Introduction to Acoustics Course Development

A Major Qualifying Project

submitted to the faculty of

WORCESTER POLYTECHNIC INSTITUTE

in partial fulfillment of the requirement for the

Degree of Bachelor of Science in

Mechanical Engineering and Interdisciplinary Studies

By: Jacob Bendick (Mechanical Engineering), Samuel Lambert (Mechanical Engineering), Lauren Meinhold (Mechanical Engineering), and James Obermaier (Interdisciplinary – Acoustics)

May 2024

Abstract

The study of Acoustics has been of relative discussion dating back to the 6th century. Since then, the topic has expanded into different fields and degrees of complexity. Topics such as wave phenomena, ultrasonics in medical science, effects on architecture, and SONAR/underwater applications are presently being studied and applied in today's world. Seeing the glimpse of modern-day topics within acoustics, the study of acoustics itself continues to become more prominent as society grows, gets louder, and technology advances. With this, Worcester Polytechnic Institute (WPI) does not have a course on acoustics, hence the creation of this project. Therefore, the end goal of this project was to create a foundation for a course that professors, like our advisor, would be able to teach. In completing this goal, the student team researched topics within acoustics following an ideation exercise to determine what was to be included in this project. Once the research on a topic was complete, the student team collaborated to create course material that applied the teaching methods the team researched and believed in. Hence, course material was created via lecture slide decks, electronic modeling applications and examples, and live demonstrations to assist in the conceptualization of topics.

Table of Contents

Acknowledgements

Completion of this project would not have been possible without the notable commitment of our project advisor Professor Joe Stabile. Throughout the project, he provided the students with intuitive comments that helped shape the course to be understood and received by future students. Professor Stabile’s previous experience with both the topic of acoustics and course development overall formulated a workspace for us to learn from him as an advisor. Additionally, Professor Stabile gave freedom to the team to work and bring new ideas to the table. In the end, he helped the team to foster creativity and maintain endurance when producing the final course for this project.

List of Figures

List of Tables

1.0 Introduction

1.1 Overview

This research project’s purpose is to create a widely adaptable course that will introduce acoustic studies into the mechanical engineering department at Worcester Polytechnic Institute (WPI). The group created a course that can be altered and taught by any professor knowledgeable in acoustics leaving room for personal knowledge and experiences to be incorporated into the course.

This MQP went through multiple iterations throughout the academic year. Planning the best course of action to deliver information clearly and concisely was an overarching challenge that caused the group to have to rework the course layout. While the initial plan was to have lectures be created completely by the end of the first semester, then finalized in the second, the group rerouted and restructured lectures to implement teaching methods proven to enhance students understanding of topics presented.

Through researching mechanical engineering programs at alternative universities, it was discovered that acoustics was taught at a variety of them. Therefore, the implementation of this course may enhance students' understanding of mechanical engineering, but also diversify WPI’s mechanical engineering department. Moreover, the primary objective of this project was to create an acoustic course that will:

1. Provide students with introductory information about acoustic science.
2. Create a course that conveys information clearly and efficiently.
3. Incorporate real-world examples and lab opportunities that will enhance students' ability to apply information in the workforce.
4. Allow professors the flexibility to tailor this course to their own experiences and knowledge.

1.2 Objectives

At the beginning of the project, the group created a “global map”. This allowed us to build lectures based on topics. In turn, our first design method iteration of a course layout was created. Overtime, however, it lacked lecture structure and did not focus on building knowledge throughout the course. Initial slide deck creations did not have a cohesive theme, and the content did not build from lecture to lecture. Despite its downfalls, the first course layout was helpful in determining our course structure of three lectures and one lab per week.

