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MEDICAL IMPULSE TURBINE CUTTING DEVICE

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________________________________________
Professor Cosme Furlong
Abstract

Tissue sampling is becoming a more common diagnosis method in the clinical field because of its diagnosis accuracy; however, some current methods involve at least two medical instruments for cutting and sampling, which makes procedures time consuming, invasive, and expensive.

To overcome these imperfections our group has designed a millimeter scale medical device, which can cut and suction tissue samples simultaneously at rotation speeds ranging from 5,000 to 40,000 RPM. The device can be broken into three portions: power, cutting, and sample delivery. The impulse turbine, a major component of the power portion, is used to generate a high-speed rotation to the shaft. With a high rotation speed, the cutting tip of the shaft can cut the tissue with the required torque and cutting force. This project focuses on the structural analysis of the existing device by analytical and experimental methods to draw conclusions that could help in future designs.
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1. Introduction

Debridement is a medical technique used to remove dead, damaged, or infected tissue from in and around wounds. This medical procedure is implicated in cases of infected wounds that result from injury, burns, bedsores, or infected incisions. Debridement improves the healing potential of remaining healthy tissue because dead, damaged, or contaminated tissue can compromise circulation to your wound. Devitalized tissue also invites bacteria to grow, which compete with growing cells for nutrients and interfere with the healing process.

There are a number of tissue removal techniques currently being used for debridement. Of these techniques includes autolytic, enzymatic, mechanical, and surgical debridement. Autolytic debridement uses the body’s own enzymes and moisture to rehydrate, soften, and liquefy the dead eschar and slough. This method is effective, however, wounds must be monitored closely for signs of infection. The next debridement technique is enzymatic debridement. This method involves the use of chemical enzymes that produce necrotic tissue. Disadvantages of enzymatic debridement are that it is very expensive, requires a prescription, and inflammation or discomfort may occur. Mechanical debridement is a technique that has been used for decades that involves covering the wound with a moist dressing, then manually removing the dressing. Unfortunately this technique is non-selective, time consuming, painful, and may traumatize healthy or healing tissue.

Surgical debridement is the preferred method of dead tissue removal because it is fast and selective. Also use of local or general anesthetics make the procedure painless. The surgical debridement procedure begins with anesthesia to numb the infected area,
then, saline or disinfectant solutions are used to wash or irrigate the wound. Next, the wound is evaluated to determine the depth of the dead tissue, and finally, the dead tissue is removed using a variety of surgical hand tools. Different types of hand tools are used for different types of wounds, but the most common tool is the scalpel. Surgical debridement is an effective and fast method of dead tissue removal, however, the doctors precision is limited by the tool he uses.

Microdebridment is a relatively new surgical procedure that was introduced to aid in the extraction of internal damaged tissue and remove microscopic cellular debris that causes inflammation. The most common medical condition treated with microdebridment is Achilles tendonitis. Achilles tendinitis causes inflammation and pain along the back leg near the heel, which is usually a result of repetitive stressing of the tendon. This inflammation is the body’s natural response to stress induced injury. In cases where the tendon has been damaged less than 50%, debridement is used to remove the damaged portion to promote growth of the healthy tissue.

The impulse turbine cutter designed by our group is an excellent candidate for these medical applications. Use of a medical cutter with suction capabilities allows the procedure to be completed with relatively small incisions to gain access to the infected area. Also, with a smaller cutting portion than historical tools, microdebridment can be attained with greater levels of precision. The turbine’s suction capability allows the doctor to remove damaged tissue continuously, reducing operation time while keeping the cut tissue debris out of the wound.
2. Description of Impulse Turbine Cutter

Historical methods of tissue removal involve the use of at least two medical instruments, one for cutting the tissue and another for extraction. Since these surgical methods have proven time consuming and inadequate, our group has designed a first generation prototype impulse turbine medical cutting device with suction capabilities. The design of the medical impulse turbine was targeted to both cut and suction the tissue sample simultaneously. This device is unique in that it is the world’s smallest medical device with cutting and suction capabilities. The device consists of three major components, which can be identified as the cutting, power, and delivery portions, as depicted in Figure 2.1.

![Overall Conceptual Design](image_url)
The cutting portion of the device is based on an orthopedic blade that is used in sinus and throat surgery. Operation of the blade is dependent on the annulus, or spacing between the two cutting surfaces. The cutting action is generated by the closing of the two sharpened surfaces on the annulus. The cutting portion is capable of cutting tissue samples with a maximum diameter of 0.6mm, which is constrained by the inner diameter of the cutting head. The cutting portion is a part of the spindle and rotates inside the cannula. Cutting teeth align both sides of the cutting head allowing samples to be chopped and suctioned with each rotation of the spindle.

The power portion’s most essential component is the impulse turbine. The operating principle of this turbine is based on the air pressure driving a jet of air directly against the turbine blades. This pneumatically driven turbine is used to generate spindle rotation speeds in the range of 5,000 to 40,000 RPM. The turbine has an outer diameter of only 6mm and has eight turbine blades. The turbine blades were modeled using similar turbine geometry found in dental drills. The spindle delivers torque from the turbine to the cutting portion. With a high rotation speed, the cutting tip of the shaft can cut the tissue with the required torque and cutting force.

The suction portion of the device draws tissue samples through the hollow spindle. The vacuum supply is external to the device and has variable operating pressure. The suction capabilities of this device provide a number of benefits. One benefit is that the doctor can continuously make incisions without stopping to extract tissue. Another benefit of the suction portion is that once the tissue is suctioned it is automatically stored in a sterile ampule. This is beneficial because contamination of samples affects diagnostic testing accuracy.
We worked closely with the engineers at the professional machine shop where the device was manufactured to better understanding their capabilities and limitations in manufacturing. When designing a device of this scale it’s important to design for manufacturability. One of the manufacturing challenges with this device was the 0.6 mm hole that extends 25 mm, the length of the spindle. Another design feature that was a problem was the air input and output. The machine shop needed to use an adhesive to attach these features because the parts were too small to be welded. The machine shop also reported that they experienced difficulty press fitting the turbine to the spindle. This was because the spindles diameter is very small and experienced bending from the pressure. Since all of these manufacturing challenges directly influence the cost of the product, it’s important to design for manufacturability.

