

CALCULATING WOODEN BASEBALL BAT EFFICIENCY

A Major Qualifying Project Report

Submitted to the Faculty

of the

WORCESTER POLYTECHNIC INSTITUTE

in partial fulfillment of the requirements for the

Degree of Bachelor of Science

in Mechanical Engineering

by

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Date: April 29, 2010

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Abstract

The purpose of this project is to develop a calculational method to quantify in a simple way the performance of wooden baseball bats. The method we propose is based on the rigid body dynamics of the ball/bat collision, which predicts the outgoing velocity of the ball as a function of the location of the point of impact along the bat. Two measures of bat performance that we propose are the maximum outgoing velocity of the ball, and the span of the bat along which the outgoing velocity exceeds the incoming velocity. A third measure we introduce is a hybrid that accounts for both the span of the second measure and the size of the outgoing velocities along that span. Interestingly, we found that the shape of the bat that optimizes one measure of performance is different than the shape that optimizes another.

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Nomenclature Used

x	Location along the bat measured from the handle
\bar{x}	Location of the center of mass in the x coordinate system
x_1^*, x_2^*	Location along the bat that the desired ball velocity is achieved in x coordinate
y	Location along the bat measured from the center of mass
y_1^*, y_2^*	Location along the bat that the desired ball velocity is achieved in y coordinate
x_{max}, y_{max}	Location along the bat that maximum velocity is achieved in x and y scale
v_1	Velocity of the baseball bat prior to impact
v_2	Velocity of the ball prior to impact
v_c	Velocity of the center of mass of the bat prior to impact
v_1'	Velocity of the baseball bat after impact
v_2'	Velocity of the baseball after impact
v_c'	Velocity of the center of mass of the bat after impact
V_2^*	Desired outgoing ball velocity
ρ	Density of the baseball bat
ω	Angular velocity of bat prior to impact
ω'	Angular velocity of bat after impact
h	Distance from handle to point of rotation
m_1	Mass of the baseball bat
m_2	Mass of the baseball

m	Mass ratio of ball to bat
I_c	Moment of inertia about the center of mass of the bat
I_p	Moment of inertia about the center of rotation of the bat
$R(x)$	Baseball bat radial profile as a function of x
r_0	Handle diameter
b, c, d	Cubic coefficients
l	Length of the baseball bat
e	Coefficient of restitution
V_{max}	Maximum outgoing ball velocity
L_{eff}	Effective length
P_{LW}	Large weighted performance
P_W	Weighted performance

Executive Summary

Throughout the baseball and softball world there is plenty of discrepancy on the location of maximum performance along a bat. The purpose of this project is to develop a calculational method to quantify in a simple way the performance of wooden baseball bats in turn answering this question. Previous methods of calculating bat performance used measurements taken at the “sweet spot” of a bat, which was considered to be either the area between the nodes of the first and second modes of vibration or at the center of percussion. The reasoning behind these theories is that at these points along the bat a collision results in the least amount of vibration, thus less energy is wasted on vibration and more is output onto the ball. However, bat testing agencies such as the NCAA found that bat manufacturers could move around the center of percussion which would alter the bat’s performance in the span between the nodes of the first two modes of vibration confirming that neither of these locations are accurate definitions of the “sweet spot” or spots of maximum performance. These complications have led the NCAA and other organizations to turn to new testing methods in which they find the point of maximum performance by physically testing bat/ball collisions along the entire barrel. The method we propose is similar, but based on the rigid body dynamics of the ball/bat collision we can mathematically predict the outgoing velocity of the ball as a function of the location of the point of impact along the bat.

In order to develop a model to predict outgoing ball velocity versus position along the profile for a given bat we used equations for the conservation of linear and angular momentum as well as from the definition of the coefficient of restitution. Our analysis, which can be performed using a few given diameter measurements from an actual bat or from a cubic radius profile,

outputs a curve of the outgoing velocity versus the position along the bat. It is possible to compare bats using just these curves, but we developed three measures which we think are essential parameters that can be used for the comparison of different bat profiles. The first measure, called maximum velocity, is used to quantify the performance of a bat as the maximum attainable outgoing velocity that can be achieved, without regard to the position along the bat. The second measure, referred to as effective length, indicates the length along the bat within which the ball exit velocity is greater than or equal to some chosen velocity. This can be seen as the length of the bat that is effective at producing a desired outgoing ball velocity. The third measure, called weighted performance, combines the information from the previous two measures and introduces a measure that takes both maximum and minimum velocity into account.

We compare the measures introduced above for a range of bats with radial profiles given by a cubic equation:

$$R(x) = r_0 + cx^2 + dx^3 \quad (1)$$

The cubic profile was varied while holding the mass (33 oz.) of the bat fixed - with the handle diameter equal to 1 inch and the slope at the handle flat. Fixing the mass relates the coefficients c and d in $R(x)$. Interestingly, we found that the maximum velocity increases with d while the effective length decreases. The weighted performance measure accounts for both these conflicting trends, and actually increases by more than 30 percent over the range of bats considered.

While holding mass fixed through varied bat profiles may seem intuitive, it is actually the moment of inertia about the point of rotation (known to players as swing weight) that is most felt. The cubic profile was again varied, this time holding the moment of inertia of the bat fixed. The handle diameter was again set equal to 1 inch, and slope at the handle flat. By fixing the moment of inertia a new relation between the coefficients c and d in $R(x)$ was found. Notably, while the maximum velocity and weighted performance increase with the variation of d , mass and effective length decrease. Variations in effective length are also much smaller than in the case of constant mass, resulting in a much a greater increase in weighted performance.

In conclusion, when developing our three measures we were looking to find trends when testing profiles from different bats. Interestingly, we found that the shape of the bat that optimizes one measure of performance is different than the shape that optimizes another. When choosing a bat using our measures the type of player needs to be taken into account. More advanced players may be looking for a higher maximum velocity, while a beginner may be looking for a larger effective length. In either case the weighted performance provides a good compromise for either player. Using cubic profiles with a constant mass and a constant moment of inertia gave us the chance to view the trends in our measures as the profile changed. Because it was found that the most change occurred in weighted performance followed by effective length, an efficient bat would have a good combination of the three measures with the most emphasis on weighted performance.

Chapter 1: Introduction

There have been many different theories over the years regarding the performance of wooden baseball bats. Many people refer to the “sweet spot” of a bat as the point of maximum performance. But, when asked what and where the sweet spot is you will receive a number of different answers. The “sweet spot” has been referred to as the center of percussion, the span of the bat between the nodes of the first and second modes of vibration, and the location of minimum hand sensation among others. The theories defining the “sweet spot” as the center of percussion or the location of minimum hand sensation use the reasoning that at these points there is the least vibration, and thus the least amount of energy being output into vibration instead of directly into the outgoing ball. While the theory using the nodes of the first two modes of vibration uses the reasoning that in this span vibrations from each of the first two modes essentially cancels out.

Since there is no clear cut definition of the “sweet spot” of a bat this argument will have a tough time being put to rest. The purpose of this project is not to argue what the “sweet spot” is or to determine a bats performance in this area. Its purpose is to develop a calculational method to quantify in a simple way the performance of wooden baseball bats, which could then be used to compare the performance of bats in the span of the bat realistically used in bat/ball collisions. The method we propose is based on the rigid body dynamics of the ball/bat collision, which predicts the outgoing velocity of the ball as a function of the location of the point of impact along the bat.

1.1 Background

Many experts often refer to the sweet spot of baseball bat as the center of percussion or the node of the first bending mode. In the research done at Kettering University, however, the sweet spot is considered to be between the nodes of the first and second modes of vibration. In this area the vibrations given out essentially cancels each other out. At the nodes in their experiment, they found that there was little to no vibration occurring along the bat resulting in a maximum amount of energy being given to the ball. The experiment used to find the nodes of vibration used a bat suspended by rubber bands which was tapped by a hammer every inch along the barrel in order to measure a ratio of acceleration to force. Consequently, they found that a ball hit outside these nodes results in more vibration and less energy being given to the ball.

Bat/ball collisions near the end of the bat tend to send the most vibration to the lower hand, while hits near the handle create the most in the top hand, resulting in a sharper pain in the top hand between the thumb and the forefinger. When comparing these vibrations it has been found that hits from the end of the bat cause low frequency vibration consistent with the first bending shape, while stinging in the top hand is a result of higher frequency vibrations consistent with the second bending shape.

The first bending shape has a node, which represents a point on the bat that remains still while the rest of the bat is vibrating, that was found to be approximately 7.5 inches from the end of the barrel. Impact at a node will not excite this particular mode of vibration. The second bending shape has a node that was found to be approximately 4 inches from the barrel end of the bat. Therefore, the region between these nodes about 4-7 inches from the end of the bat is what many people have considered the sweet spot. This area would be considered a sweet spot because an impact within this region will minimally excite the first two bending modes of

vibration causing the bat to vibrate much less than it would outside this region. There is one problem with this theory though. Since the collision of the bat and ball is less than a millisecond, patterns of vibration are still developing after the contact is over. The different vibrations are determined by the profile of the bat and the conditions at both ends. Because the bat/ball collision is so quick the ball is no longer in contact with the bat before any of the vibration patterns had the opportunity to be returned from the handle. Therefore, since these vibration patterns weren't given the chance to be transferred back to the ball, there would be no influence on the exit velocity due to the conditions at the handle. As a conclusion, the experimentation at Kettering University found that while the grip of a player on the bat can influence the position of these nodes there is no influence on the performance.

Since the location of the nodes of the first and second bending mode is so close to the Center of Percussion (COP) many people also believe that the COP is where the sweet spot is. The problem is that the COP shifts closer to the handle and further away from the nodes once the bat is put in the hands of the batter. The grip of the batter also changes the total mass and the moment of inertia of the system. The axis of rotation used while swinging a bat passes through the wrist and hands and because of the impact on these parameters due to the bat being held, the impact point that may feel the best to a player will occur between the newly shifted two nodes, most preferring contact at the node of the fundamental vibration mode.

The center of percussion, which is also known as the center of oscillation, is measured by pivoting a bat about a point on the handle 6 inches from the knob and measuring the period of oscillation. The COP is important because impact at that point along a bat will result in zero net force at the pivot point. When the ball is hit by the bat at a spot closer to the handle than the COP a translational force will be introduced at the pivot. While bat/ball contact closer to the end of the

bat than the COP, results in rotation about the bats center-of-mass. This rotation causes force in the opposite direction at the pivot point. Currently, many methods being used to test baseball and softball bat performance use a point 6-inches off the handle as the pivot point, which is then used in finding the COP.

Baseball bats for NCAA and high school both use the collision efficiency

$$e_A = \frac{V_{out}}{V_{in}} \quad (2)$$

to determine the Ball Exit Speed Ratio (BESR) when considering the certification of baseball and softball bats and also to calculate the Bat Performance Factor (BPF). Collision efficiency is calculated by measuring the ratio of the outgoing velocities of a baseball off a bat when fired from a cannon, compared to the incoming velocity. For many years the BPF was used to measure bats performance at the point of maximum performance, which they considered to be the COP, of a bat in reference to the 6 inch point off the handle. But, it was later discovered that bat manufacturers were able to pass these tests by moving around the moment of inertia so that a bat could remain beneath the certain performance levels at the COP but would actually perform much better outside this region. As a result of this finding, all testing now finds the point of maximum performance by testing collisions along the entire barrel.

In order to start our project we were able to use the past years MQP as a starting point and for background information on our topic. They had developed a mathematical model that predicts the outgoing ball velocity as a function of impact point along the bat, coefficient of restitution, bat density, angular swing velocity, swing geometry and bat geometry. Their model can be utilized to predict at what point along the bat the outgoing ball velocity is maximized, and the resultant velocity from an impact at this location is used as a metric for overall bat performance. They used a cubic function to represent the radial profile of the bat, and the bat

geometry was varied by substituting different boundary conditions. A relation was also developed that predicted bat angular velocity at the moment of impact as a function of the torque applied and the moment of inertia about the center of rotation. These relations were combined to study the effects of variable bat geometry on the overall bat performance. Using their models they found that the maximum allowable bat radius yielded the highest outgoing ball speed, while also noting that the optimal location for the maximum radius is a few inches in from the extreme barrel-end.

1.2 Previous work

This section looks into previous work done on determining the maximum performance of baseball bats. There have been many different opinions about the locations of maximum performance and what parameters this location is associated with, and the center of percussion, the location of minimum hand sensation, the swing weight vs. bat weight and the bending mode are some of the more common ones.

1.2.1 Center of percussion

Previous work done at Kettering University has explored the effect of the location of the center of percussion (COP) on a bats performance. Also once it was found that the COP wasn't in direct correlation with the point of maximum performance, they also explored the different manipulations that could be made to the COP changing the performance of the bat. In the end they found that a bat/ball collision at the COP may feel the best, but it is not the location of maximum performance.

1.2.2 Location of minimum hand sensation

The location of minimum hand sensation comes with some disagreement. Many people think of this as the location of maximum performance and some argue that this spot is the location between the nodes of the first and second modes of vibration while others argue that it is the spot between the node of the first node of vibration and the COP. However, work done at Kettering University and the American Journal of Physics both found that either of these locations like the COP may feel the best to the player, but it does not have a direct correlation to the location of maximum performance.

1.2.3 Swing weight versus bat weight

The swing weight of a bat is greatly affected by the moment of inertia. The bat weight is determined by weighing a bat using a scale, while the swing weight is calculated based on the moment of inertia. Tests done at Kettering University and by the people at the American Journal of Physics have shown that bats with a lower swing weight, or a lower moment of inertia, translate to a faster swing. But, in terms of maximum performance a lower moment of inertia results a less effective collision and a slower batted ball speed. In, fact they also found that often times the purpose of a corked bat was to lower the moment of inertia in order to increase the swing speed. Their testing concluded that the combination of the higher bat speed and the less effective collision effectively cancelled each other out.

1.2.4 Bending mode

The bending mode is sometimes referred to as the location of minimum hand sensation. The span of the bat between the nodes of the first and second bending modes of vibration is what many people consider the “sweet spot.” This region is found to be about 5-7 inches from the end

of the bat in most cases, and while this area does produce a minimal amount of vibration most batters prefer the feel of a ball hit off the node of the first bending mode of vibration. Just like the location of minimum hand sensation, studies at Kettering University and by the people at the American Journal of Physics have found that bat/ball collisions may feel the best to a player in this region it may not always be the spot of maximum performance.

Chapter 2: Measures of Efficiency

In order to quantify the effectiveness of a baseball bat one must first gain an understanding of the outgoing ball velocity that can be achieved at any point along the bat. It is assumed that the collision between bat and ball is instantaneous and complainer. The distance from the center of rotation to the handle of the bat is defined as h and the angular velocity at which the bat is rotating just prior to the impact is defined as ω . Given that the length along the bat from the handle to the center of the mass is \bar{x} such that

$$\bar{x} = x - y \quad (3)$$

it can then be seen that the velocity of the bat at a given point just prior to impact is given by

$$v_1 = v_c + y\omega \quad (4)$$

Where v_c is the velocity of the center of mass of the bat given by

$$v_c = (h + \bar{x})\omega \quad (5)$$

It is important to note again that y is measured from the center of gravity \bar{x} and can, in fact, be negative.

Due to the fact that the impulse forces present during this collision are much greater than any torque that a human could apply during impact, all forces other than impulse can be neglected. Now applying the conservation of linear momentum yields

$$m_2 v_2 + m_1 v_c = m_2 v_2' + m_1 v_c' \quad (6)$$

and the conservation of angular momentum gives

$$m_2 v_2 y + I_c \omega = m_2 v_2' y + I_c \omega' \quad (7)$$

where I_c is the moment of inertia of the bat about its center of gravity.

The definition of coefficient of restitution states

$$e = \frac{-(v_1' - v_2')}{(v_1 - v_2)} \quad (8)$$

The coefficient of restitution essentially dictates how much energy is dissipated during the collision, with $e=1$ being an ideal system in which no energy dissipation occurs. Using equation (8) we can derive formulas for the post impact velocities of the bat v_1' and the ball v_2' as follows.

$$v_1' = v_c' + \omega' y \quad (9)$$

such that

$$v_c' = (h' + \bar{x}) \omega' \quad (10)$$

Combining the above equations and solving for the single term v_2' yields

$$v_2' = v_2 + \frac{m(1+e)(v_c - v_2 + \omega y)}{[1 + m + (\frac{m_2}{I_c})y^2]} \quad (11)$$

where $m = m_2/m_1$ and v_c is defined in (5).

Although the resulting velocity of the bat v_1' is not of interest to this report, having a formula for the outgoing velocity of the ball v_2' is very powerful. This formula allows for a

velocity versus position along the bat graph to be produced for any given radial profile, coefficient of restitution, density, incoming ball velocity, angular speed of the bat, and location of pivot beyond the handle of the bat. One such characteristic velocity profile is shown in Figure 1.

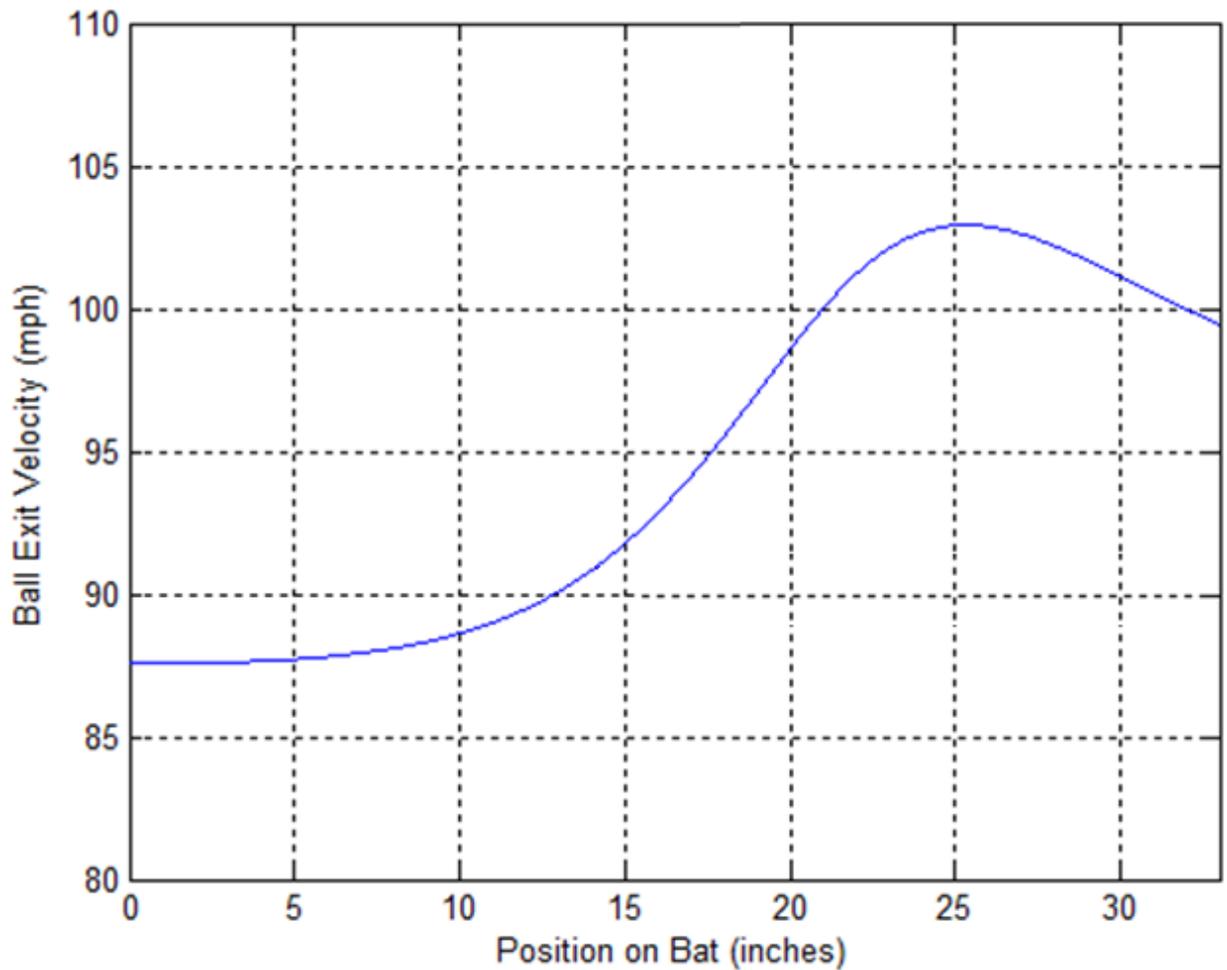


Figure 1: Typical Outgoing Velocity Curve

Using this graph we were able to create a way in which to describe the effectiveness of a baseball bat by means of 3 separate measures as described below.

2.1 Measure 1: Maximum Outgoing Ball Velocity

Measure 1 is simply the maximum outgoing ball velocity that can be achieved using a bat of the given radial profile, regardless of its location along the bat. In essence this represents what is often considered the “performance” of a bat. An individual that is only concerned with the peak outgoing ball velocity would be most interested in this measure.

The maximum outgoing velocity can be found by first identifying the y values that correspond to the minimum and maximum values of (9). This is accomplished by setting $\frac{dv'_2}{dy} = 0$, which yields two roots given by

$$y = \frac{1}{\omega} [-(v_c - v_2) \pm \sqrt{(v_c - v_2)^2 + \omega^2 (1 + m) \left(\frac{l_c}{m_1}\right)}] \quad (12)$$

The root that represents a maximum value can be found by looking at which root corresponds to a negative concavity, or $\frac{d^2v'_2}{dy^2} < 0$. This leaves one root as follows

$$y_{max} = \frac{1}{\omega} [-(v_c - v_2) + \sqrt{(v_c - v_2)^2 + \omega^2 (1 + m) \left(\frac{l_c}{m_1}\right)}] \quad (13)$$

Using equation (3) we can then find the location that yields maximum outgoing ball velocity as it would be measured from the handle of the bat for easy measurements.

$$x_{max} = \bar{x} + \frac{1}{\omega} [-(v_c - v_2) + \sqrt{(v_c - v_2)^2 + \omega^2 (1 + m) \left(\frac{l_c}{m_1}\right)}] \quad (14)$$

The location of maximum output velocity can then be used to determine the actual maximum outgoing ball velocity by substituting equation (13) into equation (11). This yields

$$V_{max} = v_2 + \frac{m(1+e)\sqrt{(v_c - v_2)^2 + \omega^2(1+m)\left(\frac{l_c}{m_1}\right)}}{1+m + \left(\frac{m_1}{l_c\omega^2}\right)[v_2 - v_c + \sqrt{(v_c - v_2)^2 + \omega^2(1+m)\left(\frac{l_c}{m_1}\right)}]^2} \quad (15)$$

A graphical representation of the maximum velocity can be seen in Figure 2.

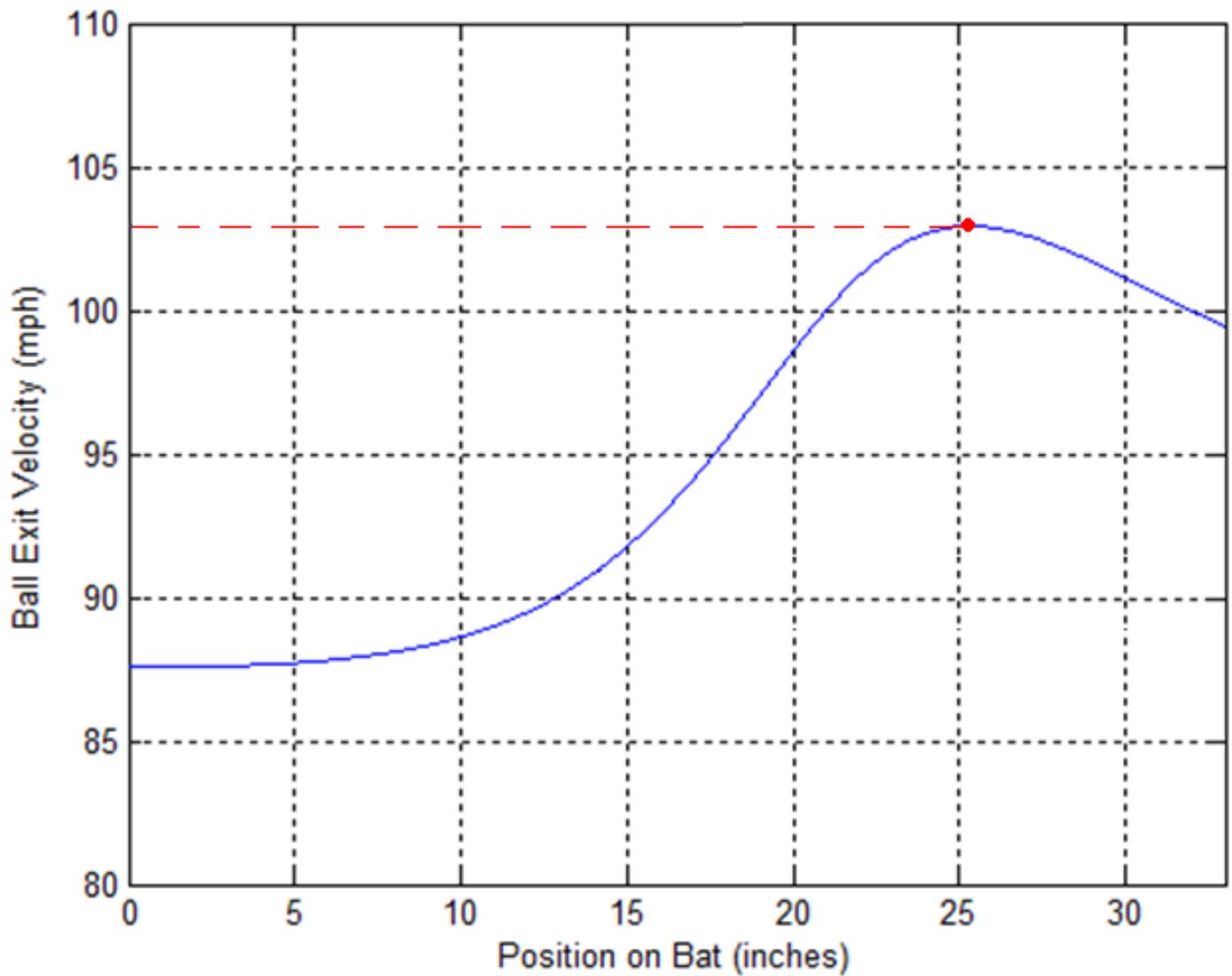


Figure 2 : Maximum Velocity

Equation (15) represents a powerful method of comparing the optimum performance of various bat profiles. Despite this it is not the only useful method of comparison. It is important to remember that this maximum outgoing ball velocity can only be achieved at one exact location along the bat. In some cases a bat that is capable of producing a very high maximum outgoing ball velocity might also achieve extremely low velocities if the point of impact varies even slightly from the ideal location. Such a bat would be extremely difficult to use, but this would not be reflected in the maximum velocity. Thus, further measures are needed for a complete comparison to be made.