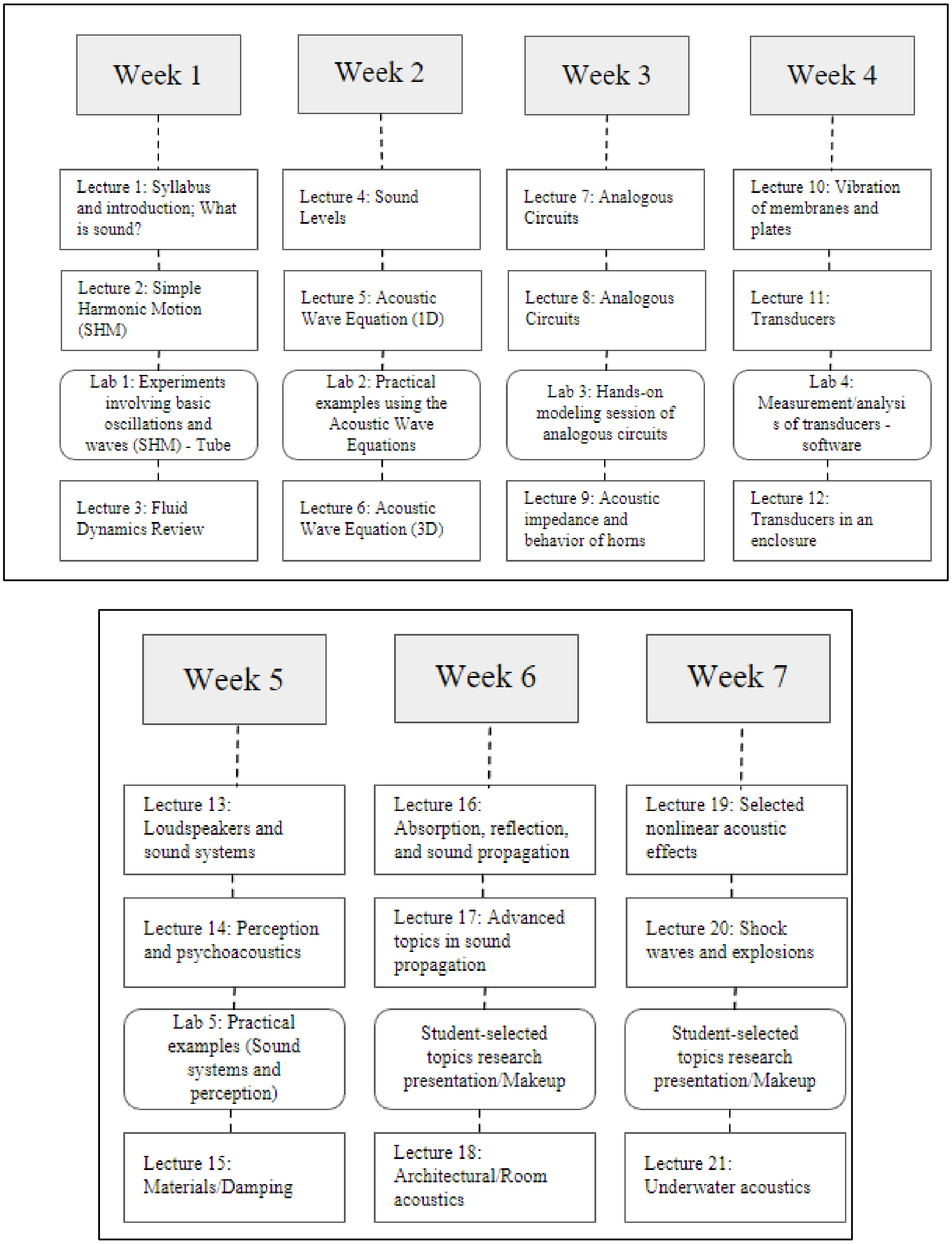


Figure 1: Initial Course Layout

In time, the course structure from Figure 1 was reworked based on the teaching methodologies that will be explained in detail in Section 2.2. Through the application of teaching methods involving multiple forms of information delivery, all students in the classroom can be reached. Therefore, our final course layout is shown below. Section 4.0 describes in depth the purpose of the final course design, and the application of the teaching methodologies.