The device is currently configured in such a way that the spindle rotates inside the cannula with a clearance of only 6μm. Deflections of the spindle greater than 6μm will cause interference with the cannula. This becomes an area of concern because contact between these two components at high rotation speed will cause friction, and thermal expansion. If the spindle thermally expands, there’s a good chance that it will prevent the turbine from rotating. To closer examine the threats of interference between components, a thorough structural analysis was conducted on the spindle.

A dynamic analysis was also conducted to determine the spindle’s factor of safety as well as its fatigue limit. We are interested in the factor of safety because it describes the structural capacity of the system beyond the expected loads or actual loads. The spindle’s ability to handle loading must be determined within reasonable accuracy because it directly influences the spindle’s fatigue limit.
In addition to stress analysis and fatigue failure theory analysis, a modal analysis was also conducted to determine the Eigenfrequencies or natural frequencies of the shaft. Natural frequencies are an area of concern because at each natural frequency, the shaft will transversely oscillate, due to centrifugal forces, causing deflection. We will examine the frequencies at which this phenomenon occurs and compare them to the operation speeds. Furthermore, the theoretical Eigenfrequencies will be validated using finite element analysis.
3. Static Structural Analysis

Structural analysis is a branch of mechanical engineering that determines the effects of loads on physical structures. Structural analysis incorporates applied mechanics and other fields to compute a structure’s deformations, internal forces, stresses, support reactions, accelerations, and stability. Section 3 covers the procedural analysis to determine the device’s structural effects of forces in static equilibrium. The analysis begins with determination of the forces acting on the turbine spindle. With all forces defined, we then move to stress analysis to determine bending and torsional stresses in the spindle. The bending stresses are a result of the cutting forces at the tip of the spindle, while the torsional stresses result from the moment created by the turbine. With these stresses calculated and maximum stresses determined, we then compute the spindle deflection. Finally, the theoretical spindle deflection is compared to computational finite element analysis to validate its accuracy.

3.1 Forces on Turbine Spindle

During operation, the impulse turbine-cutting device will experience several forces acting on the spindle. These forces, depicted in Figure 3.1, are the result of the required cutting force, the loading of the turbine, and the reaction forces at the bearings. Each of these forces is broken into component form so that the forces can be analyzed in three dimensions.
Located on the tip of the spindle, where the cutting portion is located, there are two component-cutting forces. These cutting forces are normal forces that result from the applied pressure of the cutting tip. Of these two component-cutting forces, one is acting in the negative $y$ direction $F_{cy}$, and the other in the positive $z$ direction $F_{cz}$. The magnitude of these forces was determined in a study to be 0.7 $N$. The cutting force is expected to cause bending of the spindle. The magnitude of this bending will be calculated in section 3.4 Deflection Analysis.

When air pressure is applied to the system it makes contact with the turbine blade causing the turbine to rotate. This pressure causes a force on the spindle that is distributed the length of the turbine. Again, this force is broken into component form and labeled $w_y$ and $w_z$. At the operating pressure of 10psi, the force can be determined using equation (a). Where the variable $Q$ is the mass flow rate, which can be defined using equation (b).

$$F_{air} = QV_{jet} \quad (a)$$
\[ Q = \rho V A \]  \hspace{1cm} \text{(b)}

In equation (b), \( \rho \) is the density of air at 20°C, \( V \) is the air velocity, and \( A \) is the area of channel that the fluid flows. The force of the air once calculated is broken into component form giving us \( w_y \) and \( w_z \) with values of approximately 0.1N and 0.1N respectively.

With the cutting and turbine forces calculated for both planes we can now solve for the reaction forces \( R_1 \) and \( R_2 \) located where the bearings contact the spindle. This can be done by summing the forces and moments along the spindle and then solving the system of equations for the unknown reaction forces. Since the system is assumed to be in equilibrium, we set the summing equations equal to zero. However, in order to take the sum of forces and moments, we first need to define the dimensions along the spindle. Figure 3.2 shows dimensions of the spindle measured in the positive \( x \) direction. With the dimensions defining the locations of the forces along the spindle we write equations (c) and (d) to describe the sum of forces and sum of moments respectively.

\[ a = 2.1\text{mm} \]
\[ b = 2.7\text{mm} \]
\[ c = 6.7\text{mm} \]
\[ d = 7.3\text{mm} \]
\[ L = 25\text{mm} \]

\textit{Figure 3.2: Spindle Dimensions}
\[ \Sigma F = R_1 - w(L - b) + w(L - c) + R_2 - F_C = 0 \] \quad (c)

\[ \Sigma M = R_1(L - a) - \frac{w}{2}(L - b)^2 + \frac{w}{2}(L - c)^2 + R_2(L - d) = 0 \] \quad (d)

To solve for the unknown reaction forces \( R_1 \) and \( R_2 \), we substitute the known dimensions and forces and solve the system of equations. This calculation is done for both the \( x-y \) plane and the \( x-z \) plane to obtain the following reaction forces at the bearings:

\[
R_{1y} = -0.971 \text{ N} \quad R_{2y} = 4.495 \text{ N}
\]

\[
R_{1z} = -1.511 \text{ N} \quad R_{2z} = 3.955 \text{ N}
\]

The reaction forces are then entered back into the equations to verify the obtained values. With every force on the turbine spindle defined, we proceed to analyze the bending and torsional stresses to determine their affects on the spindle.
3.2 Stress Analysis

A stress analysis was conducted to determine the maximum stresses in the spindle under loading. The spindle experiences both bending and torsional stresses due to the moment created by the cutting force as well as the torque created by the turbine. Of these stresses, we draw particular attention to the maximum shear stress, maximum bending moment, principal normal stresses, and principal shear stress.

