{1 \le j \le 2}, n_2\right) = \sum_{i=1}^{2} A_2 N S(t_i) \Delta t_i B(t_i) - \sum_{i=1}^{2} (1-R) N \left(S(t_{i-1}) - S(t_i)\right) B(t_i) +$$

$$\sum_{i=3}^{4} A_2 N S(t_i) \Delta t_i B(t_i) - \sum_{i=3}^{4} (1-R) N \left(S(t_{i-1}) - S(t_i)\right) B(t_i)$$

Rewrite as

$$V_{CDS}\left(A_2, \left(\lambda_j\right)_{1 \le j \le 2}, n_2\right) = \sum_{i=1}^{2} A_2 N e^{-\lambda_1 t_i} \Delta t_i B(t_i) - \sum_{i=1}^{2} (1-R) N \left(e^{-\lambda_1 t_{i-1}} - e^{-\lambda_1 t_i}\right) B(t_i) +$$

$$\sum_{i=3}^{4} A_2 N e^{-\lambda t_i} \Delta t_i B(t_i) - \sum_{i=3}^{4} (1-R) N \left(e^{-\lambda t_{i-1}} - e^{-\lambda t_i}\right) B(t_i)$$

Or

$$V_{CDS}\left(A_2,\left(\lambda_j\right)_{1\le j\le 2},n_2\right)=V_{CDS}\left(A_2,\left(\lambda_j\right)_{1\le j\le 1},n_1\right)+\sum_{i=3}^{4}A_2 Ne^{-\lambda t_i}\Delta t_i B(t_i)-\sum_{i=3}^{4}(1-R)N\left(e^{-\lambda t_{i-1}}-e^{-\lambda t_i}\right)B(t_i)$$

Then solve numerically for λ_2 by setting $V_{CDS}\left(A_1,\left(\lambda_j\right)_{1\le j\le 2},n_2\right)=0$.

Repeat the steps until $m_{K_{10}}$

We can generalize this process in the following:

Solve for λ_1 by setting

$$V_{CDS}\left(A_1,\left(\lambda_j\right)_{1\le j\le 1},n_1\right)=\sum_{i=1}^{n_1}A_1 NS(t_i)\Delta t_i B(t_i)-\sum_{i=1}^{n_1}(1-R)N\left(S(t_{i-1})-S(t_i)\right)B(t_i)=0$$

Then iteratively solve for λ_k given $\lambda_1,...,\lambda_{k-1}$ in the following equation:

$$V_{CDS}\left(A_k,\left(\lambda_i\right)_{1\le i\le k-1},n_{k-1}\right)+$$
$$\sum_{i=n_{k-1}}^{n_k}A_k Not(t_i)e^{-\lambda t_i}\Delta t_i B(t_i)-\sum_{i=n_{k-1}}^{n_k}(1-R)Not(t_i)(e^{-\lambda t_{i-1}}-e^{-\lambda t_i})B(t_i)=0$$

For $k=2..K_T-1$

So that at the end, the hazard rate function is a given as a step function of the form:

$$\begin{cases}\lambda(t)=\lambda_k \\ m_{k-1}\le t\le m_k \\ k=1,..,K_T\end{cases}$$

3.2. RESULTS AND DISCUSSION

We now present a numerical example to illustrate the valuation process under various scenarios. For that purpose, we consider a CDS contract with the following specifications:

Start Date: 29/04/05

Next Pay Date: 25/05/05

Prev Pay Date: 25/04/05

Business Days: New York Banking

Discounting: 1 month LIBOR

Scenario: PPC 100/call

Notional: $7,500,000

Recovery: 40%

Premium (bps): 190

Curr Mkt Spread: 175

Maturity: 29/04/12

We consider 3 cases to evaluate the effect of the hazard rate curve on the value of the CDS. In the first case, we assume one hazard rate, therefore ignoring the dependence between spreads and maturities. In the second case we consider only one market price available, i.e. A_{K_T}, we assume a straight line hazard rate term

structure. A 3rd case is considered in which we assume the availability of a full market spread structure, i.e. $\left(A_k\right)_{1 \le k \le K_T}$.

Case 1

Then using equation (20), we solve for the constant hazard rate λ and find that

$\lambda = 0.032801778$ and the swap value is $V_{CDS} = \$159,299.00$

While this might work for short-term maturities, it is clearly not realistic to use a single point hazard rate to price CDS. It is well know that spreads are dependent on maturities.

Case 2

In this case, we assume the same market price for all maturities and a straight line hazard rate term structure. We then use this information to obtain the implied hazard curve. The results are shown in the following table.

CDS Maturity (Years)	CDS Market Spread (bps)	Hazard curve
1	175	0.0295553
2	175	0.0295548
3	175	0.0295547
4	175	0.0295546
5	175	0.0295548
7	175	0.0295548
10	175	0.0295548

The value of the CDS is $V_{CDS} = \$107569.00$.

As we would expect, we observe that the hazard curve is almost flat .We repeat this process for various spreads and observe the same result.

Case 3

Assume we have a full market price term structure of the CDS. We feed this term into the bootstrap method to compute the hazard rate term structure.