2.2 *Measure 2: Effective Length*

Measure 2, referred to as effective length, represents the length along the bat within which a minimum desired velocity can be achieved. That is to say the effective length indicates the length of the bat that is effective at producing at least the desired velocity for the outgoing ball. For instance, if your goal was to find a bat with which it would be the easiest to hit the ball out at least as fast as it was pitched, you would be most concerned with this measure. The graphical equivalent of this measure can be seen below as the length of the red line in Figure 3.

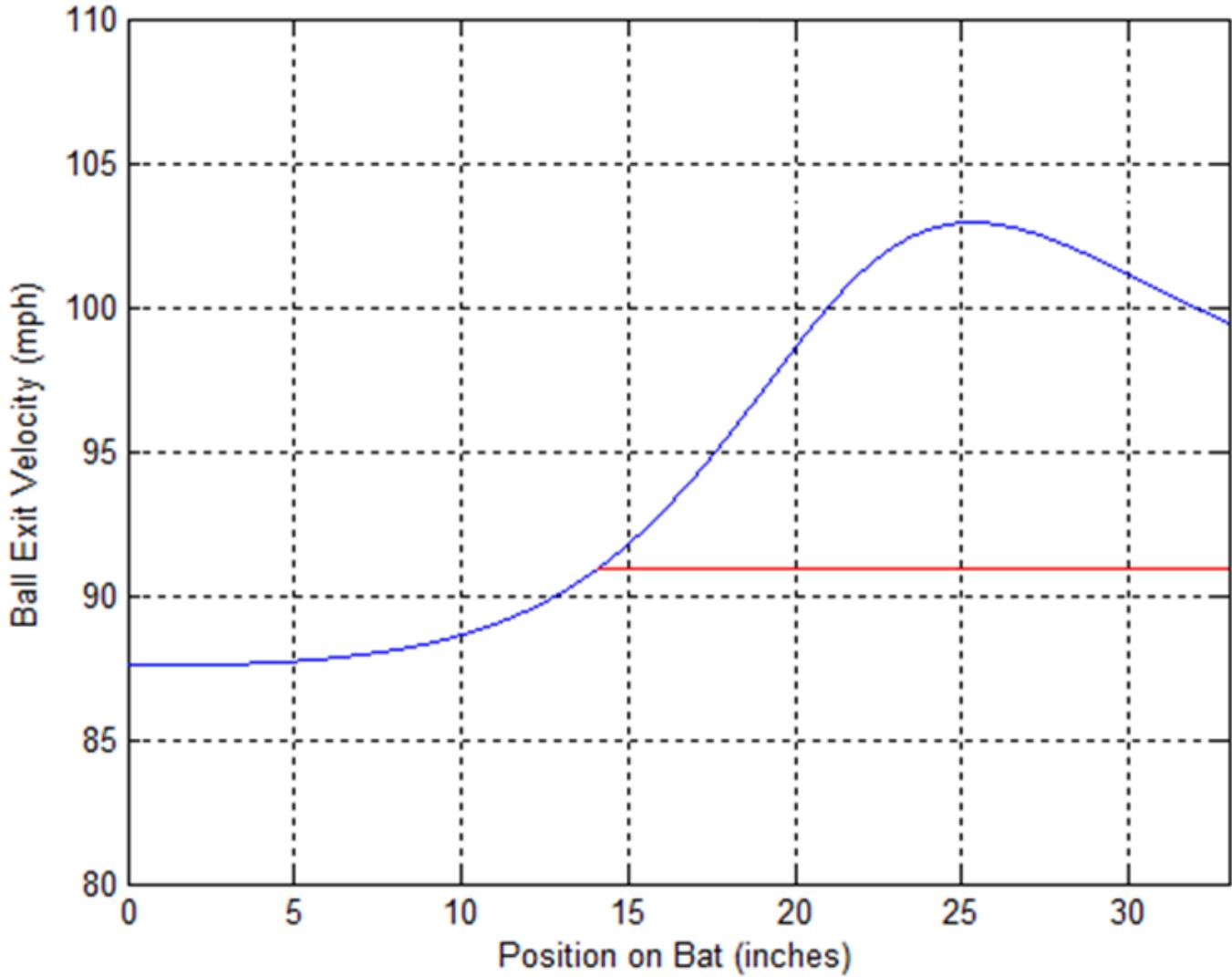


Figure 3: Effective Length

Given a minimum desired velocity v_2^* , before calculating the effective length of the bat it is first necessary to find the positions on the bat at which this exact v_2^* is attained. This can be done by putting the desired value into equation (11) and finding the values of y .

$$V_2^* = v_2 + \frac{m(1+e)(v_c - v_2 + \omega^* y)}{[1 + m + (\frac{m_2}{I_c})y^2]} \quad (16)$$

Equation (16) can be rearranged to

$$0 = Ay^2 + By + C \quad (17)$$

where

$$A = \frac{(V_2^* - v_2) * m_1}{(1 + e)I_c} \quad (18)$$

$$B = \omega \quad (19)$$

$$C = -v_c + v_2 + \frac{V_2^* - v_2}{m(1 + e)} + \frac{V_2^* - v_2}{(1 + e)} \quad (20)$$

Solving (17) using the quadratic formula to find the roots

$$y^* = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \quad (21)$$

$$y_1^* = \frac{-\omega - \sqrt{\omega^2 - 4\left(\frac{(V_2^* - v_2) * m_1}{(1 + e)I_c}\right)\left(-v_c + v_2 + \frac{V_2^* - v_2}{m(1 + e)} + \frac{V_2^* - v_2}{(1 + e)}\right)}}{2 * \left(\frac{(V_2^* - v_2) * m_1}{(1 + e)I_c}\right)} \quad (22)$$

$$y_2^* = \frac{-\omega + \sqrt{\omega^2 - 4\left(\frac{(V_2^* - v_2) * m_1}{(1 + e)I_c}\right)\left(-v_c + v_2 + \frac{V_2^* - v_2}{m(1 + e)} + \frac{V_2^* - v_2}{(1 + e)}\right)}}{2 * \left(\frac{(V_2^* - v_2) * m_1}{(1 + e)I_c}\right)} \quad (23)$$

Now converting from the y to the x scale using equation (3) so that they can be easily

measured from the handle of the bat

$$x_1^* = \bar{x} + \frac{-\omega - \sqrt{\omega^2 - 4\left(\frac{(V_2^* - v_2)^* m_1}{(1+e)I_c}\right)\left(-v_c + v_2 + \frac{V_2^* - v_2}{m(1+e)} + \frac{V_2^* - v_2}{(1+e)}\right)}}{2 * \left(\frac{(V_2^* - v_2)^* m_1}{(1+e)I_c}\right)} \quad (24)$$

$$x_2^* = \bar{x} + \frac{-\omega + \sqrt{\omega^2 - 4\left(\frac{(V_2^* - v_2)^* m_1}{(1+e)I_c}\right)\left(-v_c + v_2 + \frac{V_2^* - v_2}{m(1+e)} + \frac{V_2^* - v_2}{(1+e)}\right)}}{2 * \left(\frac{(V_2^* - v_2)^* m_1}{(1+e)I_c}\right)} \quad (25)$$

It is important to note that these two values have a lower bound of 0 and an upper bound of the length of the bat. If either number is less than 0 it must be set to 0, and similarly if either number is greater than the length of the bat it must be set be equal to the length of the bat.

These two values represent the locations along the bat at which the desired outgoing velocity can be achieved. Knowing that these two points represent points on the left and right ends of a parabola with negative concavity as shown in Figure 3 it is obvious that the effective length of the bat is given by

$$L_{eff} = x_2^* - x_1^* \quad (26)$$

In the special case that the desired outgoing ball velocity is equal to the incoming ball velocity, the value of x_2^* becomes infinite and is thus simply defined to be equal to the length of the bat. Directly solving for x_1^* , however, yields $\frac{0}{0}$ and it must therefore be solved using L'Hôpital's Rule as follows. Recalling definitions (18), (19), and (20) and further defining

$$g(a) = B - \sqrt{B^2 - 4AC} \quad (27)$$

$$h(a) = 2A \quad (28)$$

such that

$$y_1^* = \frac{g(a)}{h(a)} \quad (29)$$

Where $g(0)=0$ and $h(0)=0$ L'Hôpital's Rule states

$$\lim_{a \rightarrow 0} (y_1^*) = \frac{\frac{dg}{da}(0)}{\frac{dh}{da}(0)} \quad (30)$$

Which yields

$$\lim_{a \rightarrow 0} (y_1^*) = \frac{C}{B} \quad (31)$$

Substituting definitions (19) and (20) gives

$$y_1^* = \frac{-(v_c - v_2)}{\omega} \quad (32)$$

Equation (32) should be used along with the special case definition that y_2^* = bat length to find the effective length of a bat in the case where the desired outgoing ball velocity is equal to the incoming ball velocity.

2.3 Measure 3: Weighted Performance

The third measure used in this report for the purpose of comparing wooden baseball bats of various radial profiles is the so called weighted performance. This measure takes into account both the maximum ball exit velocity and the effective length of the bat. In essence the weighted performance indicates how fast a ball can be hit within the effective length of the bat given by the second measure. The graphical representation of the weighted performance of a bat can be seen below in Figure 4 as the green shaded area. This measure acts as a hybrid measure, taking into account both maximum velocity and effective length and determining the overall achievable performance.

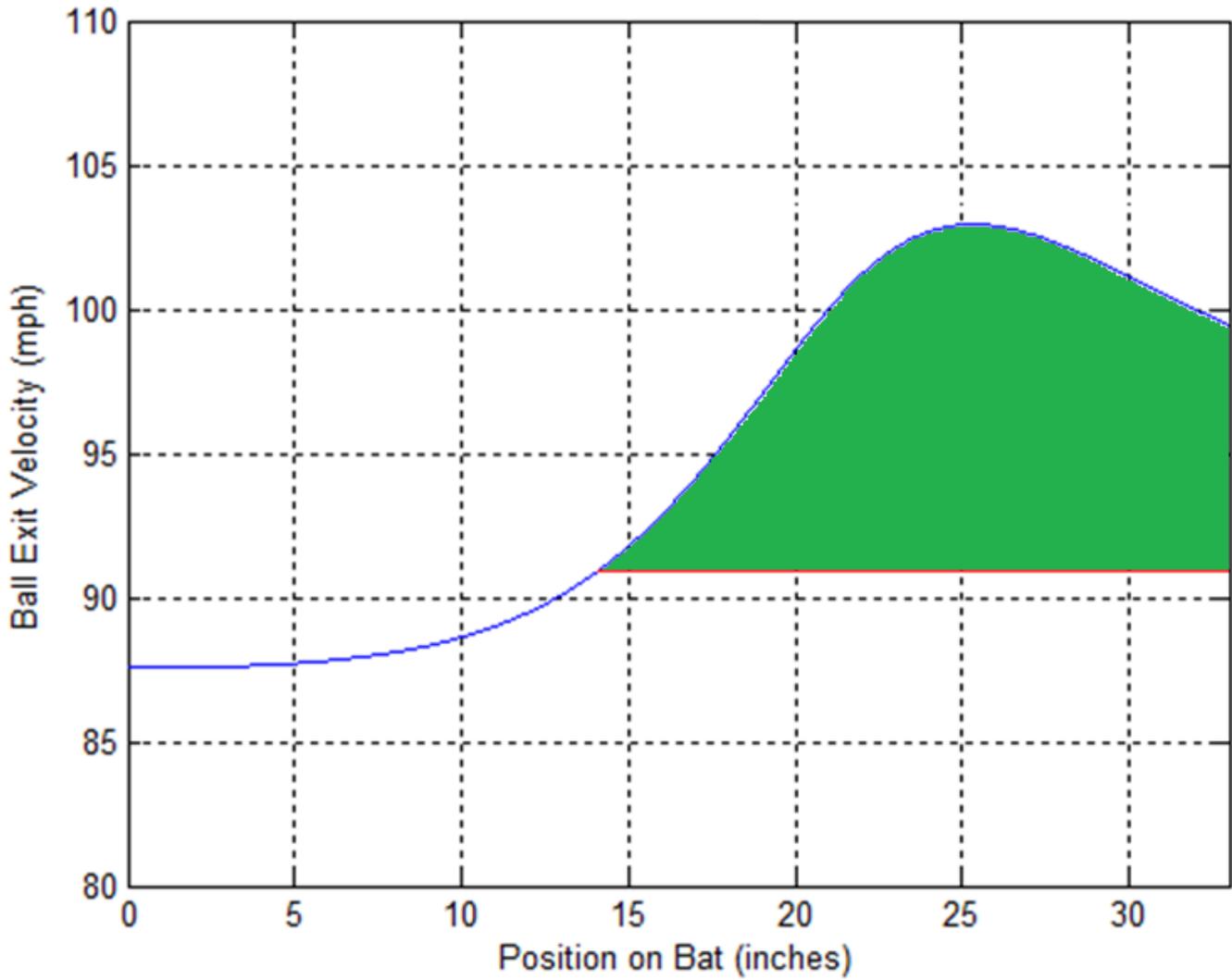


Figure 4: Weighted Performance

The derivation for the formula for weighted performance is quite intuitive. First, the formula for the “large” weighted performance is calculated starting with equation (16) rewritten as follows

$$V_2^* = v_2 + \frac{A+By}{C^2+D^2y^2} \quad (33)$$

where

$$A = m(1 + e)(v_c - v_2) \quad (34)$$

$$B = m(1 + e)\omega \quad (35)$$

$$C^2 = (1 + m) \quad (36)$$

$$D^2 = \left(\frac{m_2}{I_c}\right) \quad (37)$$

Now large weighted performance (P_{LW}) can be defined as the area under the velocity curve in the region defined by V_2^* . This area is calculated using integration of the velocity function between the two corresponding points y_2^* and y_1^*

$$P_{LW} = \int_{y_2^*}^{y_1^*} v_2^* dy = \int_{y_2^*}^{y_1^*} v_2 dy + \int_{y_2^*}^{y_1^*} \frac{A}{C^2 + D^2 y^2} dy + \int_{y_2^*}^{y_1^*} \frac{By}{C^2 + D^2 y^2} dy \quad (38)$$

Substituting in definitions (34), (35), (36), and (37) and solving yields

$$P_{LW} = v_2 * y_2^* + \frac{(1+e)\sqrt{I_c}m(v_c-v_2)atan\left(\frac{\sqrt{m_1}y_2^*}{\sqrt{I_c^*}\sqrt{m+1}}\right)}{\sqrt{m+1}\sqrt{m_1}} + \frac{(1+e)I_c m \omega \log(m_1 y_2^{*2} + I_c + I_c m)}{2m_1} - [v_2 * y_1^* + \frac{(1+e)\sqrt{I_c}m(v_c-v_2)atan\left(\frac{\sqrt{m_1}y_1^*}{\sqrt{I_c^*}\sqrt{m+1}}\right)}{\sqrt{m+1}\sqrt{m_1}} + \frac{(1+e)I_c m \omega \log(m_1 y_1^{*2} + I_c + I_c m)}{2m_1}] \quad (39)$$

Due to the way in which the large weighted performance was calculated the measure contains unnecessary information, namely the area calculated under the cutoff velocity V_2^* . This unnecessary amount is constant for all bats using the same V_2^* and simply inflates the measure. To rectify this inflation the area under V_2^* is simply removed from the measure. This area can be calculated easily as

$$Area = V_2^* * L_{eff} \quad (40)$$

Subtracting this from the large weighted performance gives the small weighted performance, or simply weighted performance.

$$P_w = v_2 * y_2^* + \frac{(1+e)\sqrt{I_c}m(v_c-v_2)atan\left(\frac{\sqrt{m_1}y_2^*}{\sqrt{I_c^*}\sqrt{m+1}}\right)}{\sqrt{m+1}\sqrt{m_1}} + \frac{(1+e)I_c m \omega \log(m_1 y_2^{*2} + I_c + I_c m)}{2m_1} - [v_2 * y_1^* + \frac{(1+e)\sqrt{I_c}m(v_c-v_2)atan\left(\frac{\sqrt{m_1}y_1^*}{\sqrt{I_c^*}\sqrt{m+1}}\right)}{\sqrt{m+1}\sqrt{m_1}} + \frac{(1+e)I_c m \omega \log(m_1 y_1^{*2} + I_c + I_c m)}{2m_1}] - v_2^* * L_{eff} \quad (41)$$

Chapter 3: Case Studies

In order to establish a better understanding of the effect of different variables on the three performance measure, the team performed several case studies holding different parameters constant. With the goal of being able to compare a large range of bats, we decided to use a cubic function to produce radial bat profiles. This cubic took the form

$$R(x) = r_0 + bx + cx^2 + dx^3 \quad (42)$$

where r_0 is the radius at the handle of the bat, b is the slope of the bat at the handle, and c and d are cubic coefficients for x^2 and x^3 respectively.

The first studies were performed on bats with a constant slope throughout their length and with either constant moment of inertia about the center of rotation in one case, or constant mass in the other case. After completing studies on constant linear profile the cases for constant mass and constant moment of inertia were repeated using a cubic bat profile. Finally, a method was developed for using point interpolation to input radial profile information from physical bats.

3.1 *Linear Profile: Constant Mass*

For the first case considered using this model a special subset of $R(x)$ was used such that the profile could be made linear. Using a linear profile greatly reduced calculations and allowed us to verify the viability of our calculations and code while also attaining a course understanding of what results to expect. This was easily achieved by setting the cubic coefficients c and d to be zero. Next we held the mass of the bat fixed and found the slope variations that correspond to different handle diameters using the equation

$$m_1 = \int_0^l \rho \pi R(x)^2 dx \quad (43)$$

Where l is the length of the bat. After substituting and simplifying, the equation becomes

$$m_1 = \pi \rho \int_0^l (r_0^2 + 2bxr_0 + b^2x^2) dx \quad (44)$$

Performing the simple integration yields

$$m_1 = \pi \rho (r_0^2 l + r_0 b l^2 + \frac{b^2 l^3}{3}) \quad (45)$$

In this equation everything is prescribed except the slope of the radial equation b . A substitution is then made for

$$A = \frac{\pi \rho l^3}{3} \quad (46)$$

$$B = \pi \rho r_0 l^2 \quad (47)$$

$$C = \pi \rho r_0^2 l - m_1 \quad (48)$$

Which gives the equation

$$0 = Ab^2 + Bb + C \quad (49)$$

This equation can be simply solved for the slope using the quadratic formula

$$b^* = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \quad (50)$$

Being as this formula yields two roots it must be determined which value of b is appropriate. In this case, the appropriate value of b is the one of higher value.

Using these equations, the constant mass linear bat profile case was completed by analyzing bat profiles and the resulting measure values across a range of realistically manufacturable bats. For this case the mass was held constant at 33 ounces and the length of the bat was held constant at 33 ounces, and the density of the bat, the rotation center of the bat, the coefficient of restitution, the angular velocity of the bat, the incoming velocity of the ball, and the mass of the ball were all held constant. In order to be able to compare the effective length and weighted performance values of the different bat profiles, the cutoff velocity was also constant at 1,600 in/s, or approximately 90.9 mi/hr.

The first bat profile analyzed is shown in Figure 5, and it produced a maximum velocity of 105.75 mi / hr, an effective length of 17.90 in, and a weighted performance of 275.24 mi * in / hr. For this bat, the moment of inertia was 24,087 oz * in². The velocity profile along the length of the bat along with the effective length is shown in Figure 6

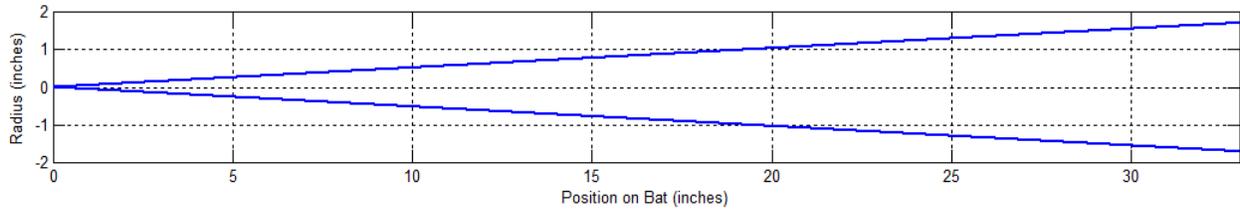


Figure 5: Linear Constant Mass Bat Profile $a = 0$

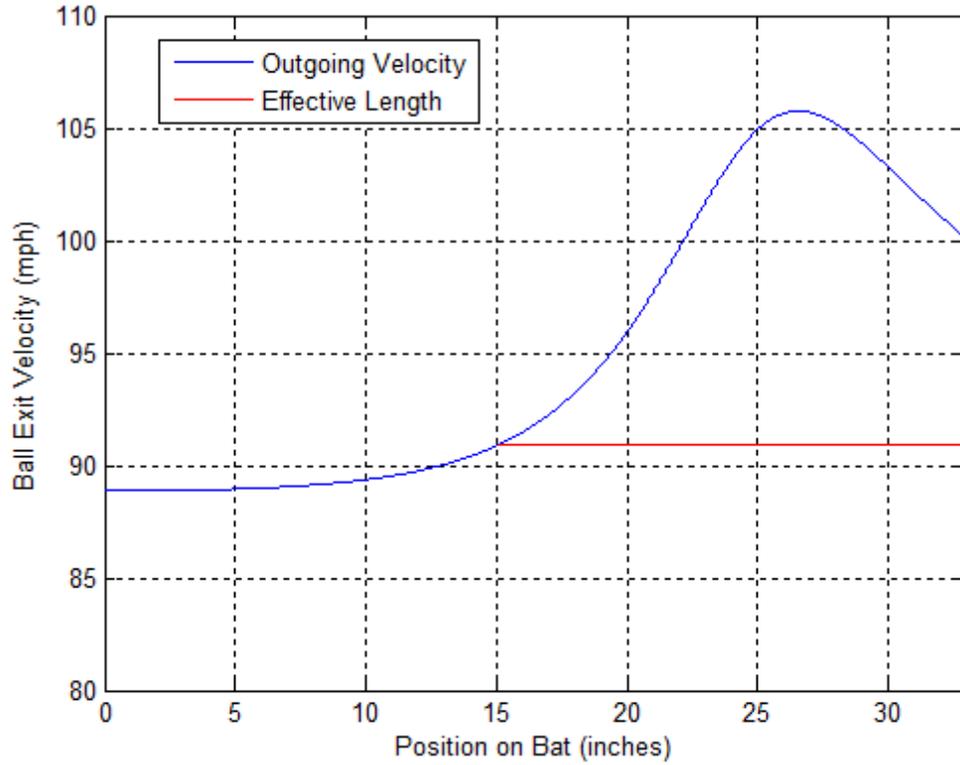


Figure 6: Velocity Curve Linear Constant Mass $a = 0$

The second bat profile analyzed is shown in Figure 7, and it produced a maximum velocity of 102.93 mi / hr, an effective length of 18.89 in, and a weighted performance of 329.74 mi * in / hr. For this bat, the moment of inertia was 19,695 oz * in². The velocity profile along the length of the bat along with the effective length is shown in Figure 8.