3.2a Maximum Shear and Moment

The maximum shear stress and bending moment were calculated at every point along the spindle to determine the section that undergoes the most stress. In order to determine the sections of the spindle that experience the greatest shear forces and bending moments, we perform a 3-dimensional analysis using singularity functions. Singularity functions allow us to graph discontinuities, which can be used to describe various loadings along a shaft. Different loading distributions are defined by different unit step functions as depicted in Figure 3.3. If the load distribution singularity function is integrated, we obtain the shear function. If the shear function is integrated, we obtain the bending moment function.
Figure 3.3: Summary of Singularity Functions

To describe the spindle loading in 3-dimensions, our singularity functions must satisfy the distributions of loads in both the $x$-$y$ and $x$-$z$ planes. To do this, we must consider loading from both component cutting forces at the tip, the component distributed loads of the turbine, as well as the component reaction forces calculated in the previous section. We write a case-specific equation for the shear function $V(x)$, which we can substitute the calculated resultant forces into to solve for the 3-dimensional loading. This singularity function is then integrated to obtain the moment function $M(x)$ as seen below in equation (3b).
\( V(x) = R_1 < x - a >^0 - w < x - b >^1 + w < x - c >^1 + R_2 < x - d >^0 - F_c < x - L >^0 \) \hspace{1cm} (a)

\( M(x) = R_1 < x - a >^1 - \frac{w}{2} < x - b >^2 + \frac{w}{2} < x - c >^2 + R_2 < x - d >^1 - F_c < x - L >^1 \) \hspace{1cm} (b)

To calculate the maximum shear and bending moment in 3-dimensions, we use the resultant forces of our component forces described in Figure 3.1. To calculate the resultant force of the each pair of component forces acting on the spindle we use equation (3c) and obtain the following values.

\[ F_R = \sqrt{F_y^2 + F_z^2} \] \hspace{1cm} (c)

\[ F_{CR} = 0.99N \quad w_R = 0.14N \quad R_{1R} = 1.8N \quad R_{2R} = 5.99N \]

Substituting these resultant forces into the singularity function and graphing allows us to determine the critical sections of the shaft with both the \( x-y \) and \( x-z \) planes considered. Figure 3.4 and Figure 3.5 show the maximum shear force and maximum bending moment respectfully.

\[ V(x) = R_1 < x - a >^0 - w < x - b >^1 + w < x - c >^1 + R_2 < x - d >^0 - F_c < x - L >^0 \]

\[ M(x) = R_1 < x - a >^1 - \frac{w}{2} < x - b >^2 + \frac{w}{2} < x - c >^2 + R_2 < x - d >^1 - F_c < x - L >^1 \]

\[ F_R = \sqrt{F_y^2 + F_z^2} \]

\[ F_{CR} = 0.99N \quad w_R = 0.14N \quad R_{1R} = 1.8N \quad R_{2R} = 5.99N \]

Substituting these resultant forces into the singularity function and graphing allows us to determine the critical sections of the shaft with both the \( x-y \) and \( x-z \) planes considered. Figure 3.4 and Figure 3.5 show the maximum shear force and maximum bending moment respectfully.

\[ V(x) = R_1 < x - a >^0 - w < x - b >^1 + w < x - c >^1 + R_2 < x - d >^0 - F_c < x - L >^0 \]

\[ M(x) = R_1 < x - a >^1 - \frac{w}{2} < x - b >^2 + \frac{w}{2} < x - c >^2 + R_2 < x - d >^1 - F_c < x - L >^1 \]

\[ F_R = \sqrt{F_y^2 + F_z^2} \]

\[ F_{CR} = 0.99N \quad w_R = 0.14N \quad R_{1R} = 1.8N \quad R_{2R} = 5.99N \]

Substituting these resultant forces into the singularity function and graphing allows us to determine the critical sections of the shaft with both the \( x-y \) and \( x-z \) planes considered. Figure 3.4 and Figure 3.5 show the maximum shear force and maximum bending moment respectfully.

Figure 3.4: Diagram of 3-Dimensional Shear Force
As determined by the graphed singularity functions, the maximum shear and maximum bending moments are located at $x = 7.3\text{mm}$ with magnitudes of $3.8\ N$ and $-12.39\ N$ respectively.

3.2b Bending and Torsional Stress

The spindle shaft is subject to simultaneous axial, torsional, and bending loads. Combined loading involves a superposition of stresses at a given point. The stresses under analysis include principal normal stresses $\sigma$, which are compressive or tensile resulting from shaft bending, and principal shear stresses $\tau$, which result from torsion. The points that experience the greatest stresses in the hollow shaft are located internally nearest the outer wall of the shaft. These points are labeled $A$, $B$, $C$, and $D$ in Figure 3.6.
Since the force is in the negative $y$ direction, we can expect a tensile stress at point $A$ and a compressive stress at point $C$. The torsion, indicated in Figure 3.6 as $\tau$, applies a torsional shear stress in the counterclockwise direction and increases in magnitude further from the center. To obtain the applied normal stresses and torsional shear stresses we need to first define the following variables:

- $d_o = 1.5 \text{ mm}$
- $d_i = 0.6 \text{ mm}$
- $\tau = 0.525 \text{ Nmm}$
- $F_c = 0.7 \text{ N}$
- $L = 17.7 \text{ mm}$
- $J = \frac{\pi(d_o^4 - d_i^4)}{32} = 4.8 \times 10^{-13} \text{ m}^4$
- $A = \pi(r_o^2 - r_i^2) = 1.5 \times 10^{-6} \text{ m}^2$
- $G = 77.2 \text{ GPa}$

At every point in a stressed body there are at least three planes. These planes are called principal planes and to graphically represent the stress components in these principal planes, we draw stress cubes. Stress cubes allow us to represent the infinitely small locations where the stresses occur. The equations for the applied normal stresses $\sigma$, and torsional shear stresses $\tau$ are expressed for each of the four points around the shaft.
The principal normal stresses are used to calculate the von Mises stress and in later steps will be used to calculate the maximum shear stress. These principal normal stresses will be denoted as $\sigma_1$, $\sigma_2$, and $\sigma_3$ and are defined by equations (3d), (3e), and (3f).

\[
\begin{align*}
\sigma_1 &= \frac{(\sigma_{xx}-\sigma_{zz})}{2} + \tau_{\text{max}} = 813 \text{ MPa} \\
\sigma_2 &= 0 \text{ MPa} \\
\sigma_3 &= \frac{(\sigma_{xx}-\sigma_{zz})}{2} - \tau_{\text{max}} = -813 \text{ MPa}
\end{align*}
\]

The principal shear stresses can be found using the values of the principal normal stresses with equations (3g) through (3i).

\[
\begin{align*}
\tau_{13} &= \frac{|\sigma_1-\sigma_3|}{2} = 813 \text{ MPa} \\
\tau_{21} &= \frac{|\sigma_2-\sigma_1|}{2} = 407 \text{ MPa}
\end{align*}
\]
\[ \tau_{32} = \frac{|\sigma_3 - \sigma_2|}{2} = 407 \text{ MPa} \quad (i) \]

The maximum shear stress, or maximum principal shear stress is equal to one-half the difference between the largest and smallest principal stresses. The maximum principal shear stress is expressed in equation (3k).

\[ \tau_{max} = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{xz}}{2}\right)^2 + \tau_{xz}^2} = 813 \text{ MPa} \quad (k) \]

The final calculation in this section is the torsional angular deflection. This angular deflection is defined in equation (3l) and is a function of the applied torque, the length of the shaft, the moment of inertia, and the material’s modulus of rigidity.

\[ \theta = \frac{\tau L}{J_G} = 0.174^\circ \quad (l) \]
3.3 Deflection Analysis

Deflection analysis was conducted to address concerns the spindle making contact with the cannula under bending stress. The current design only allows approximately 6μm of deflection before the spindle makes contact with the cannula. For this reason, machining tolerances must be carefully considered. Interference between the spindle and the cannula will cause frictional heating and thermal expansion of the shaft. This must be avoided because expansion will impede rotation of the spindle. To calculate shaft deflection we must first define the modulus of elasticity (3m) as well as the polar moment of inertia (3n).

\[ E = 77.2 \text{ GPa} \]

\[ I = \frac{\pi(d_o^4 - d_i^4)}{32} = 4.842 \times 10^{-13} \text{ m} \]

Deflection of the tip from the cutting force is measured in the y-axis in [mm].

\[ Shaft \ Deflection: Y = \frac{F_cL^3}{3EI} = 34.7 \mu m \]

It was found that induced deflections are greater than the manufacturing tolerances. To validate our theoretical results, the assembly was analyzed using Finite Elements (FE). The mesh is programmed to contain the material and structural properties, which define how the structure will react to static and dynamic loading conditions. The meshing of the spindle assembly can be seen in Figure 3.8 below. Boundary conditions were applied to the model to fix the bearings in three-dimensional space as they would be fixed to the turbine housing. After the material properties were defined and the cutting force applied to the tip, the simulation was run. The forces on the tip of the spindle are
directed in both the negative y and positive z directions with magnitudes of 0.7N and 0.7N respectively.

Figure 3.8: Spindle Assembly Meshing

Figure 3.9: FE Deflection Analysis

It was found that the deflection simulation results in Figure 3.9 compare to the theoretical analysis by a 5% error. It is assumed that the error is a result in differences in material properties.
4. Dynamic Structural Analysis

When designing any device that is subjected to stresses, it’s important to consider the mechanisms that may cause failure of that device. Failures occurring under conditions of dynamic loading are called fatigue failures. We are going to analyze the stresses experienced by both the spindle and the bearings under dynamic loading to determine the theoretical fatigue strength and safety factor for both components.

4.1 Theoretical Fatigue Failure Analysis of Spindle

In most cases a part will fail due to time-varying loads rather than static loads. To safely design the turbine spindle, dynamic failure theories must be investigated and a safety factor calculated. Because the spindle rotates 360°, stresses on the spindle are considered to be fully reversed. This will be important to determine which fatigue-failure model to choose to calculate of the number of cycles to failure, as well as the safety factor. The steps that are going to be followed to determine the system’s safety factor are outlined below in Figure 4.1.