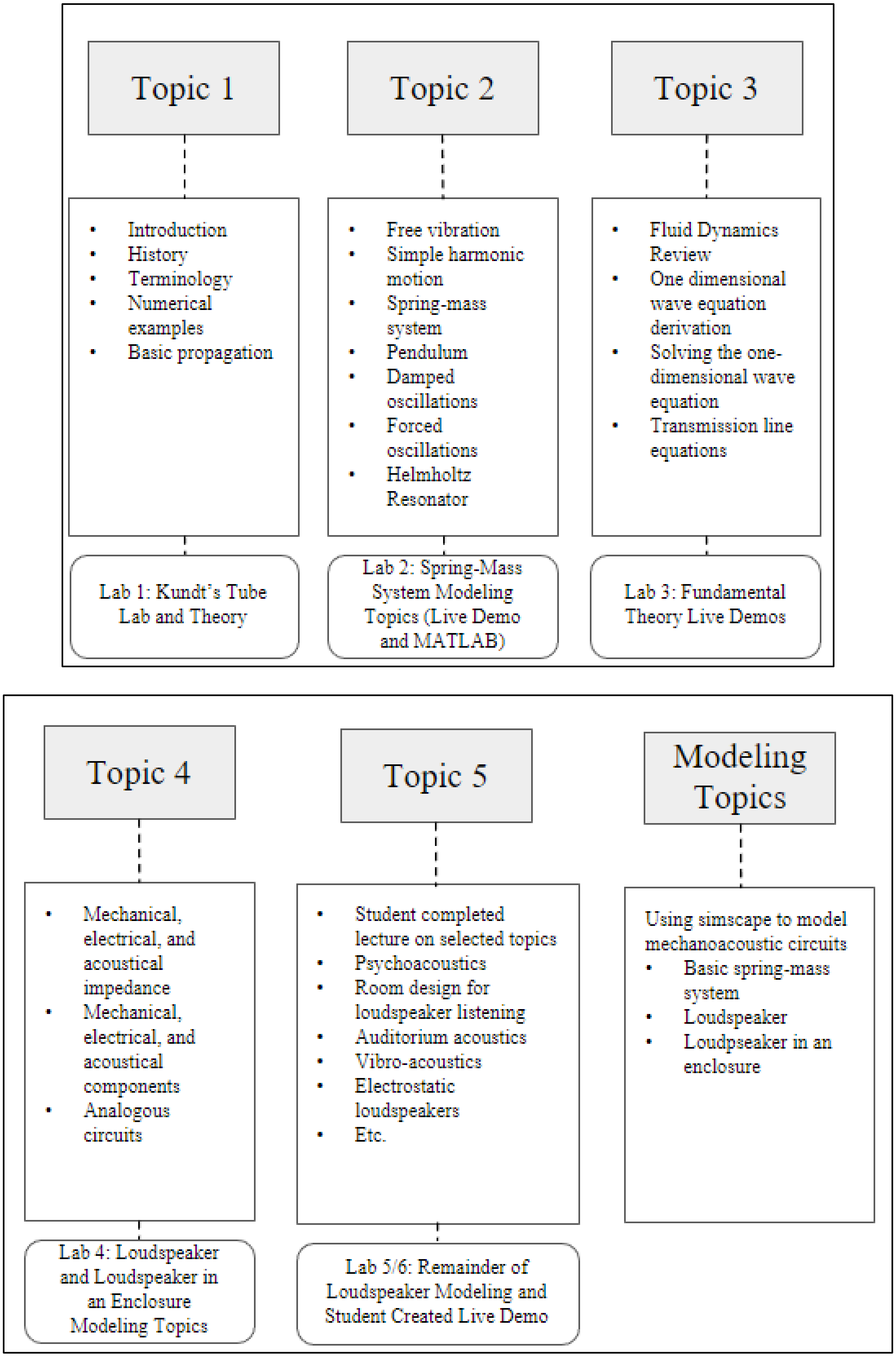


Figure 2: Final Course Layout

2.0 Background

2.1 History of Acoustics

In 6th century BC, the Greek philosopher Pythagoras experimented with properties of vibrating strings.[1] These experiments ultimately lead to the tuning system in musical instruments, as well as awarded Pythagoras to being the origin of acoustic science. In 4th century BC, Aristotle hypothesized sound wave propagation in air.[2] He based this greatly on philosophy opposed to experimental physics. He additionally hypothesized that high-frequency waves propagate faster than low frequencies, which was later determined to be incorrect. However, this false hypothesis persisted until 1st century BC. The Roman architectural engineer Vitruvius determined the correct mechanism for sound wave transmission through his design of theatres. In 6th century AD, Boethius, a Roman philosopher, suggested the human perception of pitch and frequency, based on the documentation of ideas relating science to music.[3]

Galileo elevated acoustic science in the 15th century to a modern study of sound waves in acoustics. He introduced the correlation between pitch and frequency of the sound source and created a foundation for mathematician Marin Mersenne. Mersenne studied the vibration of stretched strings and created the three Mersenne's laws which provided the basis for modern musical acoustics. The three laws summarized state:

1. Longer strings play lower notes and shorter strings play higher notes.
2. Strings that are looser play lower notes.
3. Heavier strings play lower notes, while lighter strings play higher notes.[4]

Later in the 15th century, English physicist, Robert Hook produced the first sound wave of known frequency using a rotating cog wheel as a measuring device. This was the beginning of simple harmonic motion.[5]

2.2 Teaching Methods

To effectively create a course, one must first identify the desired outcomes that students must fulfill. This is done to achieve the overall goal of an effective course, which is to present knowledge clearly, and concisely with multiple delivery methods. With this, students can begin to develop characteristics that include:[6]

---

[1] "Acoustics | Definition, Physics, & Facts | Britannica," n.d., para. 5, https://www.britannica.com/science/acoustics.