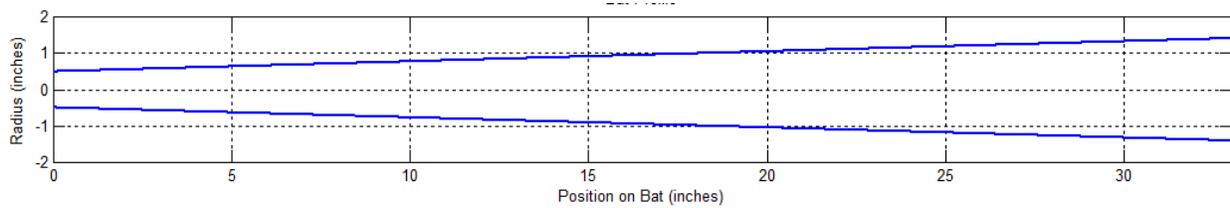


Figure 7: Linear Constant Mass Bat Profile $a = 0.49555$

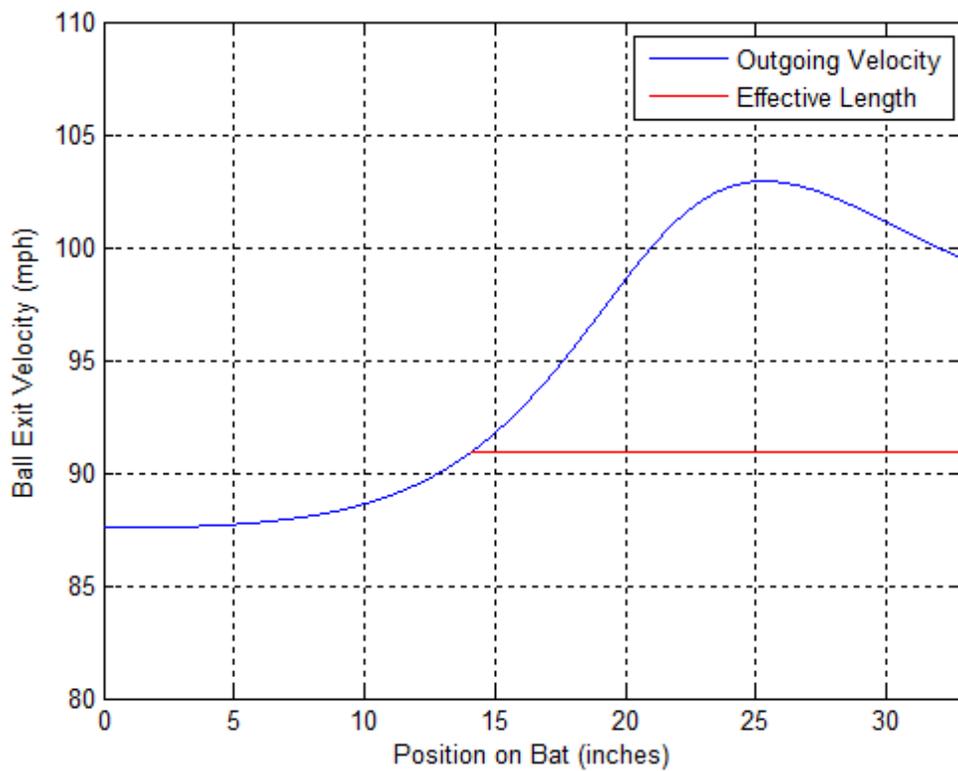


Figure 8: Velocity Curve Linear Constant Mass $a = 0.49555$

The next bat profile analyzed is shown in Figure 9, and it produced a maximum velocity of 98.90 mi / hr, an effective length of 19.31 in, and a weighted performance of 283.41 mi * in / hr. For this bat, the moment of inertia was 13,686 oz * in². The velocity profile along the length of the bat along with the effective length is shown in Figure 10.

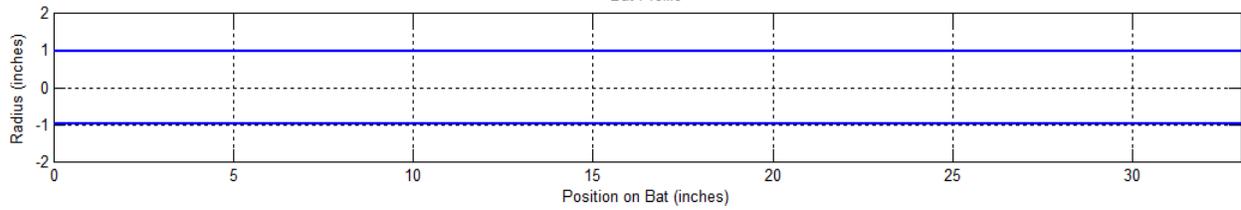


Figure 9: Linear Constant Mass Bat Profile $a = 0.9911$

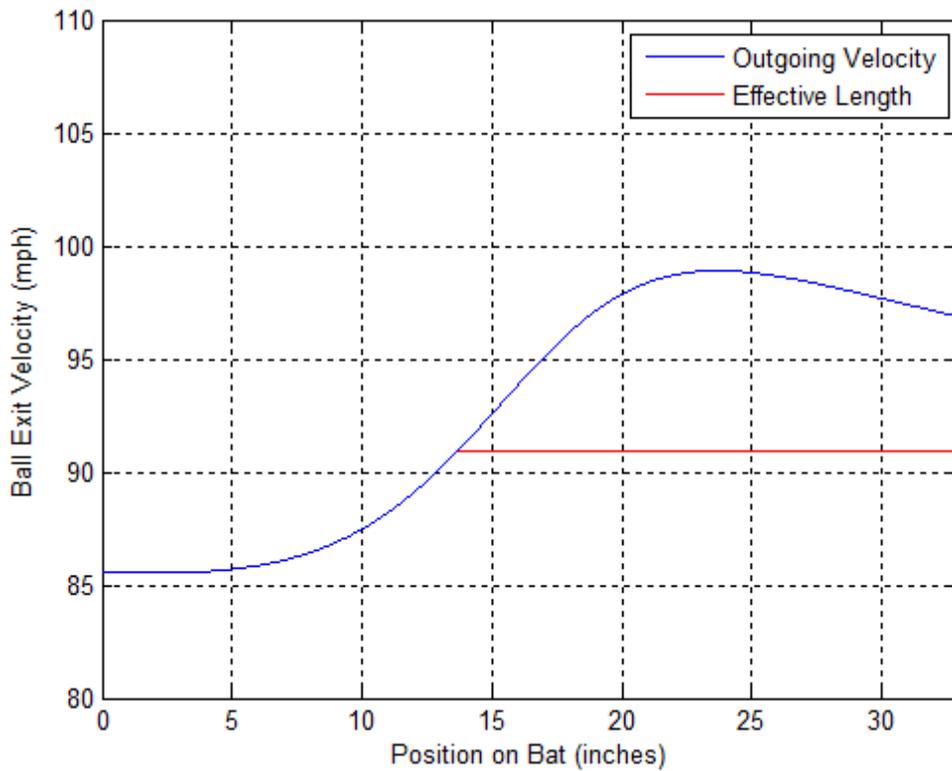


Figure 10: Velocity Curve Linear Constant Mass $a = 0.9911$

The fourth bat profile analyzed is shown in Figure 11, and it produced a maximum velocity of 95.44 mi / hr, an effective length of 19.35 in, and a weighted performance of 159.83 mi * in / hr. For this bat, the moment of inertia was 8,819 oz * in². The velocity profile along the length of the bat along with the effective length is shown in Figure 12

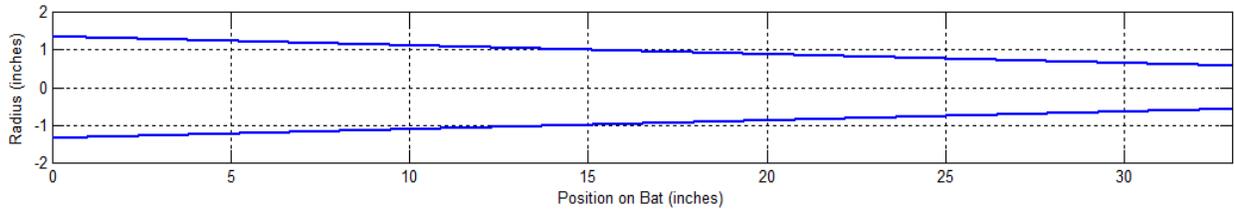


Figure 11: Linear Constant Mass Bat Profile a = 1.353828

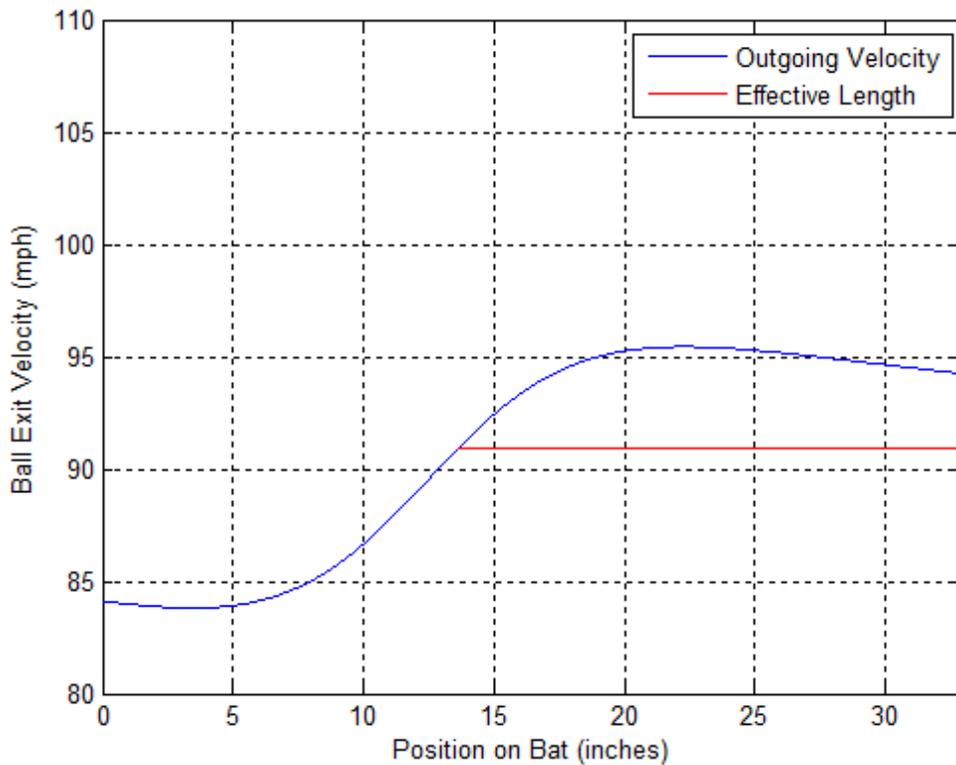


Figure 12: Velocity Curve Linear Constant Mass a = 1.353828

The final bat profile analyzed is shown in Figure 13, and it produced a maximum velocity of 92.24 mi / hr, an effective length of 18.81 in, and a weighted performance of 31.61 mi * in / hr. For this bat, the moment of inertia was 4,485 oz * in². The velocity profile along the length of the bat along with the effective length is shown in Figure 14.

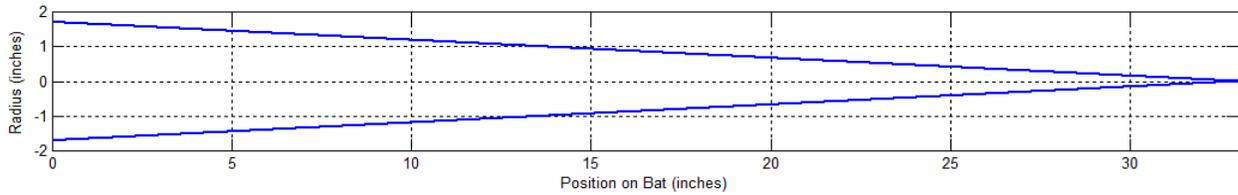


Figure 13: Linear Constant Mass Bat Profile $a = 1.716575$

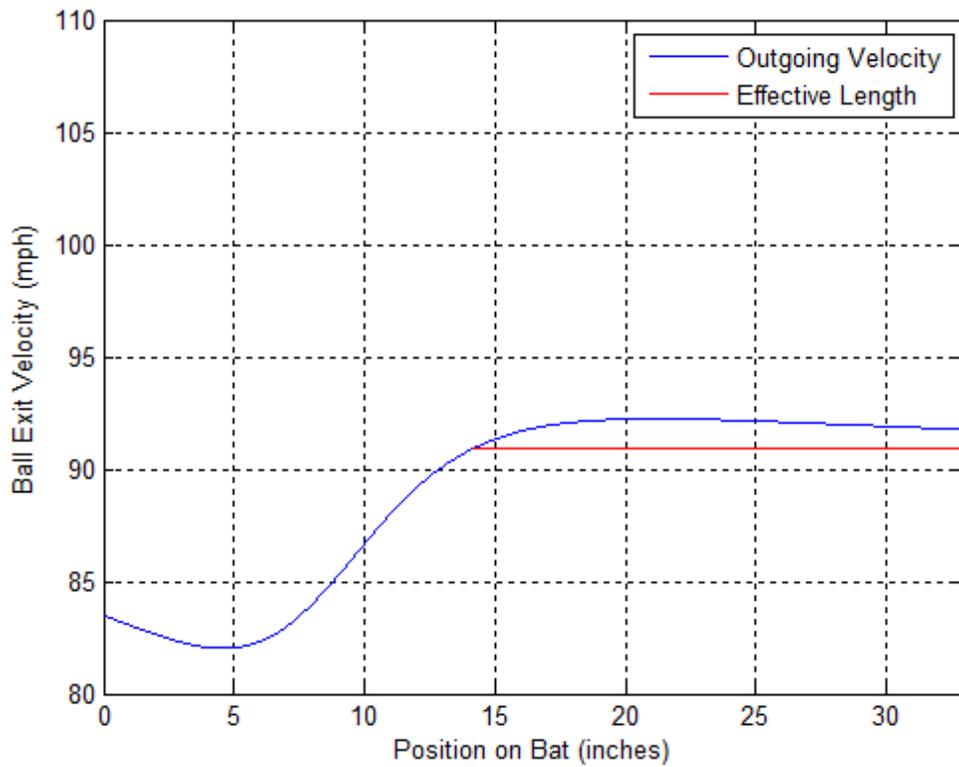


Figure 14: Velocity Curve Linear Constant Mass $a = 1.716575$

The results of all the bat profiles were compiled and the chart shown in Figure 15 shows the relationship between handle diameter and the maximum velocity performance measure. As shown the relationship is almost linear, with maximum velocity decreasing as the handle diameter increases. The chart shown in Figure 16 shows the relationship between the effective length and handle diameter. This relationship is slightly more complicated as effective length reaches a peak slightly past the point of linear bats. Since the weighted performance is a combination of the effective length and maximum velocity performance measures, the graph in Figure 16 reflects both of the relationships. The final graph, shown in Figure 18, shows the relationship between moment of inertia and handle diameter. As the handle diameter increases, moment of inertia decreases fairly linearly in the same manner as the maximum velocity, showing the relationship between the two variables. Besides impacting the performance factors, the moment of inertia greatly increases the feel of the bat as moment of inertia is often referred to as swing weight in swinging applications. The higher the moment of inertia, the heavier the bat feels to swing. This case was done under the assumption that all bats could be swung at the same angular velocity in order to show the performance of the bat under the same circumstances. However, in real applications where swing weight will vary the velocity at which the bat can be swung, the moment of inertia would be a very important factor.

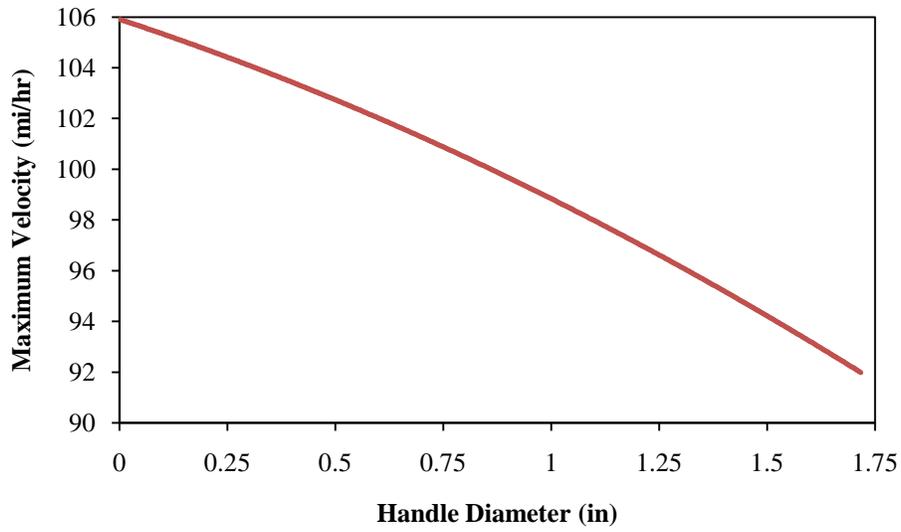


Figure 15: Maximum Velocity vs. Handle Diameter for linear constant mass bats

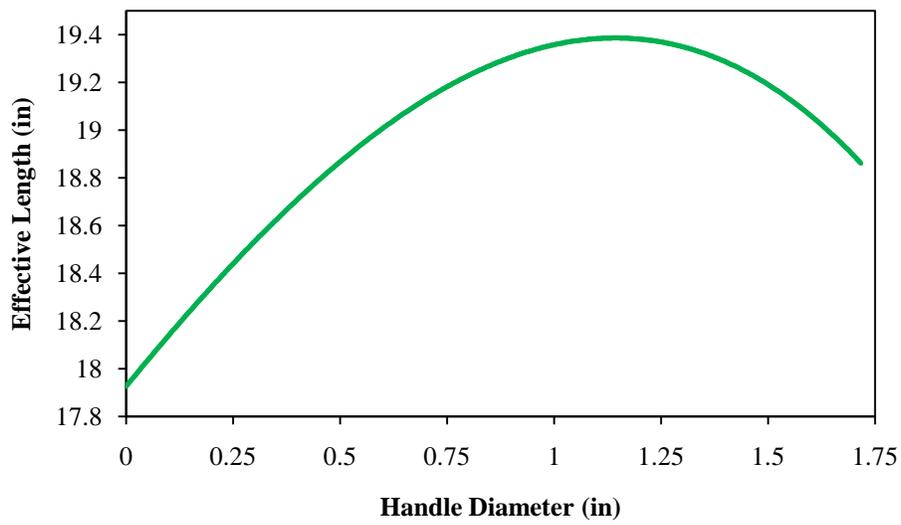


Figure 16: Effective Length vs. Handle Diameter for linear constant mass bats

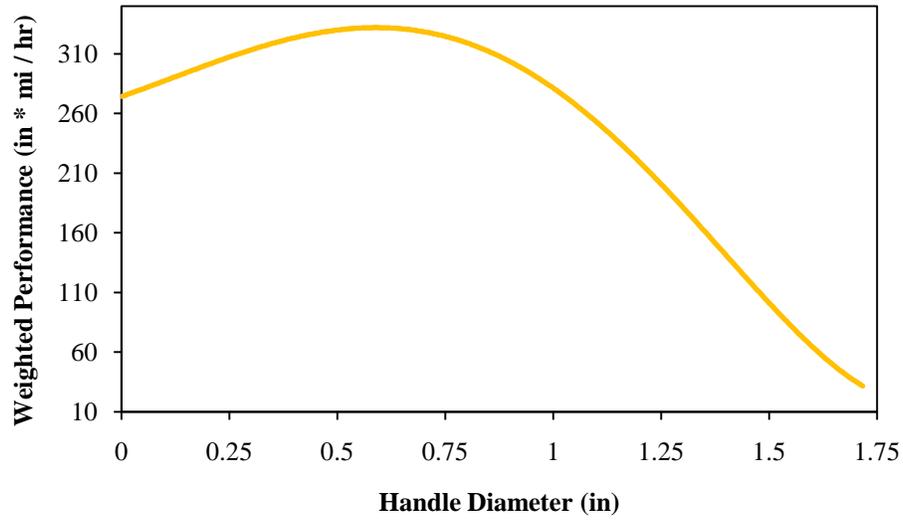


Figure 17: Weighted Performance vs. Handle Diameter for linear constant mass bats

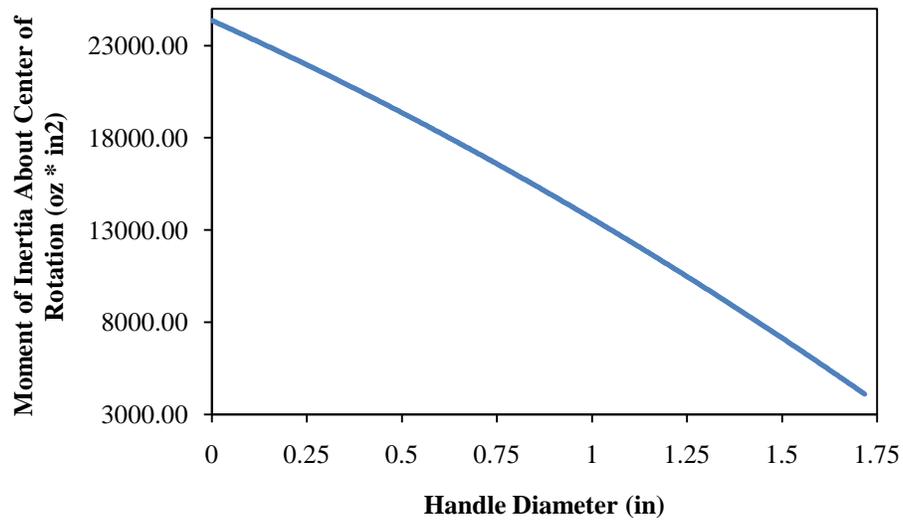


Figure 18: Moment of Inertia vs. Handle Diameter for linear constant mass bats

3.2 *Linear Profile: Constant Moment of Inertia*

After completing the case of linear profile with constant mass, we decided to allow mass to vary, and instead use the moment of inertia about the point of rotation to relate the slope and handle diameter. We felt that using this constant moment case would be beneficial in that while the mass of the bat would be allowed to change, the “swing weight” would remain the same. This swing weight is how the bat feels to the player during the swing. Being as this case was also performed using a linear radial profile $R(x)$, the cubic coefficients c and d were once again set to zero.

To find the proper relationship between r_0^2 and b the equation for moment of inertia about the center of rotation was used as follows

$$I_p = \pi\rho \int_0^l R(x)^2 (h+x)^2 dx \quad (51)$$

where h is the length from the handle of the bat to the center of rotation. Substituting $R(x)$ and expanding yields

$$I_p = \pi\rho \int_0^l (r_0^2 + 2br_0x + b^2x^2)(h^2 + 2hx + x^2)dx \quad (52)$$

Further simplifying and grouping x terms gives the equation

$$I_p = \pi\rho \int_0^l (r_0^2h^2 + (2r_0^2h + 2br_0h^2)x + (r_0^2 + 4hbr_0 + b^2d^2)x^2 + (2br_0 + 2hb^2)x^3 + b^2x^4)dx \quad (53)$$

Performing the integration and regrouping in terms of b yields

$$0 = Ab^2 + Bb + C \quad (54)$$

Where

$$A = \pi\rho \left(\frac{h^2l^3}{3} + \frac{hl^4}{2} + \frac{l^5}{5} \right) \quad (55)$$

$$B = \pi\rho \left(r_0h^2l^2 + \frac{4hr_0l^3}{3} + \frac{r_0l^4}{2} \right) \quad (56)$$

$$C = -I_p + \rho\pi \left(r_0^2h^2l + r_0^2hl^2 + \frac{r_0^2l^3}{3} \right) \quad (57)$$

which can be solved for b using the quadratic formula as

$$b = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \quad (58)$$

As with the case of constant mass two roots are found for b for any given r_0 and the larger of these two values should be used.

The first bat profile analyzed for the constant moment linear case study is shown in Figure 19, and it produced a maximum velocity of 115.35 mi / hr, an effective length of 18.67 in, and a weighted performance of 527.39 mi * in / hr with a mass of 18.80 oz. The velocity profile along the length of the bat along with the effective length is shown in Figure 20.