![Procedural Flow Chart](image)

*Figure 4.1: Procedural Flow Chart*
Fully reversed stress-time functions have a *mean stress* value of zero, which is defined in equation (3a). The *alternating stress component* $\sigma_a$ is found from equation (3b).

\[ \sigma_m = \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2} = 0 \]  
\[ \sigma_a = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2} = 407 \text{ MPa} \]

A materials endurance limit $S_e$ is an expression that describes the range of cyclic stress that can be applied to a material without causing fatigue failure. Estimation of the theoretical endurance limit $S_e$ can be conducted using a number of correction factors which account for the physical differences between the test specimen that the data was derived from and the actual part being designed. These correction factors include the spindles loading $C_{\text{load}}$, size $C_{\text{size}}$, surface finish $C_{\text{surf}}$, reliability $C_{\text{reliab}}$, and temperature $C_{\text{temp}}$. The loading effects $C_{\text{load}} = 1$ for situations involving bending fatigue. The size effects $C_{\text{size}}$, account for the fact that larger parts fail at lower stresses because the number of flaws in the material. For shafts with a diameter less than 8mm the correction factor for size is $C_{\text{size}} = 1$. The surface effects $C_{\text{surf}}$ account for surface imperfection that act as stress raisers. The equation to calculate surface effects is $C_{\text{surf}} = A(S_{\text{ut}})^b$, where $S_{\text{ut}}$ is the material’s ultimate tensile strength and $A$ and $b$ are coefficients for the surface-factor. $C_{\text{surf}}$ was determined to be 0.753. Assuming 99.9% reliability, $C_{\text{reliab}} = 0.753$ from the reliability factor table. The last correction factor is temperature. For temperatures that are less than 450°C, $C_{\text{temp}} = 1$. 
With values for the correction factors, the last variable needed to calculate the corrected endurance limit $S_e$ is the uncorrected endurance limit $S_e'$. The uncorrected endurance limit is half the ultimate tensile strength and is calculated using equation (3c).

$$S_e' = 0.5S_{ut} = 430\text{MPa}.$$  \hspace{1cm} (c)

With all variables defined, we are now able to calculate the spindle’s endurance limit from equation (3d).

$$S_e = C_{load}C_{size}C_{surf}C_{temp}C_{reliab}S_e'$$  \hspace{1cm} (d)

The corrected endurance limit was calculated to be $244\text{MPa}$. The next procedural step is to determine the material’s strength at $10^3$ cycles $S_m$ in order to create an estimated S-N diagram. The estimated S-N diagram is used to graphically represent the failure point of the device. For bending loading we find $S_m$ using equation (3e).

$$S_m = 0.9S_{ut} = 774\text{ MPa}$$  \hspace{1cm} (e)

With known values for the material’s ultimate tensile strength $S_{ut}$, the corrected endurance limit $S_e$, the material’s strength at $10^3$ cycles $S_m$, and the alternating stress component $\sigma_a$ we can draw the Wohler S-N diagram to graphically represent how many cycles our device can endure before failing. If the alternating stress component intersects with the $S_{ut}$, $S_m$, and $S_e$ line, the point of intersection denotes the number of cycles that part can endure. If the applied stress level is below the endurance limit of the material, the structure is said to have an infinite life. The S-N curve levels out at the endurance limit and exhibiting a knee. Endurance limit knees are characteristic for steels and
titanium, but not for non-ferrous metals, such as aluminum, magnesium, and copper alloys.

**S-N Diagram**

As determined by the Wohler S-N diagram in Figure 5.1, the alternating stress component \( \sigma_a \) intersects the alternating stress line marking the failure point. A more precise measurement of theoretical cycles to failure will be calculated to verify the data in the S-N diagram. The number of cycles of life for any alternating stress level can be found from the equation (3f).

\[
S_{(N)} = aN^b \tag{f}
\]

\( S_{(N)} \) is the fatigue strength at any \( N \) and \( a, b \) are constants defined by the boundary conditions. The variable \( b \) can be obtained from equation (3g).

\[
b = -\frac{1}{z} \log \left( \frac{S_m}{S_e} \right) \tag{g}
\]

In this equation, the variable \( z \) can be defined as
\[ z = \log N_1 - \log N_2 \]  

(\textit{h})

It was determined that \( b = -0.167 \). The value for \( a \) can be obtained by solving

\[ \log(a) = \log(S_m) - 3b. \]  

(\textit{i})

With the value of \( a = 2453.25 \) and the value obtained for \( b \), we can calculate the number of cycles the spindle can endure with the given fatigue strength. Substituting the values back into equation (3f) it can be determined that the number of cycles \( N = 10^{4.67} \) cycles.

The maximum number of cycles that would be experienced by this device, assuming an average operation speed of 15,000 RPM and operation duration of 1 hour with continuous use, is \( 10^{5.95} \) cycles.

Notch sensitivity is the extent to which the sensitivity of a material to fracture is increased by the presence of a surface inhomogeneity such as a notch, a sudden change in section, a crack, or a scratch. The notch sensitivity of the material must be found to calculate the fatigue stress concentration factors. In the case of the turbine spindle, the notches of interest are located to the inside of the bearings. At this location, the diameter of the spindle is increased and provides a position for the turbine to be press fit. The notch radius at these diameter changes is \( r = 0.003\text{mm} \). Equation (3j) defines the notch sensitivity \( q \). In this equation, \( a \) is Nurber’s constant\(^v\) and \( r \) is the notch radius.

\[ q = \frac{1}{1 + \left( \frac{\sqrt{a}}{\sqrt{r}} \right)} \]  

(\textit{j})
Substituting values in for \( q \) we find \( q = 0.4384 \). This value for \( q \) will later be used to determine the fatigue stress concentration factor. The next value we need to obtain is \( K_t \), the geometric stress concentration factor. \( K_t \) is defined in equation (3k), where \( r/d \) is the ratio of the notch radius and the smaller diameter of the shaft. \( A \) and \( b \) are obtained by linear interpolation of table data from *Figure 4-36 of Norton’s “Machine Design.”*vi

\[
K_t = A \left( \frac{r}{d} \right)^b
\]  

Substituting and solving proves \( K_t = 5.26 \). The same equation is used to solve the geometric stress concentration factor for torsion, \( K_{ts} = 3.58 \). With the geometric stress concentration factors for bending and torsion and notch sensitivity \( q \) we can solve for the fatigue stress concentration factors \( K_f \) and \( K_{fs} \). The fatigue stress concentration factor allows us to find values for the estimated local-stress components, which becomes influential in calculating the safety factor. These fatigue stress concentration factors are defined by equations (3l) and (3m).