CDS Maturity (Years)	CDS Market Spread (bps)	Hazard rate
1	130	0.0219575
2	135	0.0228179
3	140	0.023698
4	145	0.0246013
5	150	0.025531
7	160	0.0274247
10	175	0.0304582

We obtained the values shown in the table with a swap value of $V_{CDS} = \$108434.00$. Interestingly, the swap value is very close to the previous case, while the first case seems to overestimate the CDS value. This suggests that using a single market spread to generate the hazard curve for the purpose of valuing a CDS is more reasonable that assuming a flat term structure. This practice seems to be used by data vendors like Bloomberg [1].

We graph both curves on the same figure as shown below. The hazard term structure when we assume the market spread curve shows a behavior that we would expect. The hazard rate is dependent on maturities, lower maturity CDS presenting less hazard than their longer maturity counterparts.
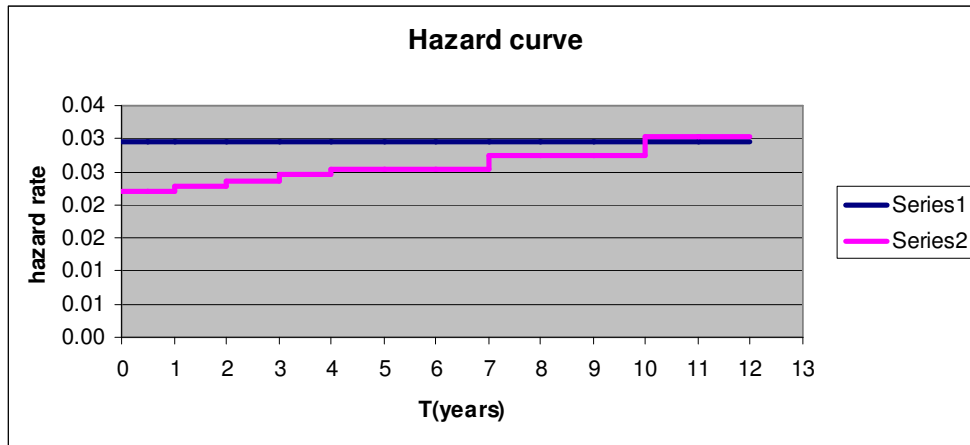
Figure 4 Hazard curves for the flat (blue) and full (pink) market price term structures.

We now focus on analyzing the sensitivity of the hazard term structure to other parameters such as the recovery rate. We generate the hazard curve using constant recovery rates, 0%, 20%, 40%, 50%, 60% and 80%. The graph of the curves is shown below.
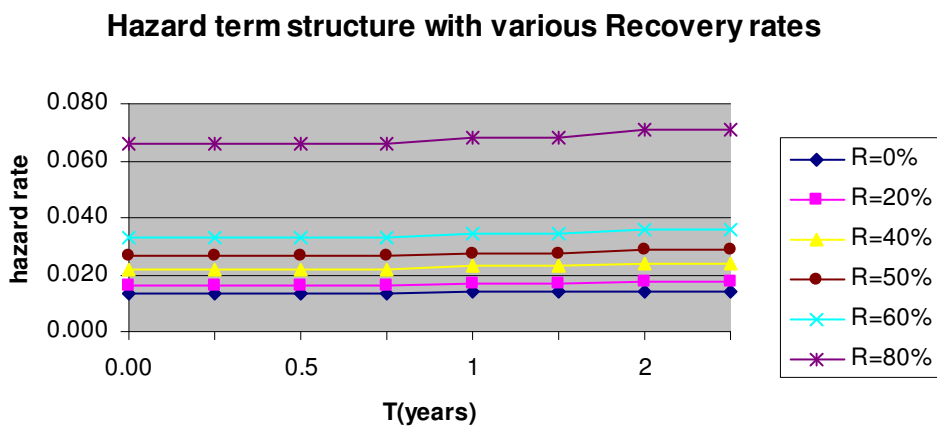

Figure 5 Hazard term structures for various recovery rates.

While we observe a similar increasing trend in all curves, we notice that for lower recovery rates, the spread between curves is minimal and that spread widens as we go to higher recovery rates. This result is in agreement with what we would expect. We notice that curves are ordered by the recovery rates. Indeed hazard rates are higher for higher recovery rates than lower ones as illustrated in fig. 5. This result is somewhat counterintuitive and evidences the need for further work, with different models for the recovery rate. For example, one could model the recovery rate as a stochastic process.

Conclusion

The credit derivatives market has known an incredible growth in the last five years. Since its infancy, along with corporate credit default swaps more complex products have been developed. One of these complex products is Asset-backed credit default swaps. In this work, we tried to value the latter using one of the two approaches used to model credit risk that is a key element in pricing credit derivatives. The question we tried to answer is whether this approach, the hazard rate or reduced form approach is applicable to price ABCDS. We have seen that default payments in ABCDS contract are contingent to the type of credit event, making it difficult to apply the hazard rate method as in the case of corporate CDS. To go around this problem, we considered one type of credit event only. We then implemented a numerical example and obtained a hazard curve that is consistent with what we expected. Thus the method seems applicable if we simplify the set of credits events. Further research will require considering each of the types of credit event as a single jump process and model eventually their joint distribution as well. We also tested the sensitivity of the method to the recovery rate and obtained very interesting because of counterintuitive results. This suggests that further research is needed in this area.

It is evident that pricing ABCDS still presents a challenge. Besides the various types of credit event in question, another issue is that by the nature of structured products, there is no direct and linear relationship between default at the tranche stratum and the collaterals level. This makes it difficult to properly model the total credit risk, which involves the credit risk at the tranche level but also at the collaterals echelon. Presently there is no easy way to do take both risks into account.

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