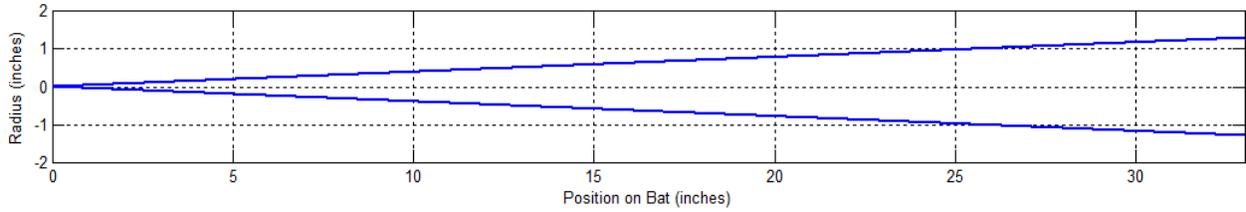


Figure 19: Linear Constant Moment Bat Profile a = 0

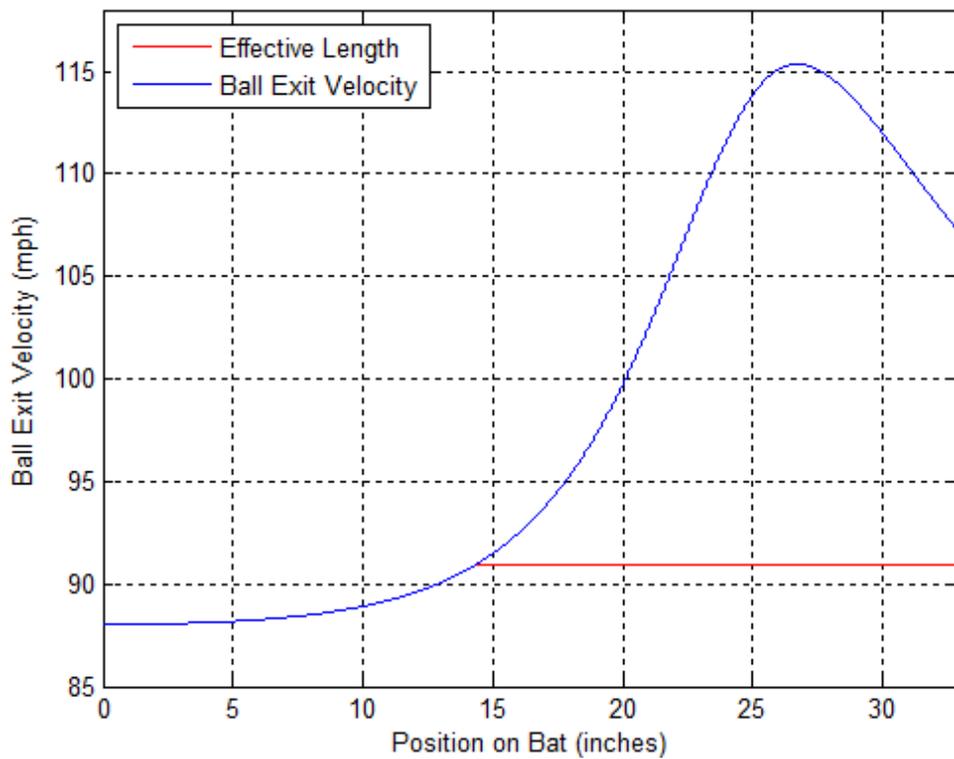


Figure 20: Velocity Curve Linear Constant Moment a = 0

The second bat profile analyzed for the constant moment linear case study is shown in Figure 21, and it produced a maximum velocity of 106.24 mi / hr, an effective length of 19.25 in, and a weighted performance of 496.23 mi * in / hr with a mass of 24.10 oz. The velocity profile along the length of the bat along with the effective length is shown in Figure 22.

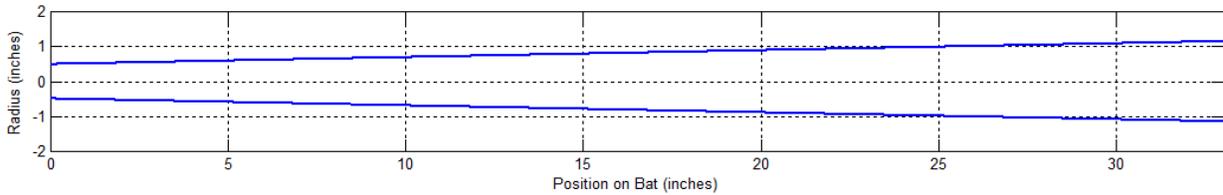


Figure 21: Linear Constant Moment Bat Profile $a = 0.49555$

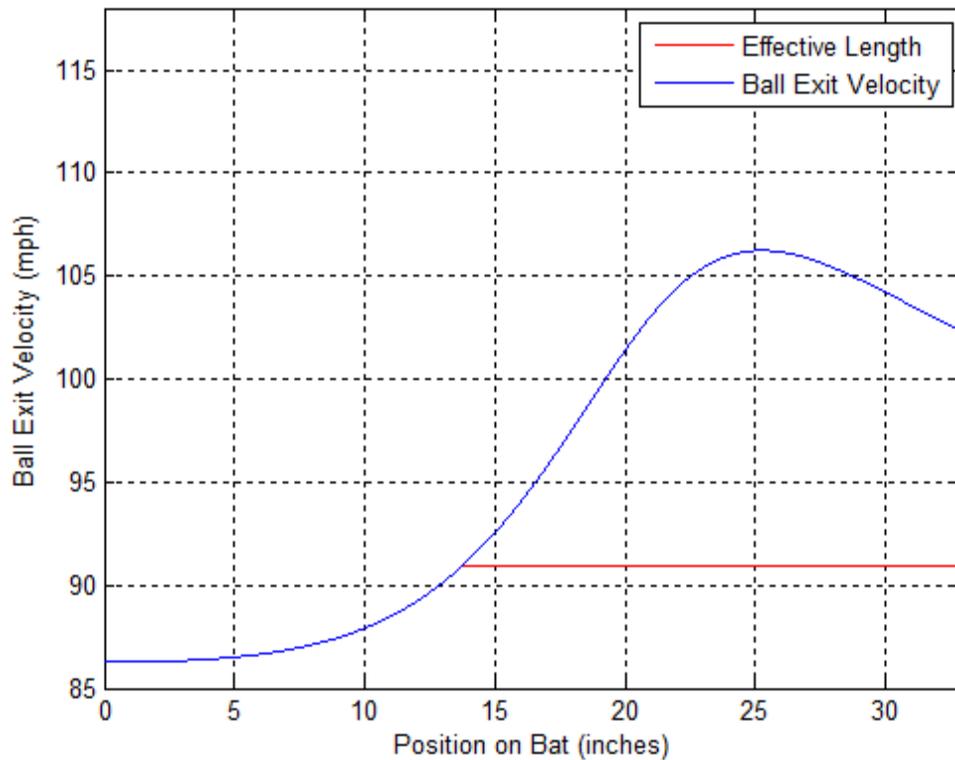


Figure 22: Velocity Curve Linear Constant Moment $a = 0.49555$

The third bat profile analyzed for the constant moment linear case study is shown in Figure 23, and it produced a maximum velocity of 98.90 mi / hr, an effective length of 19.31 in, and a weighted performance of 283.22 mi * in / hr with a mass of 33.02 oz. The velocity profile along the length of the bat along with the effective length is shown in Figure 24.

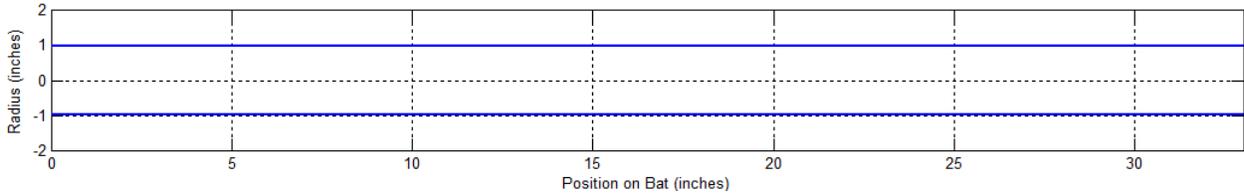


Figure 23: Linear Constant Moment Bat Profile $a = 0.9911$

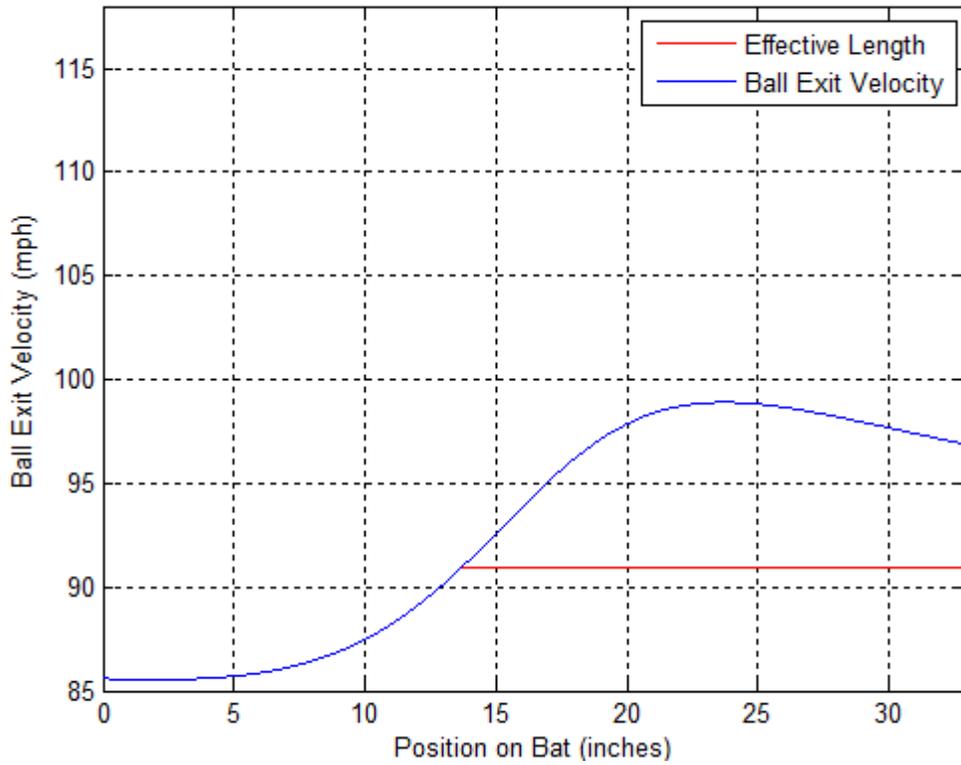


Figure 24: Velocity Curve Linear Constant Moment $a = 0.9911$

The fourth bat profile analyzed for the constant moment linear case study is shown in Figure 25, and it produced a maximum velocity of 95.54 mi / hr, an effective length of 19.19 in, and a weighted performance of 151.02 mi * in / hr with a mass of 41.73 oz. The velocity profile along the length of the bat along with the effective length is shown in Figure 26.

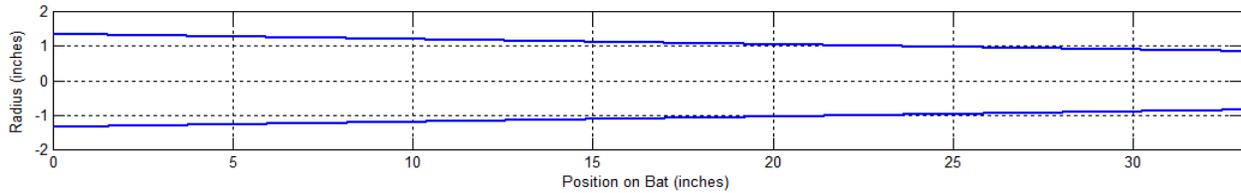


Figure 25: Linear Constant Moment Bat Profile $a = 1.353828$

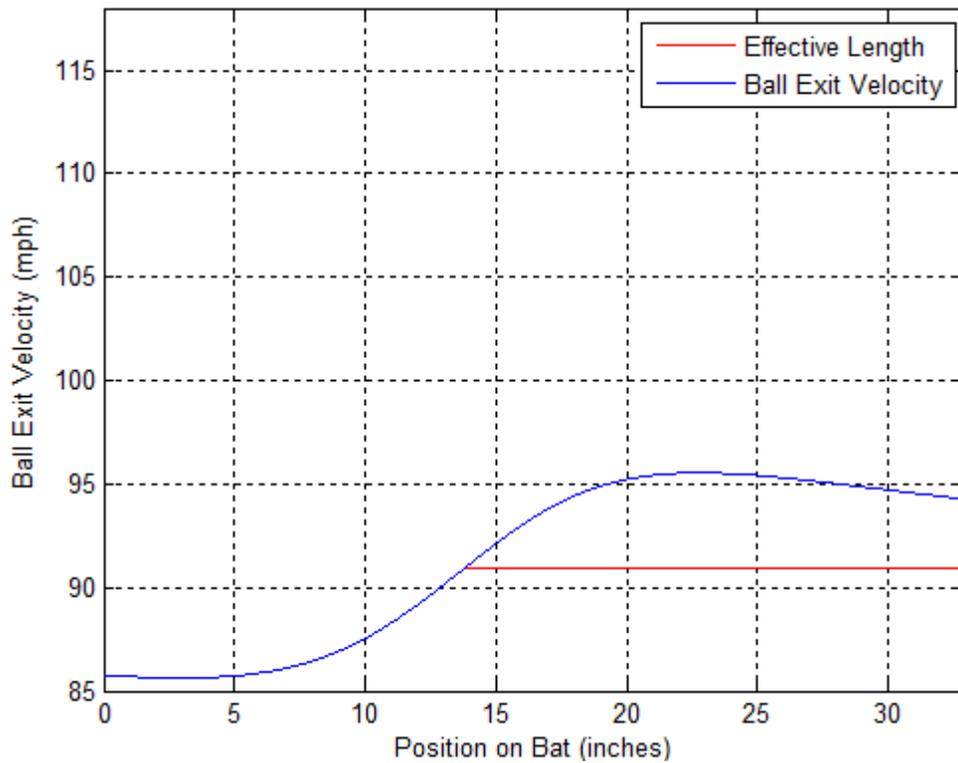


Figure 26: Velocity Curve Linear Constant Moment $a = 1.353828$

The final bat profile analyzed for the constant moment linear case study is shown in Figure 27, and it produced a maximum velocity of 93.45 mi / hr, an effective length of 18.94 in, and a weighted performance of 69.52 mi * in / hr with a mass of 52.16 oz. The velocity profile along the length of the bat along with the effective length is shown in Figure 28.

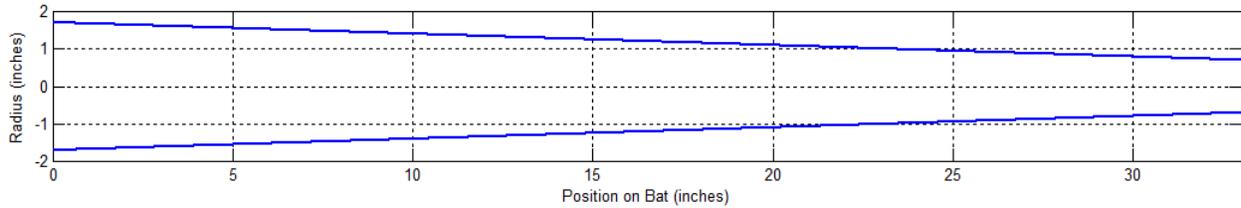


Figure 27: Linear Constant Moment Bat Profile $a = 1.716575$

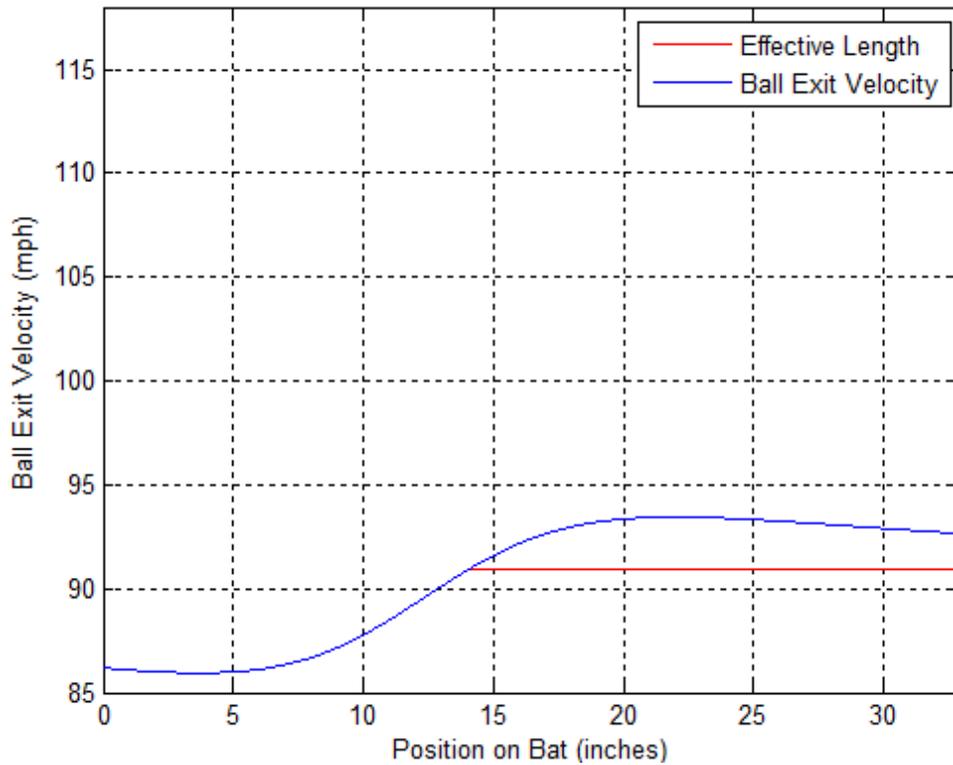


Figure 28: Velocity Curve Linear Constant Moment $a = 1.726575$

The results of all the bat profiles were compiled and the chart shown in Figure 29 shows the relationship between handle diameter and the maximum velocity performance measure. As shown the relationship appears to approach a limit as the handle diameter increases, with maximum velocity decreasing as the handle diameter increases. The chart shown in Figure 30 shows the relationship between the effective length and handle diameter. This relationship is slightly more complicated as effective length reaches a peak at the diameter that produces a linear bat. Since the weighted performance is a combination of the effective length and maximum velocity performance measures, the graph in Figure 31 reflects both of the relationships. The final graph, shown in Figure 32, shows the relationship mass and handle diameter. As the handle diameter increases, mass increases in an exponential manner, showing the relationship between the two variables. The mass appears to have no impact on the other performance factors created, further showing that the driving factor in baseball bat performance is the moment of inertia, or “swing weight.” After completing these basic linear cases that displayed the general relationships between the different performance factors and variables, the team decided that a more realistic approach was needed. To do this, cubic profiles were analyzed, and are discussed in the upcoming sections.

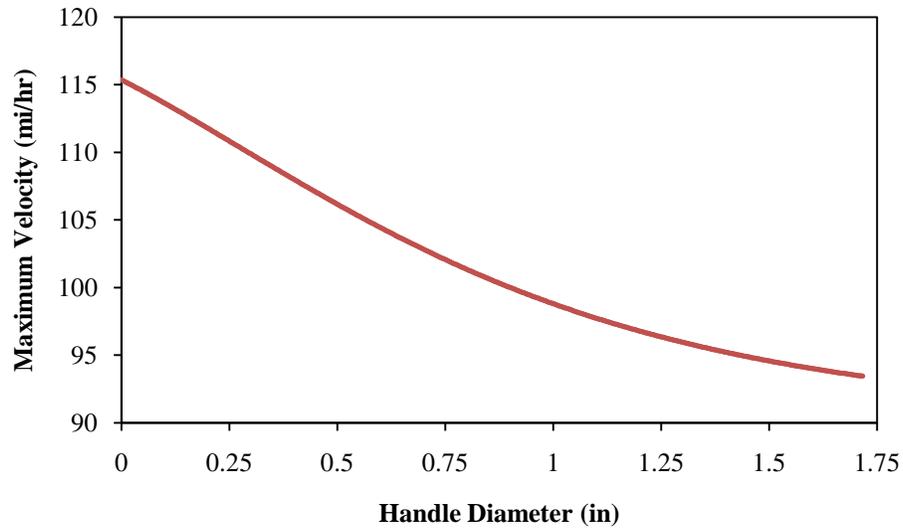


Figure 29: Maximum Velocity vs. Handle Diameter for linear constant moment bats

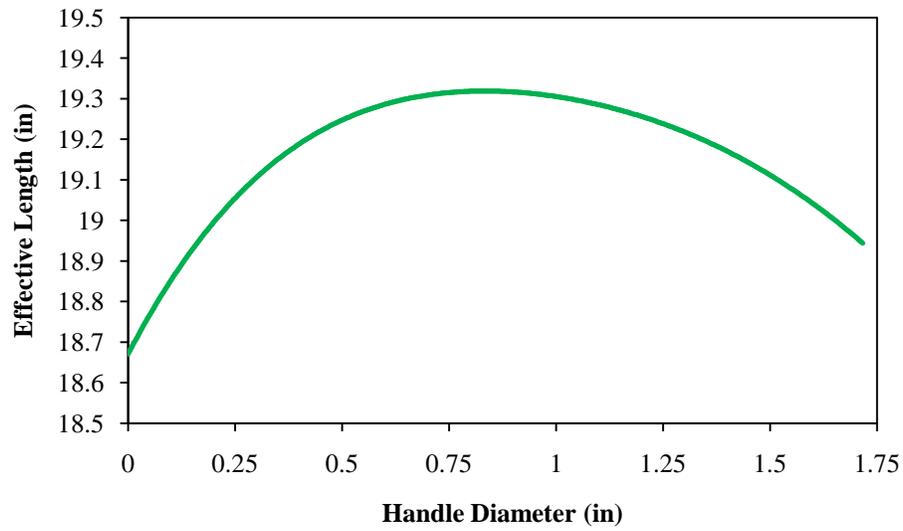


Figure 30: Effective Length vs. Handle Diameter for linear constant moment bats

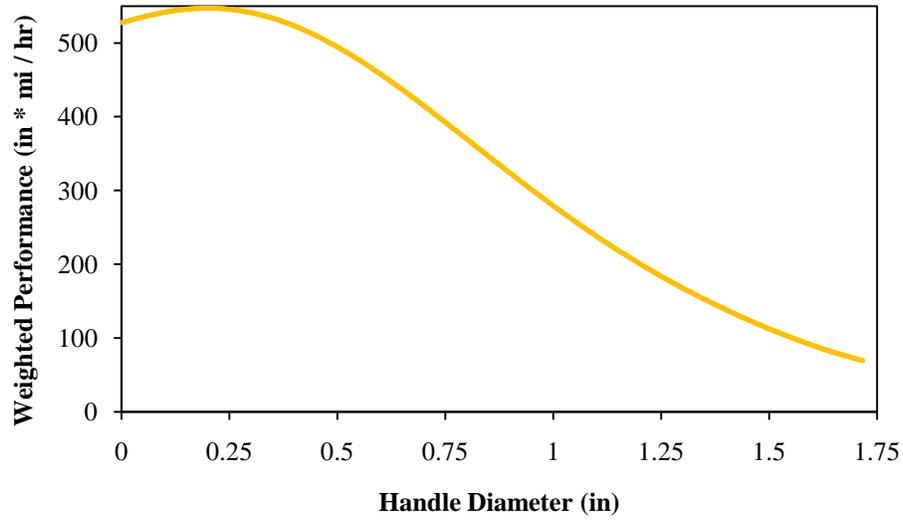


Figure 31: Weighted Performance vs. Handle Diameter for linear constant moment bats

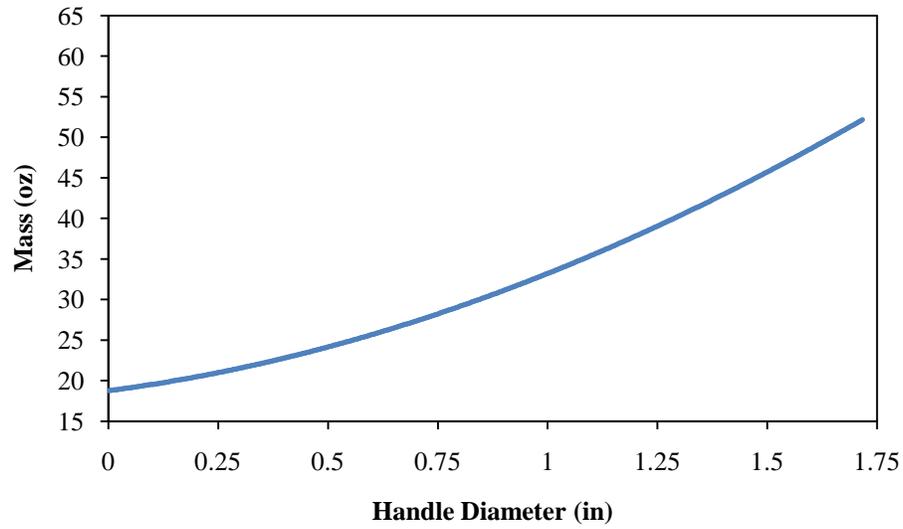


Figure 32: Mass vs. Handle Diameter for linear constant moment bats

3.3 Cubic Profile: Constant Mass

After completing the cases of linear profile the team created a series of “realistic” baseball bat shapes using the complete cubic profile for $R(x)$. In both of the following cubic profile cases two parameters deemed to be necessary in a “realistic” bat were used in creating the profiles. These parameters were a handle diameter of 1 inch and a handle slope of 0. This insures the existence of an appropriate handle for use by the player. It can be easily shown that this condition for 0 slope at the handle defines the coefficient b as 0 using the derived equation for slope along the bat

$$R'(x) = b + 2cx + 3dx^2 \quad (59)$$

Which reduces $R(x)$ for these cases to

$$R(x) = r_0 + cx^2 + dx^3 \quad (60)$$

For the first cubic profile case the cubic coefficients c and d were related using the condition of constant mass. In this way a variety of bat profiles with constant mass could be produced for a given range of the cubic coefficient d . Again using the equation for mass

$$m_1 = \int_0^l \rho\pi R(x)^2 dx \quad (61)$$

Substituting $R(x)$ and expanding yields

$$m_1 = \rho\pi \int_0^l (r_0^2 + 2r_0cx^2 + 2r_0dx^3 + c^2x^4 + 2cdx^5 + d^2x^6) dx \quad (62)$$

Performing the integration and grouping terms of c gives the form

$$0 = Ac^2 + Bc + C \quad (63)$$

where

$$A = \pi\rho \left(\frac{l^5}{5}\right) \quad (64)$$

$$B = \pi\rho \left(\frac{2r_0l^3}{3} + \frac{dl^6}{3}\right) \quad (65)$$

$$C = -m_1 + \pi\rho \left(r_0^2l + \frac{r_0dl^4}{2} + \frac{d^2l^7}{7}\right) \quad (66)$$

Which can again be solved using the quadratic formula

$$c = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \quad (67)$$

Once this relation had been formed we found by trial and error an appropriate range of values for the cubic coefficient d to be from -0.0001 to 0. Any values of d outside this range produced profiles that were deemed to be “unrealistic” and were therefore excluded from this case.