\[
K_f = 1 + q(K_t - 1) = 2.868 \quad \text{(l)}
\]
\[
K_{fs} = 1 + q(K_{ts} - 1) = 2.131 \quad \text{(m)}
\]

Illustration of a Modified-Goodman Diagram is used to graphically represent the boundaries at which the device will fail. The “Augmented” Modified-Goodman Diagram vii in *Figure 5.2* shows a plot of the alternating stress \( \sigma_a \) versus mean stress \( \sigma_m \). The shaded area bounded by ABCDEA defines the failure envelope. Any combination of alternating and mean stress that falls in this grey-colored envelope can be considered safe.
Figure 4.3: “Augmented” Modified-Goodman Diagram

Understanding the bounds of our failure envelope we continue to calculate a safety factor for the spindle shaft. To determine the safety factor for a fluctuating-stress state we first need to compute the von Mises alternating stress $\sigma'_a$, but before we can calculate that, we need to analyze the bending and torsion at two critical locations on the spindle. These two locations, if we refer back to the stress cubes in section 3.2, are labeled A and B. The fatigue stress-concentration factor for the mean stresses depends on the relationship between the maximum local von Mises stress in the notch and the yield strength\(^viii\). If $K_f|\sigma_{\text{max}}| < S_y$ then $K_f = K_f' = 2.868$ and $K_f = K_f' = 2.131$. Equations (3n) and (3o) define the maximum local von Mises stress in both bending and torsion respectively.
The alternating components of the normal bending stress and of the torsional shear stress on point A and B can be expressed by the following equations (3p) and (3q).

\[
\sigma_a = K_f \frac{M_a c}{I} = 38.7 \text{ MPa} \quad (p)
\]

\[
\tau_a = K_{fs} \frac{T_a r}{J} = 10.9 \text{ MPa} \quad (q)
\]

The alternating von Mises stresses at points A and B are defined by equations (3r) and (3s) respectively.

\[
\sigma'_{aa} = \sqrt{\sigma_{xa}^2 + \sigma_{ya}^2 - \sigma_{xa} \sigma_{ya} + \tau_{xya}} = 43.06 \text{ MPa} \quad (r)
\]

\[
\sigma'_{ab} = \sqrt{\sigma_{xa}^2 + \sigma_{ya}^2 - \sigma_{xa} \sigma_{ya} + \tau_{xya}} = 18.88 \text{ MPa} \quad (s)
\]

With the alternating von Mises stress and corrected endurance limit, the safety factor can be determined for both points A and B using equations (3t) and (3u). This equation is specific to cases in which the alternating stress varies and the mean stress is constant.

\[
N_{fa} = \frac{S_e}{\sigma_{a'}} \left( 1 - \frac{\sigma_{m'}}{S_{ut}} \right) = 5.667 \quad (t)
\]

\[
N_{fb} = \frac{S_e}{\sigma_{a'}} \left( 1 - \frac{\sigma_{m'}}{S_{ut}} \right) = 12.92 \quad (u)
\]
We calculate the safety factor of both points to determine which is the least. The safety factor for point A will be used, as the device will fail from stresses acting on point A first.

4.2 Theoretical Fatigue Failure Analysis of Bearings

To analyze the fatigue life of the ball bearings in the system we must first define certain parameters, including the dynamic load rating $C$, and the constant applied load $P$. There are two bearings in the system. The bearing located nearest the spindle tip experiences the greatest reaction forces due to the bending moment caused by the cutting force. The resultant reaction forces on the bearings were calculated in section 3.1 and identified as $R_{1R} = 1.8N$ and $R_{2R} = 5.99N$. These reaction forces are going to be the applied loads $P_1$ and $P_2$ respectively. The dynamic load rating is defined by the bearing manufacturer and in the case of the bearings used in this system, $C = 57.83 \, N$. With the information above we can define an expression for the fatigue life $L_{10}$ expressed in millions of revolutions. Through extensive testing by bearing manufacturers, the fatigue life $L_{10}$ for ball bearings can be expressed by equation (3v).

$$L_{10} = \left( \frac{C}{P} \right)^3$$

Substituting values $P_1$ and $P_2$ we obtain fatigue life for bearings 1 and 2 defined: $L_{10} = 3.3E4 \, million \, revolutions$ and $L_{20} = 8.9E2 \, million \, revolutions$.

4.3 Hydrodynamic Lubrication Analysis

Since the cutting force will cause interference between the spindle and cannula, I suggest the introduction of hydrodynamic lubrication between these surfaces. Hydrodynamic lubrication was first researched by Osborne Reynolds. His research showed that the liquid pressure caused by a wedge of lubricant beneath a rotating shaft
was great enough to keep two bodies from having contact with one another. With hydrodynamic lubrication, the surfaces can be completely separated by a lubricant film. The load of the cutting force on the spindle will be entirely supported by fluid pressure. The benefit of this phenomenon is greatly reduced friction between the rotating members resulting in reduced heating and surface wear.