[2] "Acoustics | Definition, Physics, & Facts | Britannica," para. 5.

[3] "Acoustics | Definition, Physics, & Facts | Britannica," para. 5.

[4] "Sound - Overtones, Frequency, Wavelength | Britannica," n.d., para. 5, https://www.britannica.com/science/sound-physics/Overtones.

[5] "Acoustics | Definition, Physics, & Facts | Britannica," para. 6.

[6] Eric Forcael, Gonzalo Garcés, and Francisco Orozco, "Relationship Between Professional Competencies Required by Engineering Students According to ABET and CDIO and Teaching–Learning Techniques," *IEEE Transactions on Education* 65, no. 1 (February 2022): 46–55, https://doi.org/10.1109/TE.2021.3086766.

- Understanding the societal responsibility of their actions.
- Behaving under high ethical precepts.
- Being committed, autonomous, and reliable.
- Having the necessary competencies to use, transform, and create technology.
- Working effectively in teams.
- Updating themselves in terms of current engineering problems and continuously learning in the long term.
- Knowing how to communicate efficiently.
- Having negotiation and decision-making skills.
- Incorporating the attitude toward service in the engineering profession, among other characteristics.

Forcael et al. explains the relationship of professional competencies required by engineering students according to the Accreditation Board of Engineering and Technology (ABET) outcomes, the Concept-Design-Integration-Operation (CDIO) syllabus, and teaching-learning techniques.[7] This project's course has taken the CDIO syllabus method to determine the learning techniques that are applied. The CDIO syllabus method is divided into "syllabus levels" which target four unique aspects of learning that are dispersed throughout the entirety of this project's course. The specific aspects of learning that the syllabus levels cover are as follows:

- Syllabus Level 1 (SL1): Fundamental Knowledge
- Syllabus Level 2 (SL2): Personal and Professional Skills
- Syllabus Level 3 (SL3): Interpersonal Skills
- Syllabus Level 4 (SL4): Application of Knowledge

Table 1 below depicts the summarized student syllabus levels of the CDIO syllabus teaching method.

---

[7] Forcael, Garcés, and Orozco, para. 10.