The first bat profile analyzed for the constant mass cubic case study is shown in Figure 33, and it produced a maximum velocity of 101.77 mi / hr, an effective length of 18.91 in, and a weighted performance of 268.21 mi * in / hr with a moment of inertia of 18,140 oz * in². The velocity profile along the length of the bat along with the effective length is shown in Figure 34.

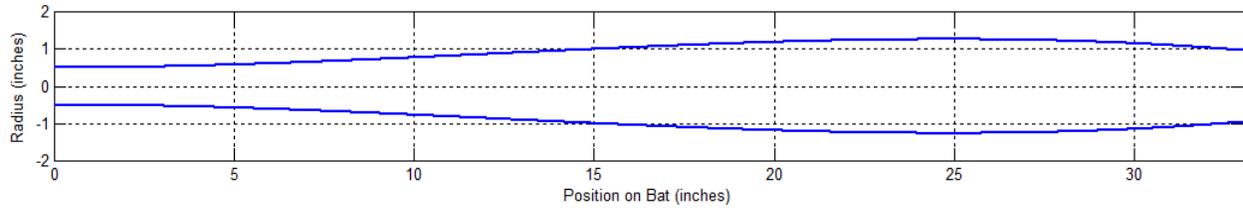


Figure 33: Cubic Constant Mass Bat Profile $d = -0.0001$

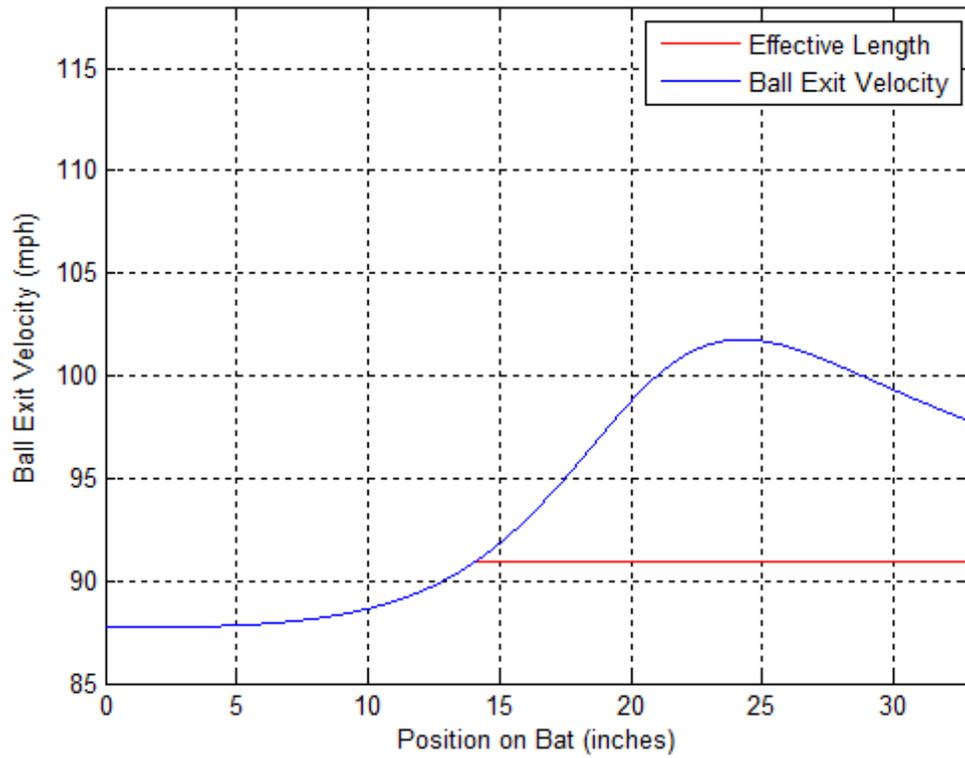


Figure 34: Velocity Curve Cubic Constant Mass $d = -0.0001$

The second bat profile analyzed for the constant mass cubic case study is shown in Figure 35, and it produced a maximum velocity of 102.61 mi / hr, an effective length of 18.84 in, and a weighted performance of 306.41 mi * in / hr with a moment of inertia of 19,298 oz * in². The velocity profile along the length of the bat along with the effective length is shown in Figure 36.

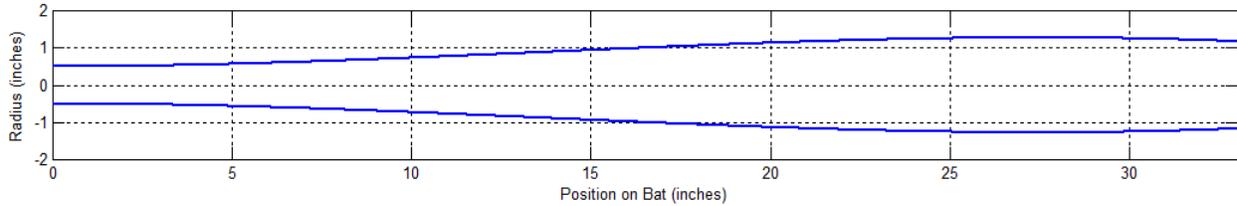


Figure 35: Cubic Constant Mass Bat Profile $d = -0.000075$

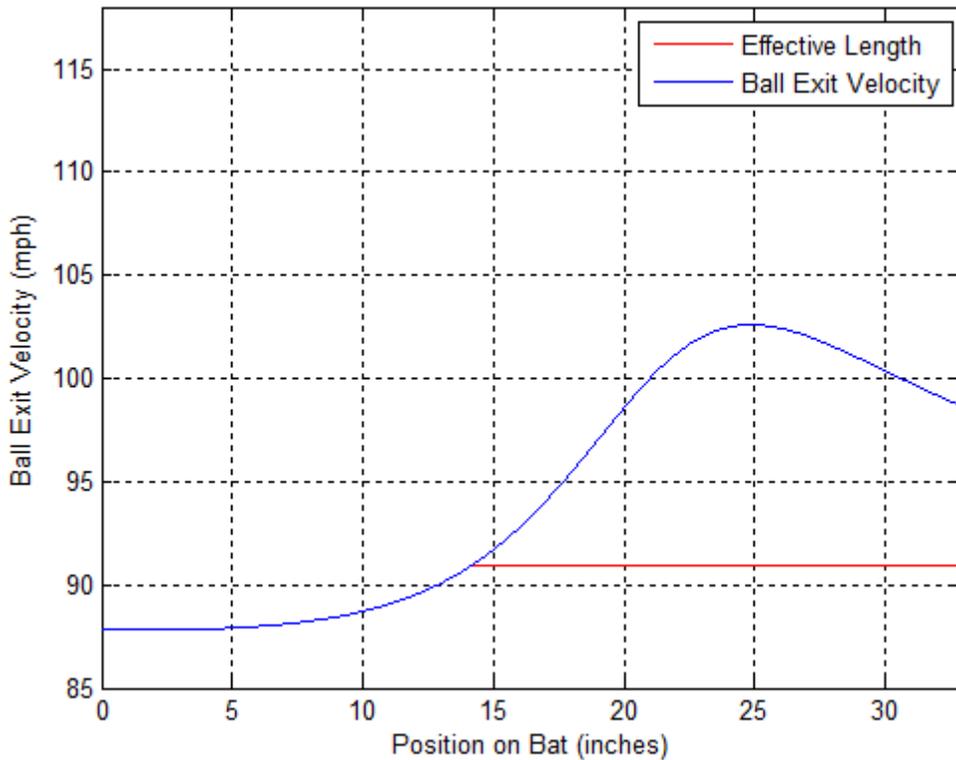


Figure 36: Velocity Curve Cubic Constant Mass $d = -0.000075$

The third bat profile analyzed for the constant mass cubic case study is shown in Figure 37, and it produced a maximum velocity of 103.43 mi / hr, an effective length of 18.76 in, and a weighted performance of 318.00 mi * in / hr with a moment of inertia of 20,477 oz * in². The velocity profile along the length of the bat along with the effective length is shown in Figure 38.

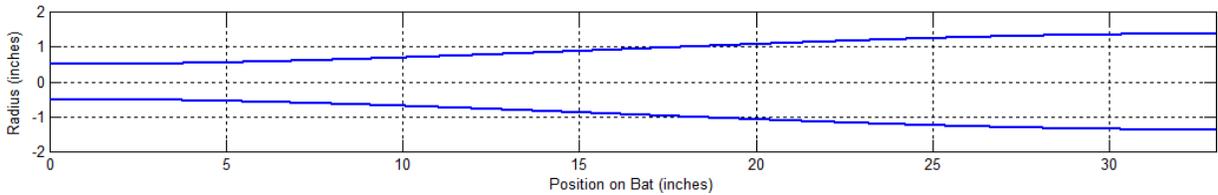


Figure 37: Cubic Constant Mass Bat Profile $d = -0.00005$

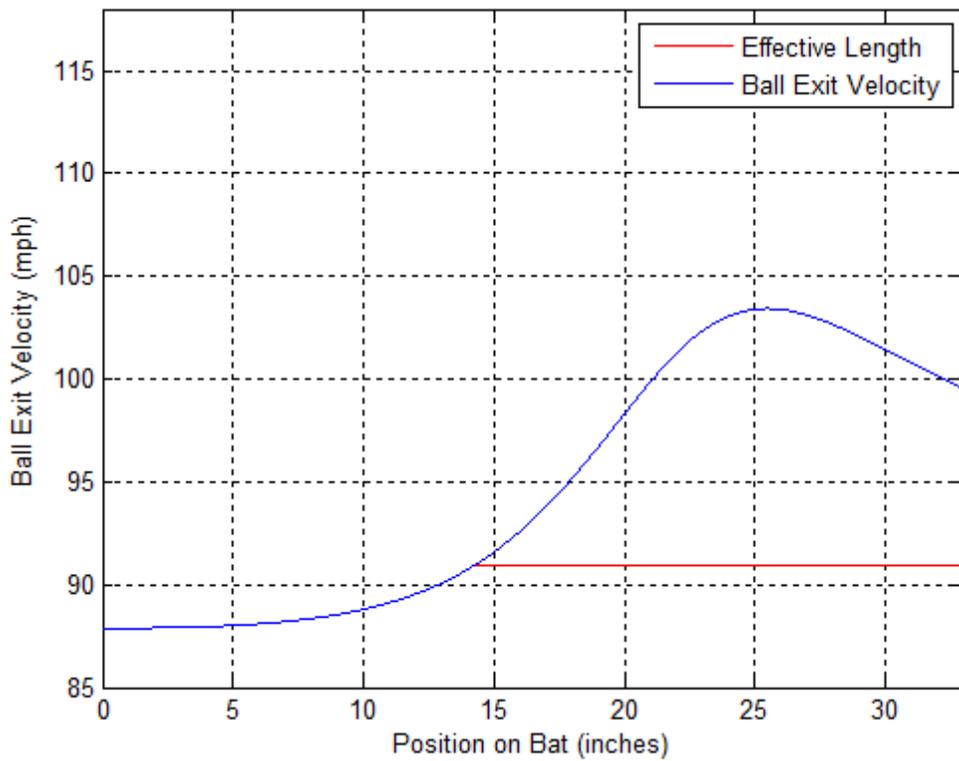


Figure 38: Velocity Curve Cubic Constant Mass $d = -0.00005$

The fourth bat profile analyzed for the constant mass cubic case study is shown in Figure 39, and it produced a maximum velocity of 104.24 mi / hr, an effective length of 18.66 in, and a weighted performance of 338.92 mi * in / hr with a moment of inertia of 21.662 oz * in². The velocity profile along the length of the bat along with the effective length is shown in Figure 40.

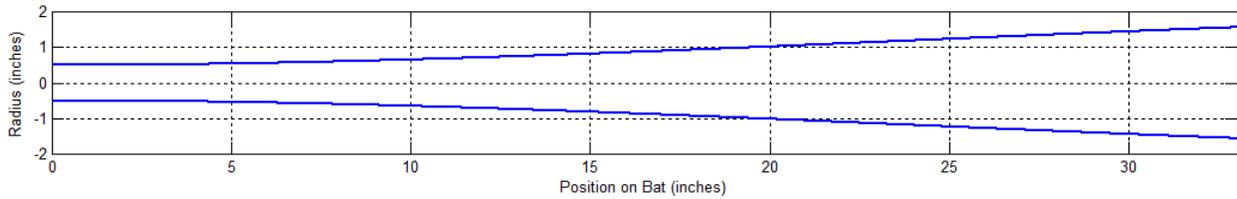


Figure 39: Cubic Constant Mass Bat Profile $d = -0.000025$

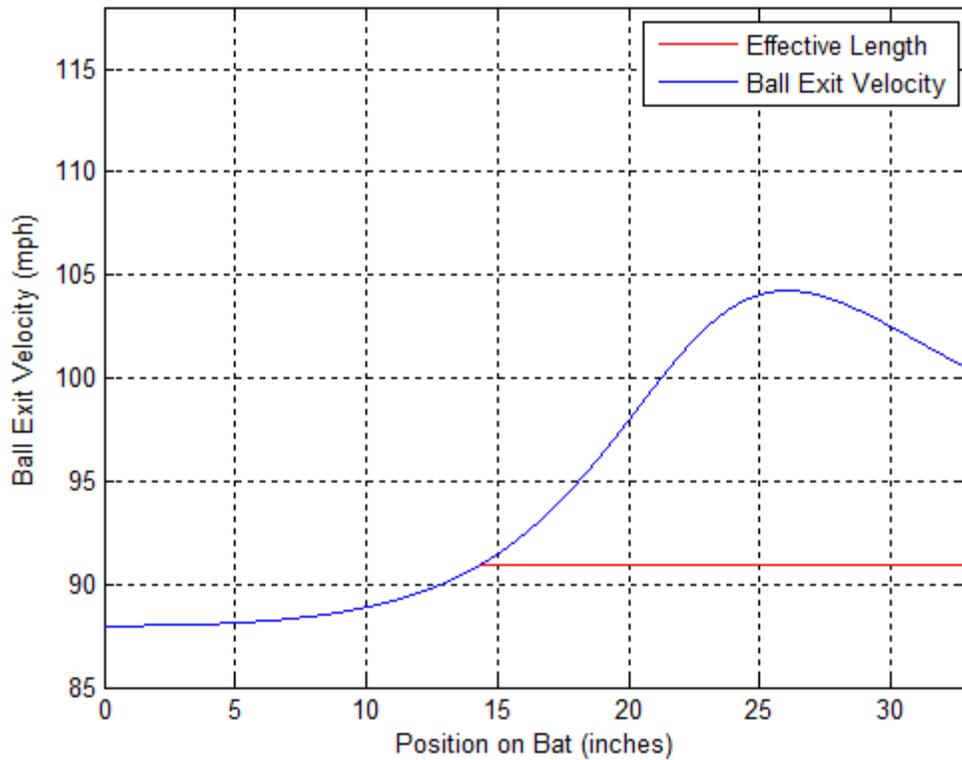


Figure 40: Velocity Curve Cubic Constant Mass $d = -0.000025$

The final bat profile analyzed for the constant mass cubic case study is shown in Figure 41, and it produced a maximum velocity of 105.03 mi / hr, an effective length of 18.55 in, and a weighted performance of 356.40 mi * in / hr with a moment of inertia of 22,841 oz * in². The velocity profile along the length of the bat along with the effective length is shown in Figure 42.

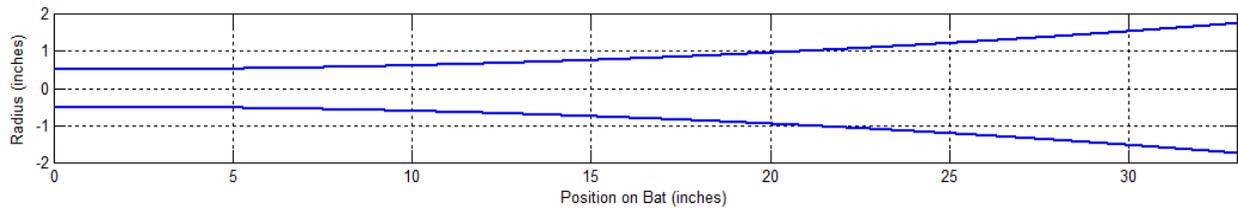


Figure 41: Cubic Constant Mass Bat Profile $d = 0$

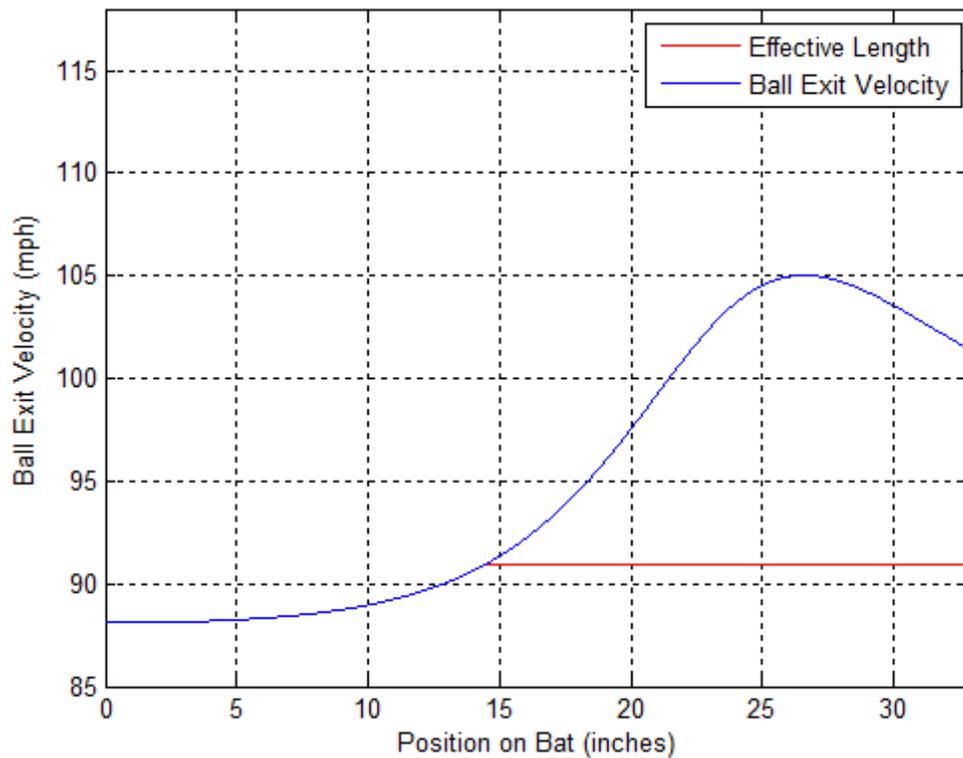


Figure 42: Velocity Curve Cubic Constant Mass $d = 0$

The graph in Figure 47 displays the percent change of all the performance measures along with the percent change of the moment of inertia. This graph provides a synopsis of the data displayed in Figure 43, Figure 44, Figure 45, and Figure 46 on a common coordinate system. It shows how the change maximum velocity and effective are fairly small, less than a five percent increase and decrease respectively. However, it also shows that the moment of inertia and weighted performance both vary in relation to each other; both have a large increase of 26 percent and 33 percent, respectively. These are very large increases given the constraints put on the bat profiles and the relatively small changes in the profile across the range of the parameter d. As discussed before, the moment of inertia has a very large impact not only on the performance measures, but also on the feel of the bat and the ability of the player to swing the bat at a high enough velocity to exploit the performance advantages.