Calculation of friction between the spindle and cannula can be achieved by treating the two surfaces as a concentric shaft and bearing. By doing this we can apply Petroff’s equation for bearing friction which is derived from Newton’s law of Viscosity in equation (3w). In this equation $F = \frac{\text{friction torque}}{\text{shaft radius}} = \frac{2T_f}{d}$, $A = \pi dl$, $U = \pi dn$, and $h =$ the radial clearance $= 0.5(d_o - d_i)$.

$$F = \mu \frac{AU}{h}$$  \hspace{1cm} (w)

Substituting and solving for friction torque we obtain equation (3x).

$$T_f = \frac{4\pi^2 \mu nl r^2}{c}$$  \hspace{1cm} (x)

Considering the cutting force a small radial load $W$, we set equation (3x) equal to $T_f = fW$ and solve for the coefficient of friction. This yields equation (3y). In this equation $\rho = \frac{F}{dl}$, $\mu = \text{absolute viscosity of water}$, $n = \text{rev/s}$ and $c = \frac{(d_o - d_i)}{2}$.

$$f = 2\pi^2 \left( \frac{\mu n}{\rho} \right) \left( \frac{r}{c} \right)$$  \hspace{1cm} (y)

Solving the equation with the known values, we determine that the coefficient of friction for this particular system at 15,000 RPM is $f = 0.04726$. With the coefficient of friction calculated we can apply this value to the friction torque equation $T_f = fF\rho = 0.0249 \, Nm$. To determine the power loss of the system we use equation (3z) where $w$ is revolution speed in rad/s.
\[ N_{loss} = T_f w = 0.659W \] (z)

Figures 4.4 and 4.5 show the relationships between the coefficient of friction vs. RPM and power loss vs. RPM respectively.
Figure 4.5: Power Loss vs. RPM
5. Modal Analysis

Modal analysis is defined as the study of the dynamic properties of structures under vibrational excitation. These dynamic properties are defined by the structure’s mass and stiffness and geometry. Any systems that are subject to vibrational excitation, whether from oscillation, rotation, or other, should undergo modal analysis.

Modal analysis was conducted to determine the natural frequencies of the spindle. We are concerned about these natural frequencies because when these frequencies are reached, relatively large deflections occurs. These deflections, or transverse oscillations are the result of centrifugal forces.

5.1 Analytical

To calculate the theoretical natural frequencies in a rotating hollow shaft we must define the material’s density $\rho$, the shaft’s mass moment of inertia $I$, and the weight of a one-millimeter-length of the shaft $w$. The other variables needed for calculation of the natural frequencies of a rotating hollow shaft are as follows.

\[
\begin{align*}
    d_o &= 1.5 \text{ mm} \\
    d_i &= 0.6 \text{ mm} \\
    L_{\text{shaft}} &= 17.7 \text{ mm} \\
    g &= 9.8 \text{ m/s}^2 \\
    I_{zz} &= 54.07 \text{ mm}^4 \\
    \rho &= 8000 \text{ kg/m}^3 \\
    I &= \frac{(d_o^4 - d_i^4)}{64} \\
    w &= 0.466 \text{ mN} \\
    g &= 9.8 \text{ m/s}^2 \\
    E &= 203 \text{ GPa}
\end{align*}
\]
Natural frequencies occur in modes, meaning there is more than one natural frequency for a vibrating member. The first mode is called the fundamental frequency. The succeeding modes are functions of the fundamental frequency and can be calculated as follows.

\[ f_n = \frac{\pi}{2} \sqrt{\frac{gEI}{WL_{shaft}}} \]

(a) 1\textsuperscript{st} Mode: \[ f_1 = 10.2\text{kHz} \]

(b) 2\textsuperscript{nd} Mode: \[ f_2 = (2^2) \frac{\pi}{2} \sqrt{\frac{gEI}{WL_{shaft}}} = 40.8\text{kHz} \]

(c) 3\textsuperscript{rd} Mode: \[ f_3 = (3^2) \frac{\pi}{2} \sqrt{\frac{gEI}{WL_{shaft}}} = 91.9\text{kHz} \]

5.2 Computational

Using FE analysis in Comsol Multiphysics, we are able to validate our analytical modal analysis. The bearings, spindle, and turbine assembly as seen in Figure 4.1 was meshed using a “fine mesh.” Fine mesh is used to obtain more accurate results, however the simulation will take longer to run. An Eigenfrequency analysis was run to determine the first three modes of natural frequencies. The results of the FE analysis can be found in Figures 5.2-5.4.
Figure 5.1: Modal Analysis Meshing

Figure 5.2: Mode 1 Eigenmode

Figure 5.3: Mode 2 Eigenmode
The results of the FE analysis compare to the theoretical analysis with a maximum percent error of 3.9%. With the theoretical results of the modal analysis validated, we can now draw conclusions regarding the data. As stated earlier, the maximum operating speed of the turbine is approximately 40,000 RPM. Comparing this to the calculated fundamental frequency we can rule out forced vibration as a cause of shaft deflection. The calculated fundamental frequency of the spindle was about 15 times greater than the maximum operating speed \( \approx 612,000 \) RPM, thus we can rule out vibrational deflection as a potential source of failure.

Figure 5.4: Mode 3 Eigenmode
6. Experimental Demonstrations

To determine the performance specifications of the cutting turbine, a test configuration was designed as seen in Figure 6.1. This preliminary drawing allowed us to make an inventory of parts that would be needed. The test configuration for the impulse turbine was designed to measure the rotation speed, mass airflow, input pressure, and maximum torque and power. The air filter/regulator [5] is used to insure that any debris, including oil, from the air supply does reach turbine. The manometer [19] allows us to measure power loss by comparing input pressure to the output pressure. To obtain the maximum power and torque at any given rotation speed we use a dynamometer [18]. The vacuum was also included in the test configuration to determine its effects on the turbine’s performance.
To measure the spindle rotation speed we used laser stroboscopy. This was achieved by pulsing a laser at the same frequency as the spindle rotation using a laser controller [14] and a function generator [15]. The physical configuration for the rotation speed measurement system can be seen below in Figure 6.2.