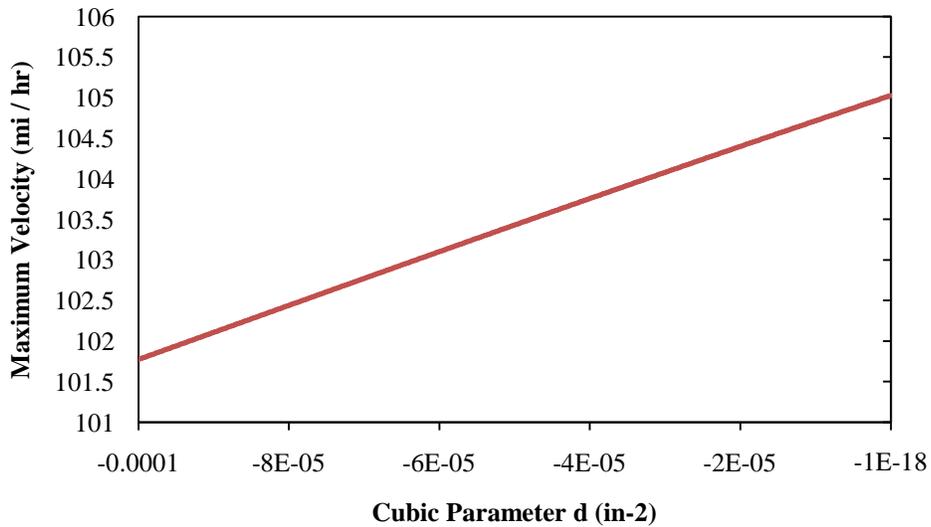


Figure 43: Maximum Velocity vs. Handle Diameter for cubic constant mass bats

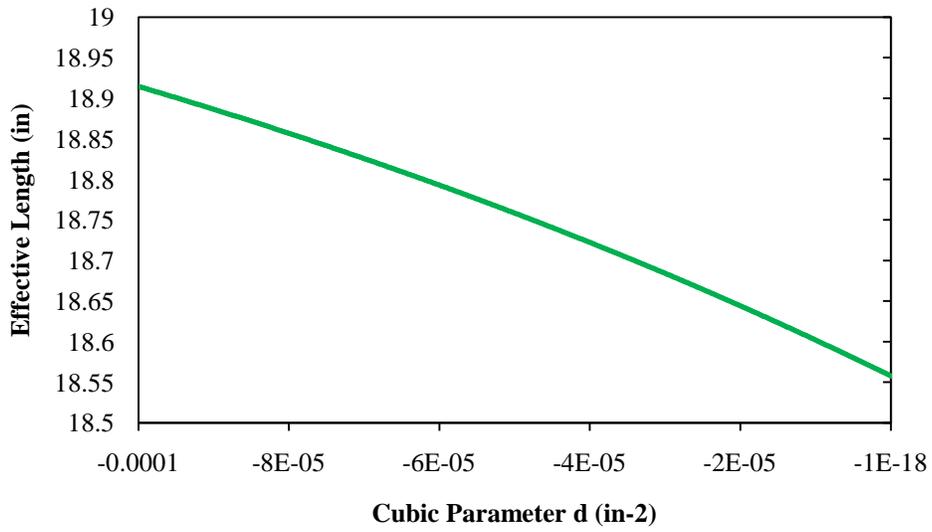


Figure 44: Effective Length vs. Handle Diameter for cubic constant mass bats

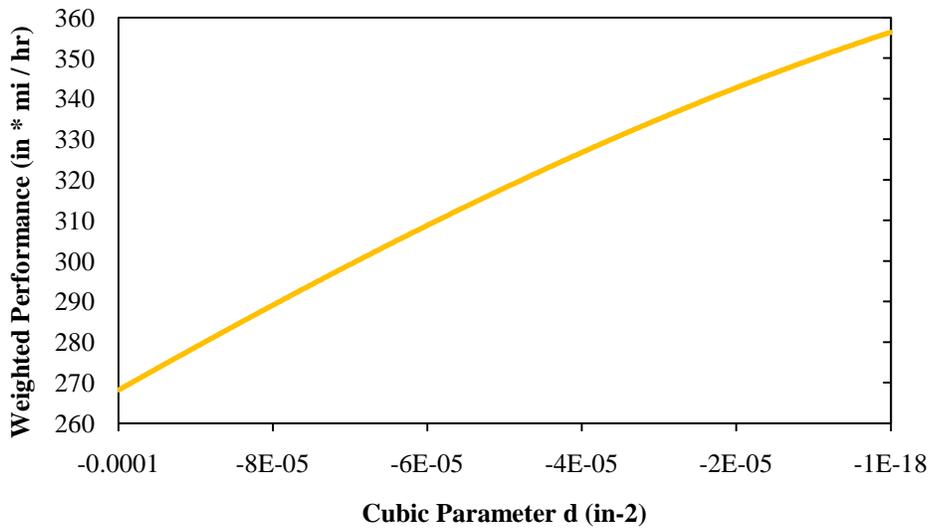


Figure 45: Weighted Performance vs. Handle Diameter for cubic constant mass bats

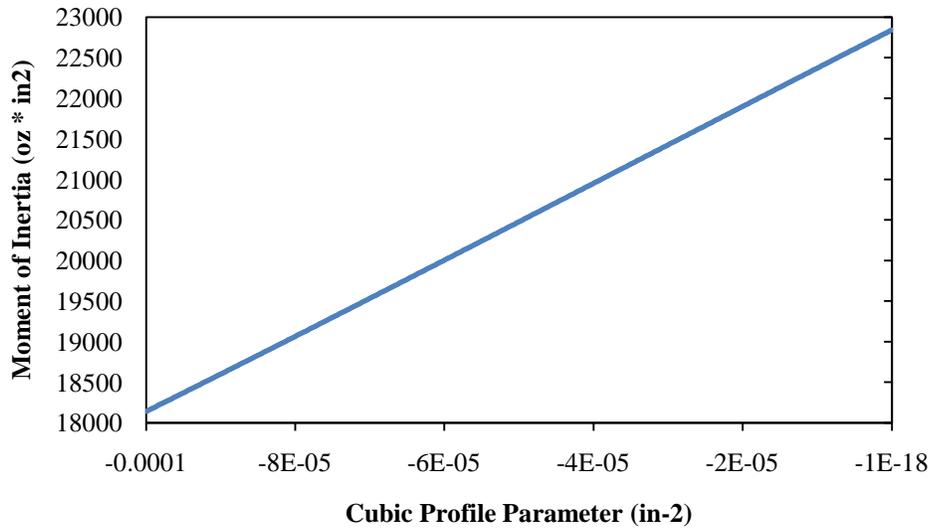


Figure 46: Moment of Inertia vs. Handle Diameter for cubic constant mass bats

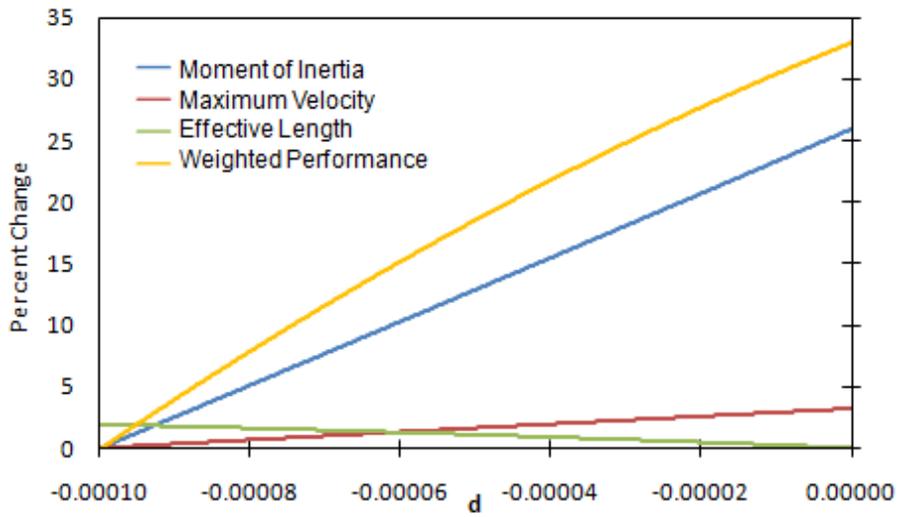


Figure 47: Percent Change vs. Cubic Parameter d for cubic constant mass bats

3.4 Cubic Profile: Constant Moment of Inertia

The final case considered by the team used a radial profile generated by a cubic equation was that of constant moment of inertia about the point of rotation. As stated previously the profiles considered were constrained to “realistic” bats where r_0 equals 0.5 inches and the slope of the profile at the handle is zero. Once again, the condition of zero slope at the handle reduces the profile equation to

$$R(x) = r_0 + cx^2 + dx^3 \quad (68)$$

For this case the cubic coefficients c and d were related using a constant moment of inertia about the point of rotation, or “swing weight.” This relationship was determined using the equation for moment of inertia about the point of rotation as follows

$$I_p = \pi\rho \int_0^l R(x)^2(h+x)^2 dx \quad (69)$$

Substituting, expanding and grouping by x yields

$$\begin{aligned} I_p = \pi\rho \int_0^l & [r_0^2 h^2 + 2r_0^2 hx + (2r_0 ch^2 + r_0^2)x^2 + (2r_0 dh^2 + 4r_0 ch)x^3 \\ & + (c^2 h^2 + 2r_0 c + 4hr_0 d)x^4 + (2cdh^2 + 2hc^2 + 2r_0 d)x^5 \\ & + (d^2 h^2 + 4hcd + c^2)x^6 + (2hd^2 + 2cd)x^7 + d^2 x^8] dx \end{aligned} \quad (70)$$

Next, the integration is performed and arranged to the form

$$0 = Ac^2 + Bc + C \quad (71)$$

where

$$A = \pi\rho \left[\frac{h^2 l^5}{5} + \frac{h l^6}{3} + \frac{l^7}{7} \right] \quad (72)$$

$$B = \pi\rho \left[\frac{2r_0 h^2 l^3}{3} + \frac{2r_0 h l^4}{2} + \frac{2r_0 l^5}{5} + \frac{d h^2 l^6}{3} + \frac{4d h l^7}{7} + \frac{d l^8}{4} \right] \quad (73)$$

$$C = \pi\rho \left[h^2 r_0^2 l + r_0^2 h l^2 + \frac{r_0^2 l^3}{3} + \frac{r_0 d h^2 l^4}{2} + \frac{4r_0 d h l^5}{5} + \frac{r_0 d l^6}{3} + \frac{h^2 d^2 l^7}{7} + \frac{d^2 h l^8}{4} + \frac{d^2 l^9}{9} \right] - I_p \quad (74)$$

This equation can be solved using the quadratic formula

$$c = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \quad (75)$$

It was concluded, as with the case of cubic profile and constant mass that the appropriate values of d used to create “realistic” profiles ranged from -0.0001 to 0. In the same manner as used in the previous case, any values of d outside this range were discarded as “unrealistic” data.

The first bat profile analyzed for the constant moment cubic case study is shown in Figure 48, and it produced a maximum velocity of 101.22 mi / hr, an effective length of 18.77 in, and a weighted performance of 243.80 mi * in / hr with a mass of 36.13 oz. The velocity profile along the length of the bat along with the effective length is shown in Figure 49.

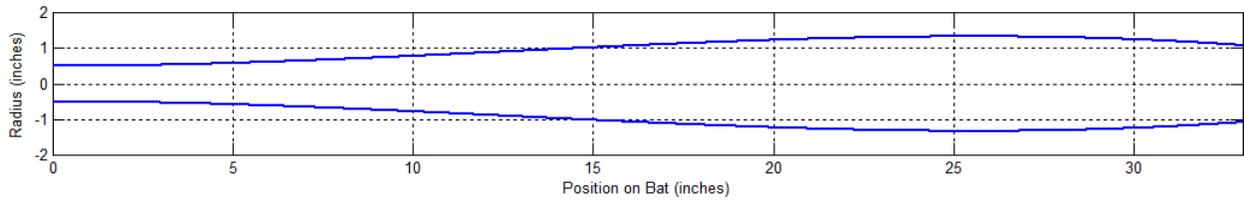


Figure 48: Cubic Constant Moment Bat Profile $d = -0.0001$

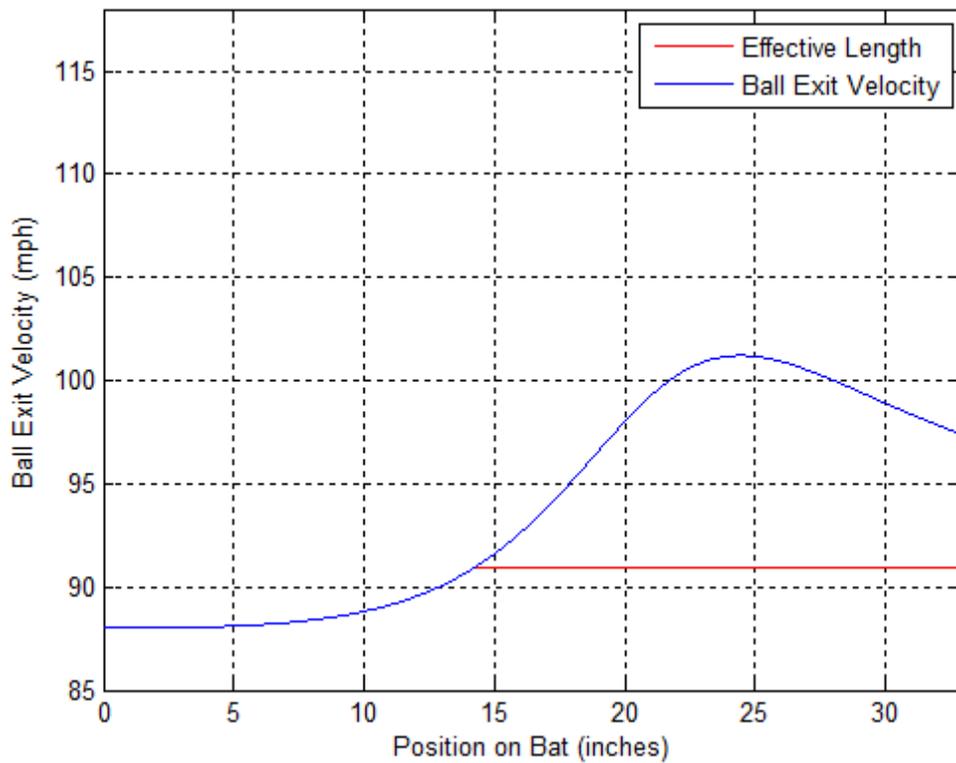


Figure 49: Velocity Curve Cubic Constant Moment $d = -0.0001$

The second bat profile analyzed for the constant moment cubic case study is shown in Figure 50, and it produced a maximum velocity of 102.28 mi / hr, an effective length of 18.77 in, and a weighted performance of 279.31 mi * in / hr with a mass of 34.53 oz. The velocity profile along the length of the bat along with the effective length is shown in Figure 51.

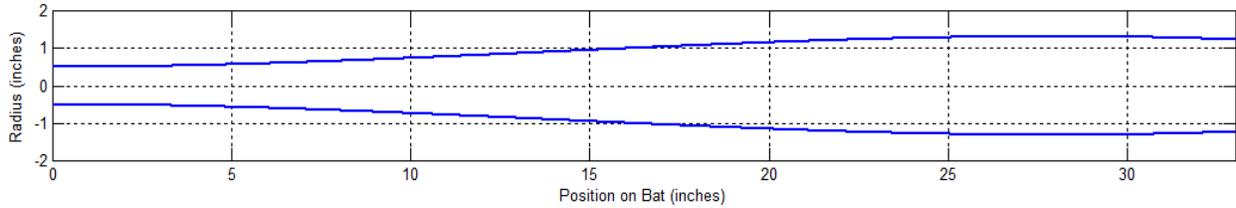


Figure 50: Cubic Constant Moment Bat Profile $d = -0.000075$

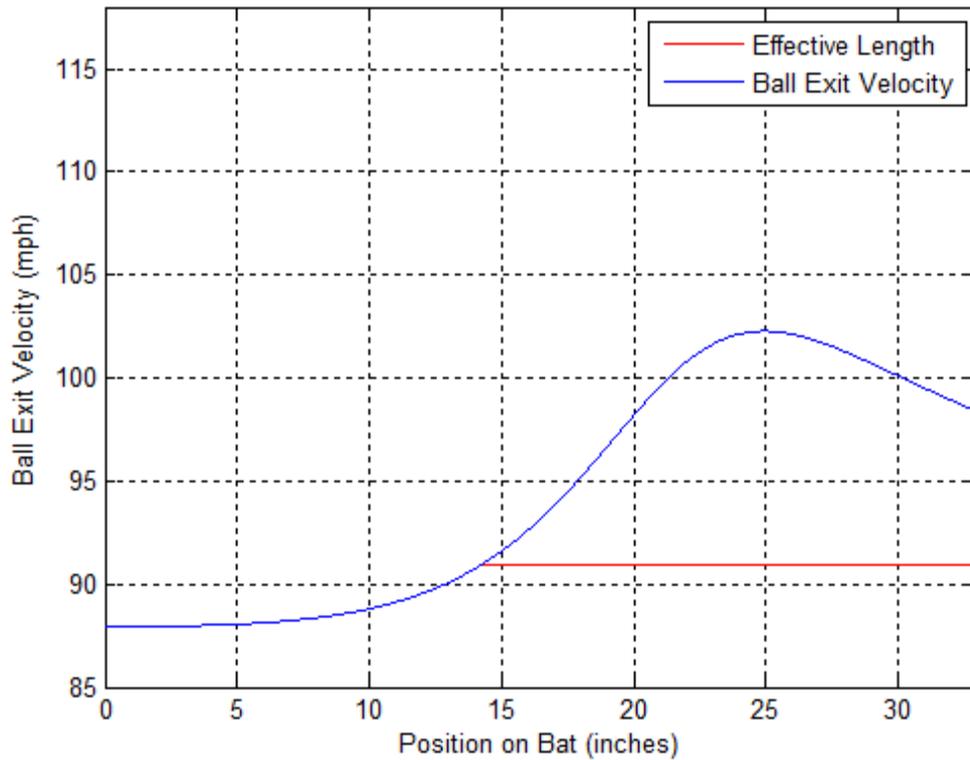


Figure 51: Velocity Curve Cubic Constant Moment $d = -0.000075$

The third bat profile analyzed for the constant moment cubic case study is shown in Figure 52, and it produced a maximum velocity of 103.44 mi / hr, an effective length of 18.75 in, and a weighted performance of 318.20 mi * in / hr with a mass of 32.98 oz. The velocity profile along the length of the bat along with the effective length is shown in Figure 53.

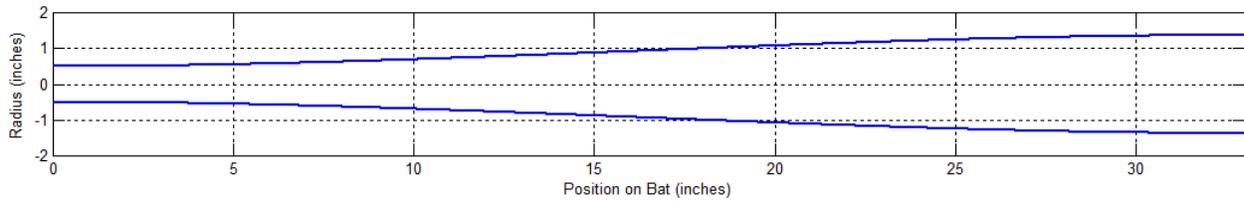


Figure 52: Cubic Constant Moment Bat Profile $d = -0.00005$

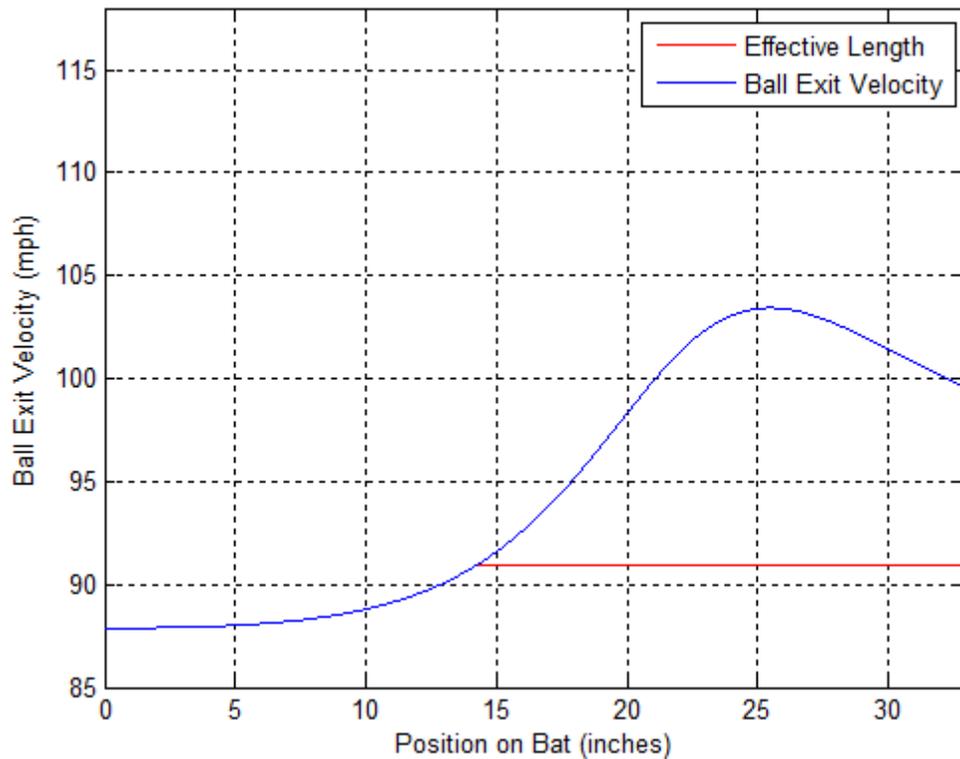


Figure 53: Velocity Curve Cubic Constant Moment $d = -0.00005$

The fourth bat profile analyzed for the constant moment cubic case study is shown in Figure 54, and it produced a maximum velocity of 104.05 mi / hr, an effective length of 18.75 in, and a weighted performance of 360.06 mi * in / hr with a mass of 31.49 oz. The velocity profile along the length of the bat along with the effective length is shown in Figure 55.

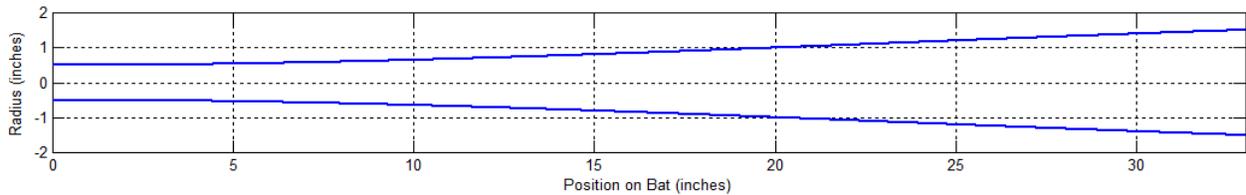


Figure 54: Cubic Constant Moment Bat Profile $d = -0.000025$

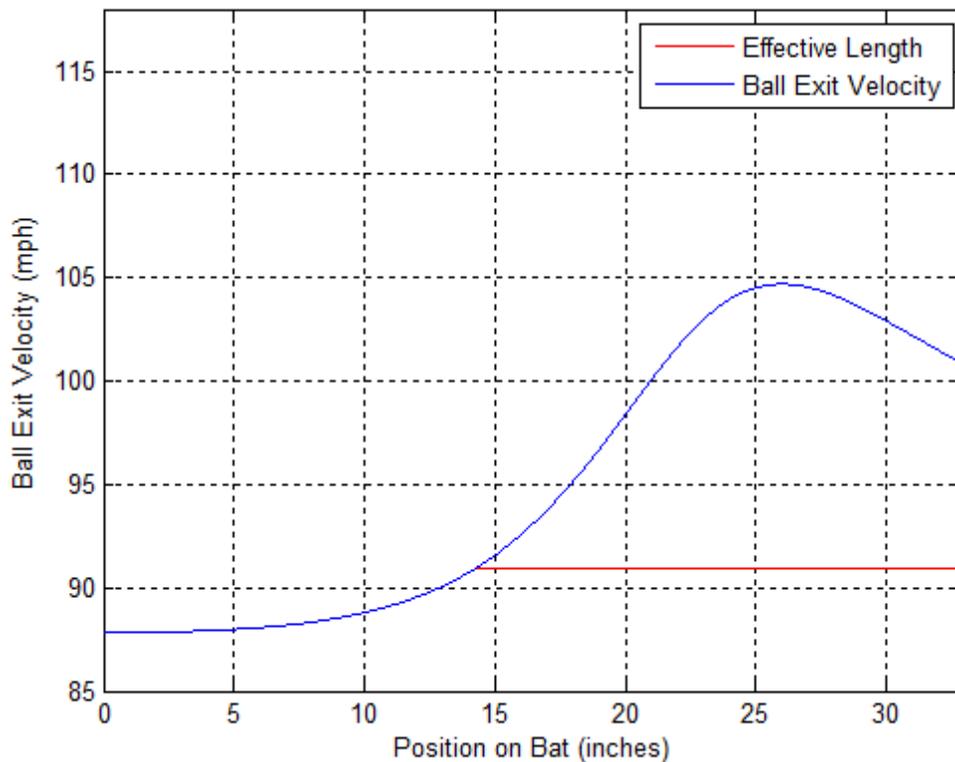


Figure 55: Velocity Curve Cubic Constant Moment $d = -0.000025$

The last bat profile analyzed for the constant moment cubic case study is shown in Figure 56, and it produced a maximum velocity of 106.05 mi / hr, an effective length of 18.73 in, and a weighted performance of 404.22 mi * in / hr with a mass of 30.06 oz. The velocity profile along the length of the bat along with the effective length is shown in Figure 57.

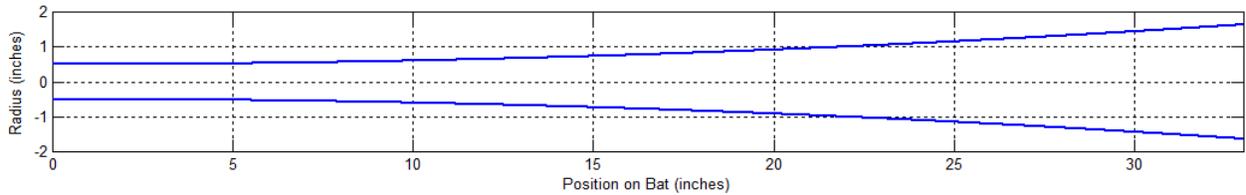


Figure 56: Cubic Constant Moment Bat Profile $d = 0$

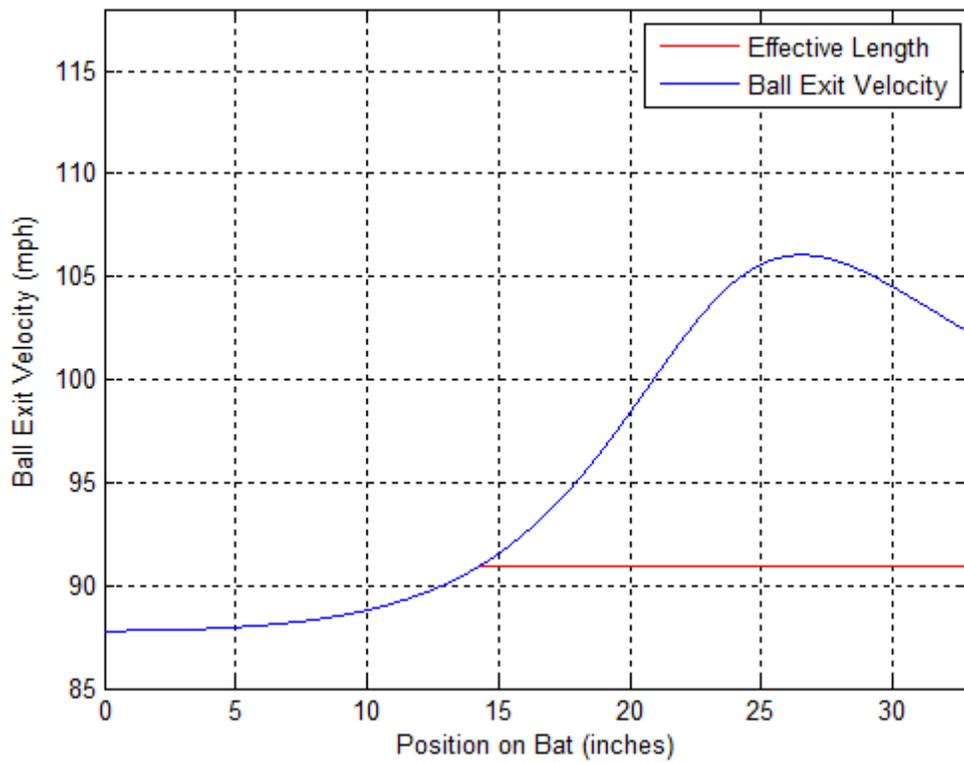


Figure 57: Velocity Curve Cubic Constant Moment $d = 0$

The graph in Figure 58 displays the percent change of all the performance measures along with the percent change of the moment of inertia. This graph provides a synopsis of the data displayed in Figure 59, Figure 60, Figure 61, and Figure 62 on a common coordinate system. It shows how the change in maximum velocity increases approximately five percent of the range of the cubic parameter d . This is almost double the change that occurred in the constant mass case. The effective length for this case, shown in green on Figure 58, remains almost constant as shown by the lack of change, which is also different than the small decrease that occurred in the constant mass case. It also shows that weighted performance varies greatly over the range of bat profiles; the weighted performance undergoes a growth of approximately 65 percent over the range of d . Over this same span, the mass decreases slightly more than twenty percent, and is the only variable to decrease over the sample range. It appears from this sample data that the mass has no effect on the performance measures, and that the true determining factors are swing weight and the mass distribution along the bat.

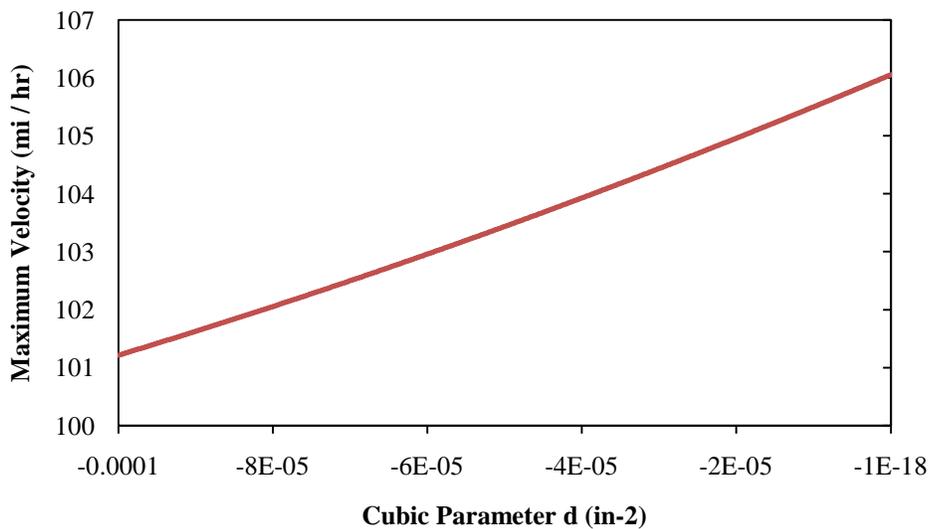


Figure 58: Maximum Velocity vs. Handle Diameter for cubic constant moment bats

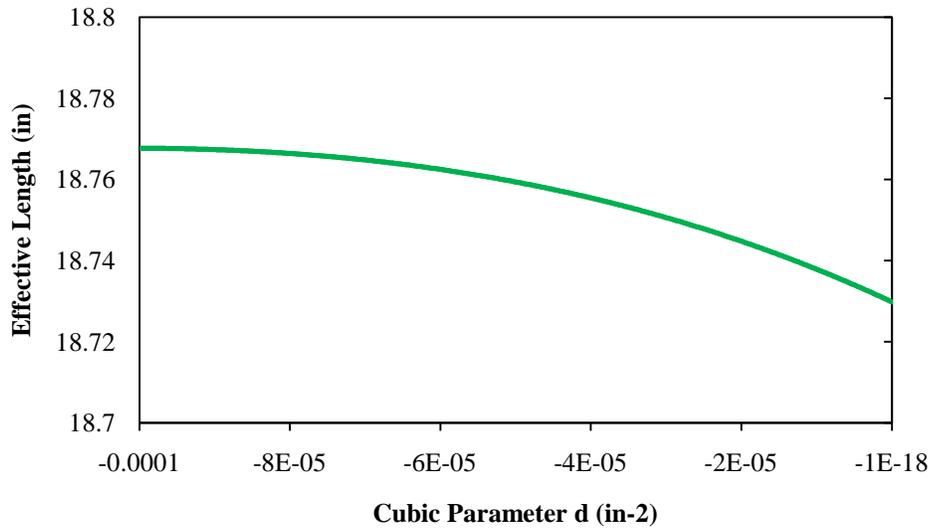


Figure 59: Effective Length vs. Handle Diameter for cubic constant moment bats

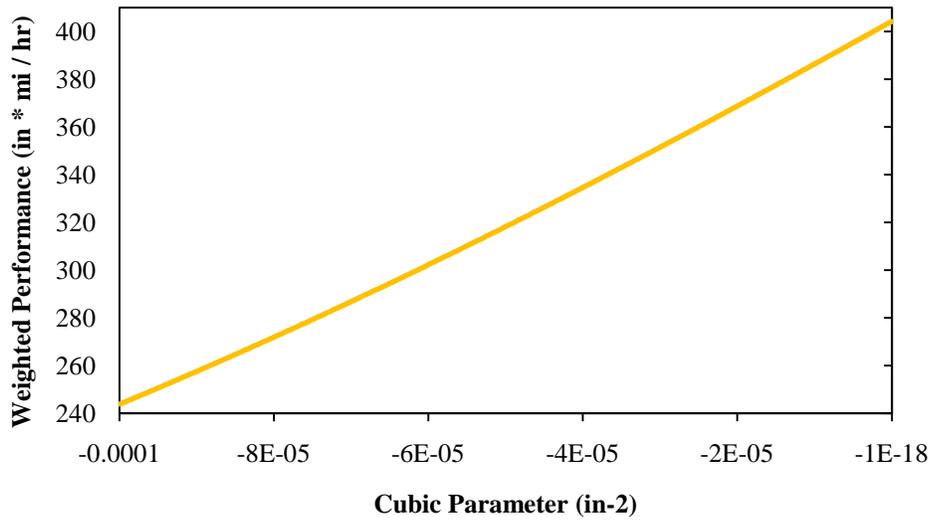


Figure 60: Weighted Performance vs. Handle Diameter for cubic constant moment bats

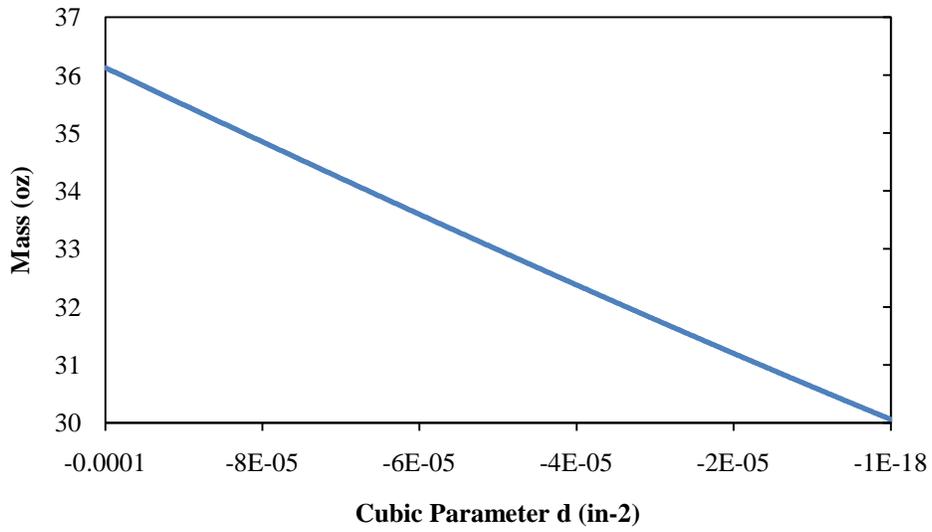


Figure 61: Mass vs. Handle Diameter for cubic constant moment bats

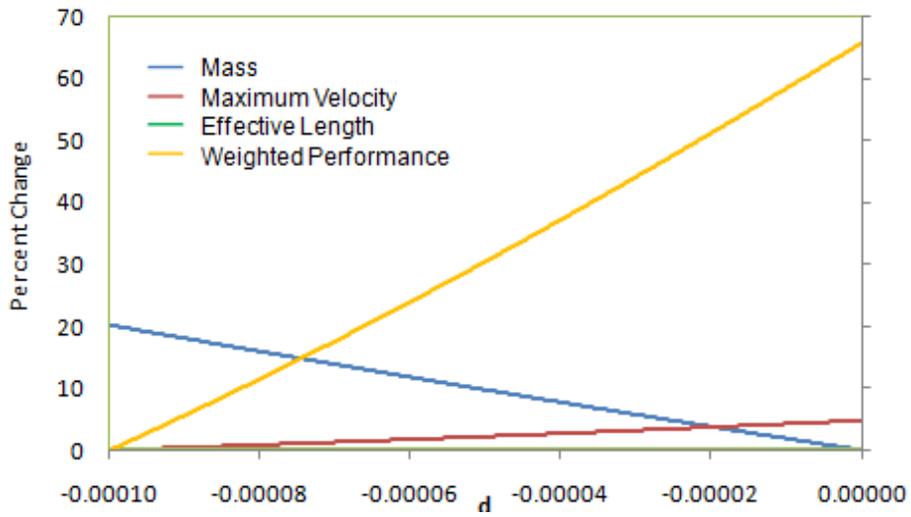


Figure 62: Percent Change vs. Cubic Parameter d for cubic constant moment bats

3.5 General Profile: Point Interpolation

After successfully devising methods of comparing different ranges of theoretical baseball bat profiles created using the cubic expression for $R(x)$, the team created a method for inputting an existing bat's profile for comparison. While the measures listed above can be easily calculated for finite point radial profiles instead of continuous ones generated using a cubic equation, the accuracy of the results would be affected by the number of points. In order to produce results of acceptable accuracy it would be necessary to take a large number of measurements of the bat in question. To get around this problem, the team decided that it would be necessary to create a method of taking fewer data points and interpolating them into a more complete profile before calculating the measures presented above.

The group began by taking 67 evenly spaced measurements of diameter for two different wooden baseball bats starting at the handle. This information was then written into a spreadsheet so that it could be easily accessed by an interpolation program. The first attempt used a method of Lagrange interpolation on all points simultaneously. The result of this was highly erratic, and did not closely resemble the desired radial profile.

Next the group broke the bat up into three distinct sections. The first section contained all points that did not exceed 1.01 times the value of the first handle measurement. The second group contained all points that were not below 0.99 times the measurement at the end of the barrel. The third group contained all values that were not characterized by the first two groups, which corresponded to the neck of the bat.

Once this splicing of the diameter measurements was completed a spline interpolation was performed on each section of the bat individually to raise the number of data points. These

new values within each section were then put back together into one complete profile. Using trial and error methods the group was then able to determine the minimum number of acceptable raw data points that would be able to produce an appropriate profile once the interpolation technique had been applied.

Figure 63, below, shows the raw 67 data points simply drawn connected. The Figure 64 is the result of all 67 points spline interpolated with the raw data superimposed. Figure 65 shows the result of 34 points being spline interpolated with the raw data superimposed. Figure 66 shows the result using 17 points, and Figure 67 shows the result using 9 points both once again shown with the raw data superimposed. The measured data that corresponds to each interpolation can be seen in Table 1, Table 2, Table 3, and Table 4.

The team determined that the best way to characterize a wooden baseball bat was by taking diameter measurements once every inch along the bat (starting at 0) and then performing an interpolation to bring the number of data points to an appropriate number for calculation of the measures of performance. If fewer than 25 data points are taken along the bat the interpolated profile begins to be unacceptably impacted.

x (Inches)	Diameter (inches)
0	1.51
0.5	1.508
1	1.506
1.5	1.299
2	1.189
2.5	1.116
3	1.041
3.5	1.005
4	0.979
4.5	0.963
5	0.944
5.5	0.931
6	0.925
6.5	0.93
7	0.927
7.5	0.932
8	0.94
8.5	0.948
9	0.957
9.5	0.967
10	0.974
10.5	0.988
11	1.002
11.5	1.028
12	1.045
12.5	1.07
13	1.085
13.5	1.11
14	1.137
14.5	1.161
15	1.189
15.5	1.218
16	1.249
16.5	1.291
17	1.33
17.5	1.372
18	1.426
18.5	1.465
19	1.506
19.5	1.557

20	1.6
20.5	1.66
21	1.72
21.5	1.799
22	1.845
22.5	1.899
23	1.949
23.5	2.01
24	2.052
24.5	2.092
25	2.14
25.5	2.188
26	2.234
26.5	2.271
27	2.301
27.5	2.321
28	2.349
28.5	2.376
29	2.395
29.5	2.417
30	2.429
30.5	2.439
31	2.454
31.5	2.464
32	2.473
32.5	2.494
33	2.494

Table 1: 67 Diameter Measurements

x (Inches)	Diameter (Inches)
0	1.51
1	1.506
2	1.189
3	1.041
4	0.979
5	0.944
6	0.925
7	0.927
8	0.94
9	0.957
10	0.974
11	1.002
12	1.045
13	1.085
14	1.137
15	1.189
16	1.249
17	1.33
18	1.426
19	1.506
20	1.6
21	1.72
22	1.845
23	1.949
24	2.052
25	2.14
26	2.234
27	2.301
28	2.349
29	2.395
30	2.429
31	2.454
32	2.473
33	2.474

Table 2: 34 Diameter Measurements

x (Inches)	Diameter (Inches)
0	1.51
2	1.189
4	0.979
6	0.925
8	0.94
10	0.974
12	1.045
14	1.137
16	1.249
18	1.426
20	1.6
22	1.845
24	2.052
26	2.234
28	2.349
30	2.429
32	2.473

Table 3: 17 Diameter Measurements

x (Inches)	Diameter (Inches)
0	1.51
4	0.979
8	0.94
12	1.045
16	1.249
20	1.6
24	2.052
28	2.349
32	2.473

Table 4: 9 Diameter Measurements

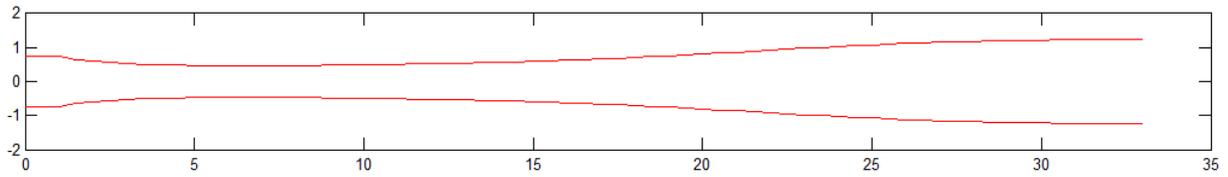


Figure 63: Raw Data

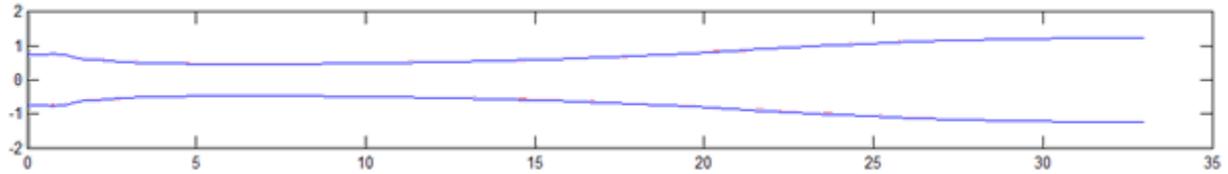


Figure 64: 67 Points Interpolated superimposed over raw data

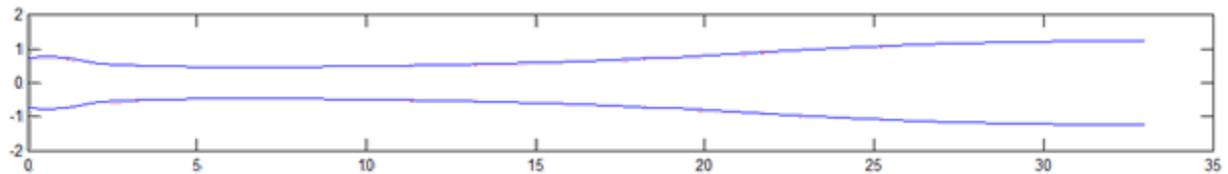


Figure 65: 34 Points Interpolated superimposed over raw data

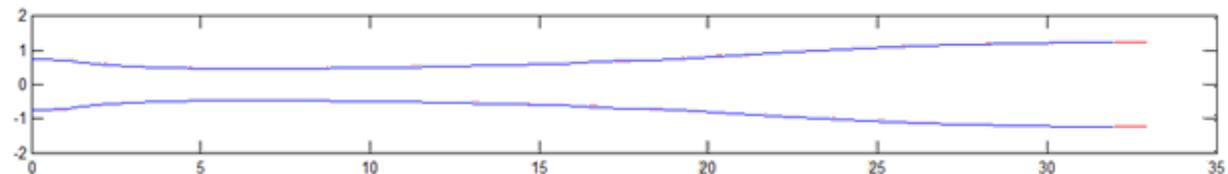


Figure 66: 17 Points Interpolated superimposed over raw data

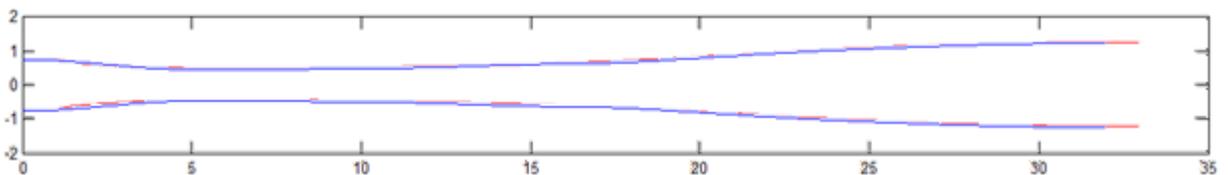


Figure 67: 9 Points Interpolated superimposed over raw data

Chapter 4: Conclusion

When developing our different cases we were looking to find trends for profiles from different bats. Interestingly, we found that the shape of the bat that optimizes one measure of performance is different than the shape that optimizes another. When choosing a bat using our measures the type of player needs to be taken into account. More advanced players may be only looking for a higher maximum velocity, due to their ability to hit the ball consistently. A beginner may be looking for a larger effective length because they want the best chance of producing at least a certain velocity. In either case the weighted performance provides a good compromise for either player.

Using cubic profiles with a constant mass and a constant moment gave us the chance to view the trends in our measures as the profile changed. What we found was that while keeping the mass constant the values of maximum velocity and weighted performance increase with the cubic coefficient d , while effective length decreases. When keeping the moment constant maximum velocity and weighted performance increase with d , and effective length continues to decrease. Because it was found that the most change occurred in weighted performance followed by effective length, an efficient bat would have a good combination of the three measures with the most emphasis on weighted performance.

Our team also successfully produced a method of comparing existing wooden bats using the three measures presented in this report. This can be done by simply taking physical measurements of the bat's diameter at regular intervals and using the appropriate code attached

in Appendix B. This could be used to aid players in selecting the best existing wooden baseball bat for their specific needs.

The models presented within this report simplify the nature of the wooden baseball bats as being made of a homogeneous material. This approach does not take into account effects such as varying densities created by the grain of the given wood. Instead these models show an optimum case, and would benefit from future work into the addition of considerations for the real material properties of wood.

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Appendix B: Matlab Code

Measure Calculator

```
close all;
clear all;
clc;

%% this block is for user inputs, uncomment for prompts
% a = input('In the form of R = a + bx + cx^2 + dx^3 please input a\n');
% b = input('Please input b\n');
% c = input('Please input c\n');
% d = input('Please input d\n');
% p = input('Density of the Bat\n');
% e = input('Coefficient of Restitution\n');
% m2 = input('Mass of the Ball\n');
% v2 = input('Incoming Velocity of the Ball\n');
% w = input('Angular Velocity of the Bat\n');
% l = input('Length of the Bat\n');
% o = input('Pivot to end of Bat length\n');

%% this block is for multiple runs with same values
a = .6301; %% in
b = .02068;
c = 0;
d = 0;
p = 35*16/1728; %% lb/ft^3 to oz/in^3
e = 0.5;
m2 = 5; %% oz
v2 = 1584; %% in/s, = 90mph
w = 110; %% rad/s
l = 33; %% in
o = 1.5;

m1 =
p*pi*(1*a^2+a*b*1^2+(b^2+2*a*c)/3*1^3+1/2*(b*c+a*d)*1^4+1/5*(c^2+2*b*d)*1^5+1/3*(c*d)*1^6+d^2/7*1^7);
xbar =
p*pi/m1*(d^2*1^8/8+2/7*c*d*1^7+1/6*(c^2+2*b*d)*1^6+2/5*(b*c+a*d)*1^5+1/4*(b^2+2*a*c)*1^4+2/3*a*b*1^3+
a^2*1^2/2);
Ic =
p*pi*(d^2*1^9/9-1/4*d*(d*xbar-c)*1^8+1/7*(c^2-4*d*xbar*c+d^2*xbar^2+2*d*b)*1^7+1/3*(-
xbar*c^2+d*xbar^2*c+b*c+a*d-2*b*d*xbar)*1^6+1/5*(b^2+2*d*xbar^2*b-4*c*xbar*b+c^2*xbar^2+2*a*c-
4*a*d*xbar)*1^5+1/2*(-xbar*b^2+c*xbar^2*b+a*b+a*d*xbar^2-2*a*c*xbar)*1^4+1/3*(a^2+2*c*xbar^2*a-
4*b*xbar*a+b^2*xbar^2)*1^3-a*xbar*(a-b*xbar)*1^2+a^2*xbar^2*1);

x = 0:0.00001:l;
m = m2/m1;
y = x-xbar;
vc = w*(o+xbar);
```

```

v2out = v2+(m.*(1+e).*(vc-v2+w.*y))./(1+m+(m1/Ic).*y.^2);
v2outm = v2+(m.*(1+e).*sqrt((vc-v2).^2+w.^2.*(1+m).*(Ic/m1)))./(1+m+(m1./(Ic.*w.^2)).*(v2-vc+sqrt((vc-
v2).^2+w.^2.*(1+m).*(Ic/m1))).^2);
v2out2 = v2out./(v2);

disp ('The Max Value of Vout is');
v2outm

disp ('The Value of Vout at L is');
v2out(end)

figure
plot(x,v2out);
title ('Vout vs. x');
xlabel ('Position on Bat (inches)');
ylabel ('Ball Exit Velocity (mph)');

figure
plot (x/1, v2out/v2);
title ('Vout vs. x');
xlabel ('Position on Bat');
ylabel ('Ball Exit Velocity');

figure
plot (x, a+b.*x+c.*x.^2+d.*x.^3);
title ('Bat Profile');
xlabel ('Position on Bat (inches)');
ylabel ('Radius (inches)');

vs= input ('Select Minimum Vout\n');

if(vs ~= v2)
ys1 = (w-sqrt(w^2-4*((vs-v2)*m1/(Ic*m*(1+e))*((vs-v2)*(1+m)/(m*(1+e))-vc+v2)))/(2*(vs-
v2)*m1/(Ic*m*(1+e)));
ys2 = (w+sqrt(w^2-4*((vs-v2)*m1/(Ic*m*(1+e))*((vs-v2)*(1+m)/(m*(1+e))-vc+v2)))/(2*(vs-
v2)*m1/(Ic*m*(1+e)));
else
ys1 = -(vc-v2)/w;
ys2 = 1;
end

xs1 = ys1+xbar;
xs2 = ys2+xbar;

if (xs2 > 1)
ys2=1-xbar;
end

measure2=ys2-ys1;

measure3big=v2*ys2+((1+e)*sqrt(Ic)*m*(vc-
v2)*atan((sqrt(m1)*ys2)/(sqrt(Ic)*sqrt(m+1)))/(sqrt(m+1)*sqrt(m1)))+((1+e)*Ic*m*w*log(m2*ys2^2+Ic+Ic*m))/(2

```

```

*m1)-(v2*ys1+((1+e)*sqrt(Ic)*m*(vc-
v2)*atan((sqrt(m1)*ys1)/(sqrt(Ic)*sqrt(m+1)))/(sqrt(m+1)*sqrt(m1))+((1+e)*Ic*m*w*log(m1*ys1^2+Ic+Ic*m))/(2
*m1));
measure3small=measure3big-vs*(xs2-xs1);

disp ('The Value of measure 2 is')
measure2
disp ('The Value of measure 3 is')
measure3small

```

Code for Constant Mass Linear Profile

```

close all;
clear all;
clc;

Range=0:.01:(0.9911*2);

measure1=[];
measure12=[];
measure2=[];
measure3=[];
Ic12=[];

for index = Range
mass = 33;
l=33;
p = 35*16/1728;
a = index;
c=0;d=0;

% z = p*pi*l^3;
quada = pi*p*l^3/3;
quadb = a*l^2*pi*p;
quadc = l*a^2*pi*p-mass;

b1 = (-quadb+sqrt(quadb^2-4*quada*quadc))/(2*quada);
b2 = (-quadb-sqrt(quadb^2-4*quada*quadc))/(2*quada);

b=max(b1,b2);

e = 0.5;
m2 = 5; %% oz
v2 = 1584; %% in/s, = 90mph
w = 110; %% rad/s
l = 33; %% in
o = 1.5;

m1
p*pi*(l*a^2+a*b*l^2+(b^2+2*a*c)/3*l^3+1/2*(b*c+a*d)*l^4+1/5*(c^2+2*b*d)*l^5+1/3*(c*d)*l^6+d^2/7*l^7);