![Figure 6.2: Rotation Speed Measurement System](image)

6.1 Results

When the impulse turbine cutter was first connected to the air supply, the turbine failed to rotate. The spindle could be turned by hand but with noticeable resistance. The impulse turbine cutter was sent back to the professional machine shop for further examination. It was determined that the clearance between the spindle and the cannula was less than the design’s allotted 6μm. After making adjustments to the inner diameter of the cannula, a second test was performed. The turbine performed well for approximately 5 minutes before the spindle stopped rotating. The cause of this was determined to be friction of the spindle and thermal expansion. The inner diameter of the cannula was adjusted one last time providing us with a functional and testable prototype.
A test was conducted to provide us with definitive data regarding the effects of input pressure on spindle rotation speed\textsuperscript{ix}. Moreover, we were interested in determining whether the suction portion’s vacuum system had negative affects on the turbine rotation speed. The spindle’s rotation speed (RPM) was measured as a function of the input pressure (psi). Increasing the pressure in intervals of one psi, the spindle’s rotation speeds were measured and recorded twice for each instance and the average was taken. The graph below in \textit{Figure 6.3} shows the trend of pressure vs. spindle RPM. Turbine speeds were measured using two different turbine assemblies. The turbine assemblies differed in the location of the input. The input on one device is located 0.8mm from the center of the turbine, while the other is 1.5mm from the center. This change in location of the input was intended to increase the torque and decrease the required pressure.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Pressure_vs_RPM.png}
\caption{Pressure vs. RPM Graph}
\end{figure}
As determined by the data collected from the test, the affects of the vacuum on the system are minimal. Also, the location of the input has small, but noticeable effects on the turbines performance. *Figure 6.4* shows the combined average of both devices disregarding the vacuum and also a logarithmic curve fit of each. You can see here that the turbine whose input is located 1.5mm from the center tends to perform better at lower RPM.

*Figure 6.4: Pressure vs. Average RPM Graph*
7. Discussion

A goal with this device is to eventually reach mass production, however, the current design will be very costly to manufacture. In order to reduce the cost of manufacturing it’s important to understand the manufacturing processes and techniques in order to design the part for manufacturability. Understanding the manufacturing process is important because there are many factors that can affect cost. These factors include raw material type, dimensioning tolerances, and post processing. The current design uses high-grade tolerances of up to $\pm 0.006 \text{ mm}$. Since the components of this design are so small, the dimensioning tolerances also need to be small. Generally speaking, smaller dimension tolerances drive up the cost.

Most surgical equipment is made out of stainless, martensitic steel, because it is much harder than austenitic steel, and easier to keep sharp. The downside to using martensitic steel is that it is relatively costly. 316 surgical steel is commonly used for medical tools because of it’s relatively inexpensive and doesn’t leave behind metallic contamination.

To eliminate almost all friction in the system, I suggest implementing hydrodynamic lubrication between the spindle and cannula. Since this is for medical use, the only lubricants feasible are water and air. A shaft with hydrodynamic lubrication touches its surfaces together when it is stopped or rotating below its “aquaplane” speed. This means that adhesive wear can occur only during startup and shutdown, greatly increasing the life of the device. There are several general rules for optimization of rotating shafts, but we will only be focusing on a few that are specific to this design. Since deflection of the shaft has proven to be a problem both theoretically and in testing,
I would suggest one of three options: 1. Eliminate the cannula from the design completely, 2. Assure that there is a minimum of 33\(\mu\text{m}\) of clearance between the spindle and cannula, or 3. Minimize the deflection of the spindle to less than 6\(\mu\text{m}\) by shortening the length of the shaft. The most practical of these three options is to increase the inner diameter of the cannula. Other suggestions for design optimization include improvement of the turbine blade geometry. More efficient blade geometry will allow the turbine to reach higher RPM’s and achieve greater torque. Another suggestion is to recalculate the computational eigenfrequencies with the cutting tip modeled. The current computational and theoretical modal analysis assumes homogeneity throughout the shaft. With a cutting portion designed we can expect lower eigenfrequencies as a result of the centrifugal forces and unevenly distributed weight at the spindle tip.

8. Conclusions

After comprehensive theoretical and FE analysis of stress, deflection, and vibration, we can draw conclusions about the system that can help future designs. As determined by the deflection analysis, the required cutting force will cause a 33\(\mu\text{m}\) deflection of the spindle. This will cause interference between the cannula and the spindle causing friction, heating, and thermal expansion. The surface roughness of the spindle and cannula should be reduced as much as possible with post processing to reduce the friction. Modal analysis determined that natural frequencies do not pose a threat to the functionality of the system. Since the fundamental frequency of the shaft is approximately 40 times greater than the maximum operating speed we can focus our
concern on other areas. The fatigue failure analysis has proven that the device’s spindle, under the given stresses, will have a factor of safety equal to 5.667.

After testing our device to determine whether or not the vacuum has effects on the spindle speed with respect to input pressure, we conclude that any effects are insignificant. It has also been determined that the differences between the spindle outer diameters ranging from 0.8mm to 1.5mm also have insignificant effect on the system. The test results provide us with important information regarding what pressure we need to achieve certain cutting speeds.

Future work on this device will involve the studying of a Tesla turbine. Tesla turbines are unique in that they are bladeless. Instead of a fluid impinging upon the blades like the first generation conventional turbine, the Tesla turbine uses the boundary layer effect. This type of turbine is usually made up of a number of disks fixed to a shaft with close spacing. An example of a Tesla turbine is depicted in Figure 8.1.
Air is forced through the disks and the resistance of the air on the disks, or boundary layer effect, causes rotation. The air then exits through holes in the shaft seconding as a means of vacuum. This design is desirable because the disks will be much easier to manufacture than complicated turbine blade geometry. Another benefit of the Tesla turbine is that the exhaust can be used to vacuum the tissue samples eliminating the need for the external vacuum.
References


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vi Fig. 73, p.98, R. E. Peterson, Stress Concentration Factors, John Wiley & Sons, 1975.