```

```

xbar
p*pi/m1*(d^2*I^8/8+2/7*c*d*I^7+1/6*(c^2+2*b*d)*I^6+2/5*(b*c+a*d)*I^5+1/4*(b^2+2*a*c)*I^4+2/3*a*b*I^3+
a^2*I^2/2);
Ic = p*pi*(d^2*I^9/9-1/4*d*(d*xbar-c)*I^8+1/7*(c^2-4*d*xbar*c+d^2*xbar^2+2*d*b)*I^7+1/3*(-
xbar*c^2+d*xbar^2*c+b*c+a*d-2*b*d*xbar)*I^6+1/5*(b^2+2*d*xbar^2*b-4*c*xbar*b+c^2*xbar^2+2*a*c-
4*a*d*xbar)*I^5+1/2*(-xbar*b^2+c*xbar^2*b+a*b+a*d*xbar^2-2*a*c*xbar)*I^4+1/3*(a^2+2*c*xbar^2*a-
4*b*xbar*a+b^2*xbar^2)*I^3-a*xbar*(a-b*xbar)*I^2+a^2*xbar^2*I);
Ip=p*pi*((a^2*o^2*I+a^2*o*I^2+a^2*I^3/3)+(a*o^2*I^2+4*o*a*I^3/3+a*I^4/2)*b+(o^2*I^3/3+o*I^4/2+I^5/5)*b^
2);
Ic12=[Ic12, Ip];

x = 0:0.00001:l;
m = m2/m1;
y = x-xbar;
vc = w*(o+xbar);

v2out = v2+(m.*(1+e).*(vc-v2+w.*y))./(1+m+(m1/Ic).*y.^2);
v2outm = v2+(m.*(1+e).*sqrt((vc-v2).^2+w.^2.*(1+m).*(Ic/m1)))./(1+m+(m1./(Ic.*w.^2)).*(v2-vc+sqrt((vc-
v2).^2+w.^2.*(1+m).*(Ic/m1))).^2);
v2out2 = v2out./(v2);

measure1 = [measure1 , v2outm*0.0568181818181818];

measure12 = [measure12, v2out(end)*0.0568181818181818];

vs= 1600;

if(vs ~= v2)
ys1 = (w-sqrt(w^2-4*((vs-v2)*m1/(Ic*m*(1+e))*((vs-v2)*(1+m)/(m*(1+e))-vc+v2)))/(2*(vs-
v2)*m1/(Ic*m*(1+e)));
ys2 = (w+sqrt(w^2-4*((vs-v2)*m1/(Ic*m*(1+e))*((vs-v2)*(1+m)/(m*(1+e))-vc+v2)))/(2*(vs-
v2)*m1/(Ic*m*(1+e)));
else
ys1 = -(vc-v2)/w;
ys2 = 1;
end

xs1 = ys1+xbar;
xs2 = ys2+xbar;

if (xs1 < 0)
xs1=0;
end

if (xs2 > l)
xs2=l;
end

measure2=[measure2 , (xs2-xs1)];

```

```

measure3big=v2*ys2+((1+e)*sqrt(Ic)*m*(vc-
v2)*atan((sqrt(m1)*ys2)/(sqrt(Ic)*sqrt(m+1))))/(sqrt(m+1)*sqrt(m1))+((1+e)*Ic*m*w*log(m1*ys2^2+Ic+Ic*m))/(2
*m1)-(v2*ys1+((1+e)*sqrt(Ic)*m*(vc-
v2)*atan((sqrt(m1)*ys1)/(sqrt(Ic)*sqrt(m+1))))/(sqrt(m+1)*sqrt(m1))+((1+e)*Ic*m*w*log(m1*ys1^2+Ic+Ic*m))/(2
*m1));
measure3small=(measure3big-vs*(ys2-ys1))*0.0568181818181818;

```

```

measure3=[measure3 , measure3small];
end

```

```

figure
plot (Range*2, measure1);
title ('Handle D vs Measure1');
xlabel ('Handle Diameter');
ylabel ('Measure1');
grid on;

```

```

figure
plot (Range*2, Ic12);
title ('Handle D vs Ip');
xlabel ('Handle Diameter');
ylabel ('Ip');
grid on;

```

```

figure
plot (Range*2, measure12);
title ('Handle D vs Vout at L');
xlabel ('Handle Diameter');
ylabel ('Vout at L');
grid on;

```

```

figure
plot (Range*2, measure2);
title ('Handle D vs Measure2');
xlabel ('Handle Diameter');
ylabel ('Measure2');
grid on;

```

```

figure
plot (Range*2, measure3);
title ('Handle D vs Measure3');
xlabel ('Handle Diameter');
ylabel ('Measure3');
grid on;

```

Code for Constant Moment Linear Profile

```

close all;
clear all;
clc;

measure1=[];
measure12=[];
measure2=[];
measure3=[];
mass2=[];

Range=[0:.01:(0.9911*2)];

for index = Range
mass = 33;
l=33;
Ip=1.8537*10^4;
p = 35*16/1728;
a = index;
o = 1.5;
c=0;
d=0;

quadra = pi*p*((o^2)*(l^3))/3+o*(l^4)/2+(l^5)/5);
quadrb = pi*p*(a*(o^2)*(l^2)+(4*o*a*(l^3))/3+(a*(l^4))/2);
quadrc = pi*p*((a^2)*(o^2)*1+(a^2)*o*(l^2)+(a^2)*(l^3))/3)-Ip;

b1 = (-quadrb+sqrt(quadrb^2-4*quadra*quadrc))/(2*quadra);
b2 = (-quadrb-sqrt(quadrb^2-4*quadra*quadrc))/(2*quadra);

b=max(b1,b2);

e = 0.5;
m2 = 5; %% oz
v2 = 1584; %% in/s, = 90mph
w = 110; %% rad/s

m1
p*pi*(1*a^2+a*b*l^2+(b^2+2*a*c)/3*l^3+1/2*(b*c+a*d)*l^4+1/5*(c^2+2*b*d)*l^5+1/3*(c*d)*l^6+d^2/7*l^7);
xbar
p*pi/m1*(d^2*l^8/8+2/7*c*d*l^7+1/6*(c^2+2*b*d)*l^6+2/5*(b*c+a*d)*l^5+1/4*(b^2+2*a*c)*l^4+2/3*a*b*l^3+
a^2*l^2/2);
Ic
= p*pi*(d^2*l^9/9-1/4*d*(d*xbar-c)*l^8+1/7*(c^2-4*d*xbar*c+d^2*xbar^2+2*d*b)*l^7+1/3*(-
xbar*c^2+d*xbar^2*c+b*c+a*d-2*b*d*xbar)*l^6+1/5*(b^2+2*d*xbar^2*b-4*c*xbar*b+c^2*xbar^2+2*a*c-
4*a*d*xbar)*l^5+1/2*(-xbar*b^2+c*xbar^2*b+a*b+a*d*xbar^2-2*a*c*xbar)*l^4+1/3*(a^2+2*c*xbar^2*a-
4*b*xbar*a+b^2*xbar^2)*l^3-a*xbar*(a-b*xbar)*l^2+a^2*xbar^2*1);
mass2 = [mass2, m1];

x = 0:0.00001:l;
m = m2/m1;
y = x-xbar;

```

```

vc = w*(o+xbar);

v2out = v2+(m.*(1+e).*(vc-v2+w.*y))./(1+m+(m1/Ic).*y.^2);
v2outm = v2+(m.*(1+e).*sqrt((vc-v2).^2+w.^2.*(1+m).*(Ic/m1)))./(1+m+(m1./(Ic.*w.^2)).*(v2-vc+sqrt((vc-
v2).^2+w.^2.*(1+m).*(Ic/m1))).^2);
v2out2 = v2out./(v2);

measure1 = [measure1 , v2outm*0.0568181818181818];

measure12 = [measure12, v2out(end)*0.0568181818181818];

vs= 1600;

if(vs ~= v2)
ys1 = (w-sqrt(w^2-4*((vs-v2)*m1/(Ic*m*(1+e))*((vs-v2)*(1+m)/(m*(1+e))-vc+v2)))/(2*(vs-
v2)*m1/(Ic*m*(1+e)));
ys2 = (w+sqrt(w^2-4*((vs-v2)*m1/(Ic*m*(1+e))*((vs-v2)*(1+m)/(m*(1+e))-vc+v2)))/(2*(vs-
v2)*m1/(Ic*m*(1+e)));
else
ys1 = -(vc-v2)/w;
ys2 = 1;
end

xs1 = ys1+xbar;
xs2 = ys2+xbar;

if (xs1 < 0)
xs1=0;
end

if (xs2 > 1)
xs2=1;
end

measure2=[measure2 , (xs2-xs1)];

measure3big=v2*ys2+((1+e)*sqrt(Ic)*m*(vc-
v2)*atan((sqrt(m1)*ys2)/(sqrt(Ic)*sqrt(m+1)))/(sqrt(m+1)*sqrt(m1))+((1+e)*Ic*m*w*log(m1*ys2^2+Ic+Ic*m))/(2
*m1)-(v2*ys1+((1+e)*sqrt(Ic)*m*(vc-
v2)*atan((sqrt(m1)*ys1)/(sqrt(Ic)*sqrt(m+1)))/(sqrt(m+1)*sqrt(m1))+((1+e)*Ic*m*w*log(m1*ys1^2+Ic+Ic*m))/(2
*m1)));
measure3small=(measure3big-vs*(ys2-ys1))*0.0568181818181818;

measure3=[measure3 , measure3small];
end

figure
plot(Range*2, mass2);
title ('Handle D vs Mass');
xlabel ('Handle Diameter');
ylabel ('Mass');

```

```
grid on;
```

```
figure  
plot (Range*2, measure1);  
title ('Handle D vs Measure1');  
xlabel ('Handle Diameter');  
ylabel ('Measure1');  
grid on;
```

```
figure  
plot (Range*2, measure12);  
title ('Handle D vs Vout at L');  
xlabel ('Handle Diameter');  
ylabel ('Vout at L');  
grid on;
```

```
figure  
plot (Range*2, measure2);  
title ('Handle D vs Measure2');  
xlabel ('Handle Diameter');  
ylabel ('Measure2');  
grid on;
```

```
figure  
plot (Range*2, measure3);  
title ('Handle D vs Measure3');  
xlabel ('Handle Diameter');  
ylabel ('Measure3');  
grid on;
```

Code for Constant Mass Cubic Profile

```
close all;  
clear all;  
clc;
```

```
measure1=[];  
measure12=[];  
measure2=[];  
measure3=[];  
Ic12=[];
```

```
Range=[-0.0001:0.00001:0.0001];
```

```
for index = Range  
mass = 33;  
l=33;  
p = 35*16/1728;  
a = .5;  
b=0;  
d=index;
```

```

quada = 1^5/5;
quadb = 2*a*1^3/3+d*1^6/3;
quadc = -mass/(p*pi)+a^2*1+a*d*1^4/2+d^2*1^7/7;

c1 = (-quadb+sqrt(quadb^2-4*quada*quadc))/(2*quada);
c2 = (-quadb-sqrt(quadb^2-4*quada*quadc))/(2*quada);

c=max(c1,c2);

e = 0.5;
m2 = 5; %% oz
v2 = 1584; %% in/s, = 90mph
w = 110; %% rad/s
l = 33; %% in
o = 1.5;

m1 =
p*pi*(1*a^2+a*b*1^2+(b^2+2*a*c)/3*1^3+1/2*(b*c+a*d)*1^4+1/5*(c^2+2*b*d)*1^5+1/3*(c*d)*1^6+d^2/7*1^7);
xbar =
p*pi/m1*(d^2*1^8/8+2/7*c*d*1^7+1/6*(c^2+2*b*d)*1^6+2/5*(b*c+a*d)*1^5+1/4*(b^2+2*a*c)*1^4+2/3*a*b*1^3+
a^2*1^2/2);
Ic =
p*pi*(d^2*1^9/9-1/4*d*(d*xbar-c)*1^8+1/7*(c^2-4*d*xbar*c+d^2*xbar^2+2*d*b)*1^7+1/3*(-
xbar*c^2+d*xbar^2*c+b*c+a*d-2*b*d*xbar)*1^6+1/5*(b^2+2*d*xbar^2*b-4*c*xbar*b+c^2*xbar^2+2*a*c-
4*a*d*xbar)*1^5+1/2*(-xbar*b^2+c*xbar^2*b+a*b+a*d*xbar^2-2*a*c*xbar)*1^4+1/3*(a^2+2*c*xbar^2*a-
4*b*xbar*a+b^2*xbar^2)*1^3-a*xbar*(a-b*xbar)*1^2+a^2*xbar^2*1);

Ip2=p*pi*(o^2*a^2*1+(a*b*o^2+o*a^2)*1^2+(2*a*c*o^2+b^2*o^2+4*a*b*o+a^2)*1^3/3+(a*d*o^2+b*c*o^2+2*a
*c*o+b^2*o+4*b)*1^4/2+(2*b*d*o^2+c^2*o^2+4*a*d*o+4*b*c*o+2*a*c+b^2)*1^5/5+(c*d*o^2+2*b*d*o+c^2*o
+a*d+b*c)*1^6/3+(d^2*o^2+4*c*d*o+2*b*d+c^2)*1^7/7+(d^2*o+c*d)*1^8/4+d^2*1^9/9);

Ic12=[Ic12, Ip2];

x = 0:0.00001:l;
m = m2/m1;
y = x-xbar;
vc = w*(o+xbar);

v2out = v2+(m.*(1+e).*(vc-v2+w.*y))./(1+m+(m1/Ic).*y.^2);
v2outm = v2+(m.*(1+e).*sqrt((vc-v2).^2+w.^2.*(1+m).*(Ic./m1)))./(1+m+(m1./(Ic.*w.^2)).*(v2-vc+sqrt((vc-
v2).^2+w.^2.*(1+m).*(Ic./m1))).^2);
v2out2 = v2out./(v2);

measure1 = [measure1 , v2outm*0.0568181818181818];

measure12 = [measure12, v2out(end)*0.0568181818181818];

vs= 1600;

if(vs ~= v2)

```

```

ys1      =      (w*sqrt(w^2-4*((vs-v2)*m1/(Ic*m*(1+e))*((vs-v2)*(1+m)/(m*(1+e))-vc+v2)))/(2*(vs-
v2)*m1/(Ic*m*(1+e)));
ys2      =      (w+sqrt(w^2-4*((vs-v2)*m1/(Ic*m*(1+e))*((vs-v2)*(1+m)/(m*(1+e))-vc+v2)))/(2*(vs-
v2)*m1/(Ic*m*(1+e)));
else
ys1 = -(vc-v2)/w;
ys2 = 1;
end

xs1 = ys1+xbar;
xs2 = ys2+xbar;

if (xs1 < 0)
    xs1=0;
end

if (xs2 > 1)
    xs2=1;
end

measure2=[measure2 , (xs2-xs1)];

measure3big=v2*ys2+((1+e)*sqrt(Ic)*m*(vc-
v2)*atan((sqrt(m1)*ys2)/(sqrt(Ic)*sqrt(m+1)))/(sqrt(m+1)*sqrt(m1)))+((1+e)*Ic*m*w*log(m1*ys2^2+Ic+Ic*m))/(2
*m1)-(v2*ys1+((1+e)*sqrt(Ic)*m*(vc-
v2)*atan((sqrt(m1)*ys1)/(sqrt(Ic)*sqrt(m+1)))/(sqrt(m+1)*sqrt(m1)))+((1+e)*Ic*m*w*log(m1*ys1^2+Ic+Ic*m))/(2
*m1));
measure3small=(measure3big-vs*(ys2-ys1))*0.0568181818181818;

measure3=[measure3 , measure3small];
end

figure
plot ( Range, measure1);
title ('D vs Measure1');
xlabel ('D');
ylabel ('Measure1');
grid on;

figure
plot (Range, Ic12);
title ('D vs Ip');
xlabel ('D');
ylabel ('Ip');
grid on;

figure
plot ( Range, measure12);
title ('D vs Vout at L');
xlabel ('D');
ylabel ('Vout at L');
grid on;

```

```

figure
plot ( Range, measure2);
title ('D vs Measure2');
xlabel ('D');
ylabel ('Measure2');
grid on;

```

```

figure
plot ( Range, measure3);
title ('D vs Measure3');
xlabel ('D');
ylabel ('Measure3');
grid on;

```

Code for Constant Moment Cubic Profile

```

close all;
clear all;
clc;

```

```

measure1=[];
measure12=[];
measure2=[];
measure3=[];
mass2=[];

```

```

Range=[-0.0001:.000001:0.0001];

```

```

for index = Range
mass = 33;
Ip=2.01*10^4;
l=33;
p = 35*16/1728;
a = .5;
b=0;
d=index;
o = 1.5;

```

```

quada = pi*p*(o^2*1^5/5+o*1^6/3+1^7/7);
quadb = pi*p*(2*a*o^2*1^3/3+b*o^2*1^4/2+2*a*o*1^4/2+4*b*o*1^5/5+2*a*1^5/5+d*o^2*1^6/3+b*1^6/3+4*d*o*1^7/7+d*1^8/4);
quadc = pi*p*(o^2*a^2*1+(a*b*o^2+a^2*o)*1^2+(b^2*o^2+4*a*b*o+a^2)*1^3/3+(a*d*o^2+b^2*o+a*b)*1^4/2+(2*b*d*o^2+4*a*d*o+b^2)*1^5/5+(2*b*d*o+a*d)*1^6/3+(o^2*d^2+2*b*d)*1^7/7+d^2*o*1^8/4+d^2*1^9/9)-Ip;

```

```

c1 = (-quadb+sqrt(quadb^2-4*quada*quadc))/(2*quada);
c2 = (-quadb-sqrt(quadb^2-4*quada*quadc))/(2*quada);

```

```

c=max(c1,c2);

```

```

e = 0.5;
m2 = 5; %% oz
v2 = 1584; %% in/s, = 90mph
w = 110; %% rad/s

```

```

m1 =
p*pi*(1*a^2+a*b*1^2+(b^2+2*a*c)/3*1^3+1/2*(b*c+a*d)*1^4+1/5*(c^2+2*b*d)*1^5+1/3*(c*d)*1^6+d^2/7*1^7);
xbar =
p*pi/m1*(d^2*1^8/8+2/7*c*d*1^7+1/6*(c^2+2*b*d)*1^6+2/5*(b*c+a*d)*1^5+1/4*(b^2+2*a*c)*1^4+2/3*a*b*1^3+
a^2*1^2/2);
Ic =
p*pi*(d^2*1^9/9-1/4*d*(d*xbar-c)*1^8+1/7*(c^2-4*d*xbar*c+d^2*xbar^2+2*d*b)*1^7+1/3*(-
xbar*c^2+d*xbar^2*c+b*c+a*d-2*b*d*xbar)*1^6+1/5*(b^2+2*d*xbar^2*b-4*c*xbar*b+c^2*xbar^2+2*a*c-
4*a*d*xbar)*1^5+1/2*(-xbar*b^2+c*xbar^2*b+a*b+a*d*xbar^2-2*a*c*xbar)*1^4+1/3*(a^2+2*c*xbar^2*a-
4*b*xbar*a+b^2*xbar^2)*1^3-a*xbar*(a-b*xbar)*1^2+a^2*xbar^2*1);

```

```

Ip2=p*pi*(o^2*a^2*1+(a*b*o^2+o*a^2)*1^2+(2*a*c*o^2+b^2*o^2+4*a*b*o+a^2)*1^3/3+(a*d*o^2+b*c*o^2+2*a
*c*o+b^2*o+4*b)*1^4/2+(2*b*d*o^2+c^2*o^2+4*a*d*o+4*b*c*o+2*a*c+b^2)*1^5/5+(c*d*o^2+2*b*d*o+c^2*o
+a*d+b*c)*1^6/3+(d^2*o^2+4*c*d*o+2*b*d+c^2)*1^7/7+(d^2*o+c*d)*1^8/4+d^2*1^9/9);
mass2 = [mass2, m1];

```

```

x = 0:0.00001:l;
m = m2/m1;
y = x-xbar;
vc = w*(o+xbar);

```

```

v2out = v2+(m.*(1+e).*(vc-v2+w.*y))./(1+m+(m1/Ic).*y.^2);
v2outm = v2+(m.*(1+e).*sqrt((vc-v2).^2+w.^2.*(1+m).*(Ic/m1)))./(1+m+(m1./(Ic.*w.^2)).*(v2-vc+sqrt((vc-
v2).^2+w.^2.*(1+m).*(Ic/m1))).^2);
v2out2 = v2out./(v2);

```

```

measure1 = [measure1 , v2outm*0.0568181818181818];

```

```

measure12 = [measure12, v2out(end)*0.0568181818181818];

```

```

vs= 1600;

```

```

if(vs ~= v2)
ys1 = (w-sqrt(w^2-4*((vs-v2)*m1/(Ic*m*(1+e))*((vs-v2)*(1+m)/(m*(1+e))-vc+v2)))/(2*(vs-
v2)*m1/(Ic*m*(1+e)));
ys2 = (w+sqrt(w^2-4*((vs-v2)*m1/(Ic*m*(1+e))*((vs-v2)*(1+m)/(m*(1+e))-vc+v2)))/(2*(vs-
v2)*m1/(Ic*m*(1+e)));
else
ys1 = -(vc-v2)/w;
ys2 = l;
end

```

```

xs1 = ys1+xbar;
xs2 = ys2+xbar;

```

```

if (xs1 < 0)

```

```

    xs1=0;
end

if (xs2 > 1)
    xs2=1;
end

measure2=[measure2 , (xs2-xs1)];

measure3big=v2*ys2+((1+e)*sqrt(Ic)*m*(vc-
v2)*atan((sqrt(m1)*ys2)/(sqrt(Ic)*sqrt(m+1)))/(sqrt(m+1)*sqrt(m1))+((1+e)*Ic*m*w*log(m1*ys2^2+Ic+Ic*m))/(2
*m1)-(v2*ys1+((1+e)*sqrt(Ic)*m*(vc-
v2)*atan((sqrt(m1)*ys1)/(sqrt(Ic)*sqrt(m+1)))/(sqrt(m+1)*sqrt(m1))+((1+e)*Ic*m*w*log(m1*ys1^2+Ic+Ic*m))/(2
*m1)));
measure3small=(measure3big-vs*(ys2-ys1))*0.0568181818181818;

measure3=[measure3 , measure3small];
end

figure
plot(Range*2, mass2);
title ('D vs Mass');
xlabel ('D');
ylabel ('Mass');
grid on;

figure
plot (Range*2, measure1);
title ('D vs Measure1');
xlabel ('D');
ylabel ('Measure1');
grid on;

figure
plot (Range*2, measure12);
title ('D vs Vout at L');
xlabel ('D');
ylabel ('Vout at L');
grid on;

figure
plot (Range*2, measure2);
title ('D vs Measure2');
xlabel ('D');
ylabel ('Measure2');
grid on;

figure
plot (Range*2, measure3);
title ('D vs Measure3');
xlabel ('D');
ylabel ('Measure3');
grid on;

```

Code for Real Bat Interpolated Profile

```
function [x,y,newx,newy]=importbat(z)

num = xlsread('bat1.xls',z);
y=[num(:,2)./2];
x=[num(:,1)];
[newx,newy]=cut2(x,y);

function [newx,newy]=cut2(x,y)

subx1=[];
subx2=[];
subx3=[];

[suby1 suby2 suby3]=sub(y);

subx1 = [x(1:length(suby1))];
subx2 = [x(length(suby1):length([suby1,suby2])-1)];
subx3 = [];

[new1] = spl(subx1,suby1);
[new2] = spl(subx2,suby2);
[new3] = [];
newy=[new1 new2 new3];
a1 = linspace(subx1(1),subx1(end),length(new1));
a2 = linspace(subx2(1),subx2(end),length(new2));
a3 = [];
newx=[a1 a2 a3];

function [new1]=spl(subx,suby)
cs=spline (subx, [0, suby, 0]);
new1=ppval (cs, linspace (subx(1), subx(end), 100));

function [part1, part2, part3] = sub(x1)

part1=[];
par2=[];
part2=[];
par3=[];
part3=[];

for index = 1:length(x1)
    if (x1(index) < 1.01*x1(1))
        part1 = [part1, x1(index)];
    elseif (x1(index) < 1.01*x1(end))
        par2 = [par2,x1(index)];
    else
        par3 = [par3,x1(index)];
    end
end
```

```
end
end
```

```
part2 = [part1(end),par2];
part3 = [par2(end), par3];
```

Stress Analysis

```
function [smax tmax] = stress (x,r,d,f)
close all;
y = -r:0.001:r;
m = f.*(x-d);
v = f;
Ic=1./4.*pi.*r.^4;
sigmax = -m.*y./Ic;
sigmay = 0;

t=2.*sqrt(r.^2-y.^2);
theta=2.*acos(y./r);
for index = 1:length(theta)
    if (theta(index) == 0)
        ybar(index) = r;
        aprime(index) = 0;
    else
        ybar(index) = 4.*r.*(sin(.5.*theta(index))).^3./(3.*(theta(index)-sin(theta(index))));
        aprime(index) = r.^2.*acos(y(index)./r)-y(index).*sqrt(r.^2-y(index).^2);
    end
end
q = ybar.*aprime;
for index = 1:length(theta)
    if (t(index) == 0)
        tau(index) = 0;
    else
        tau(index) = v.*q(index)./(Ic.*t(index));
    end
end

%mohr's circle
center = (sigmax-sigmay)./2;
rmohr = sqrt(((sigmax-sigmay)./2).^2+tau.^2);

tmax = rmohr;
smax = max(center+rmohr,center-rmohr);

figure
plot (tmax,y)
title ( 'shear max vs y');

figure
plot (smax,y)
title ( 'normal max vs y');
```