Table 1. Description of Syllabus Levels of the CDIO syllabus[8]

Descriptions of Syllabus Levels (SL) of CDIO syllabus	
SL1	Disciplinary knowledge and reasoning Knowledge of underlying mathematics and science Core fundamental knowledge of engineering Advanced engineering fundamental knowledge, methods, and tools
SL2	Personal and professional skills and attributes Analytical reasoning and problem solving Experimentation, investigation, and knowledge discovery System thinking Attitudes, thought, and learning Ethics, equity, and other responsibilities
SL3	Interpersonal skills: teamwork and communication Teamwork and communication Communications in foreign languages
SL4	Conceiving, designing, implementing, and operating systems in the enterprise, social and environmental context External, societal, and environmental context Enterprise and business context Conceiving, systems engineering, and management Designing, implementing, and operating systems

---

[8] Forcael, Garcés, and Orozco, para. 14.

Reviewing Table 1, it is observed that, through following the CDIO syllabus, a course can be structured to help students develop in a variety of areas. To this point, we used multiple learning techniques, covering the key components of each syllabus level described in Table 1, to produce this project's course. Therefore, the CDIO syllabus is addressed throughout the course overall, while the developed lecture slide decks themselves follow a philosophy that aids in fulfilling the CDIO syllabus.

Each lecture slide deck created in this course follows what Domizio refers to as a "good lecture". In "Giving a Good Lecture", Domizio explains preparation and structure are key to the delivery of a quality lecture. First, the lecture topics need to be conceptually delivered to the student. From here, the foundational math concepts can be implemented, leading to the final step of a good lecture, where the conceptual and math concepts are applied to real-world problems.[9] Below, Figure 3 shows how the individual lecture slide decks were structured to follow the good lecture format explained by Domizio.

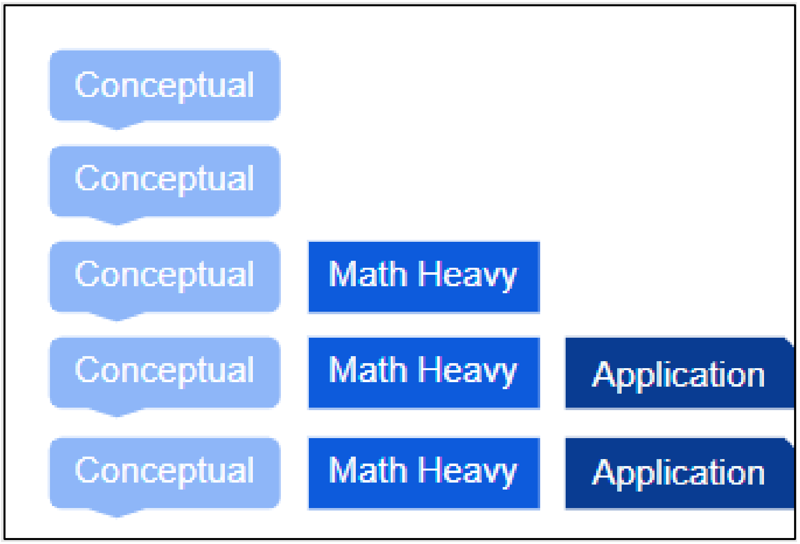


Figure 3: Ideal Structure of Balanced Lecture Slides

From Figure 3, it is observed how lectures should build both within themselves and throughout the course. Beginning with the standalone conceptual boxes, these represent the beginnings of individual topics, and the opening course slides as well. In this way, information is presented to illustrate phenomena that will be explored in greater detail. As time progresses, the material being covered will begin to grow in complexity, as Figure 3 illustrates. Through proper execution, students will therefore experience high quality individual lectures, and feel a flow as the course transitions between lecture topics.

---

[9] Paola Domizio, "Giving a Good Lecture," *Diagnostic Histopathology* 14, no. 6 (June 1, 2008): 284–88, https://doi.org/10.1016/j.mpdhp.2008.04.004.

Alongside the structure of a lecture, Domizio explains that engagement is a key aspect of a successful lecture, and in turn a fully developed course. Domizio explains "good learning outcomes are achieved by active engagement with the learning process."[10] Therefore, optimizing engagement in the classroom setting can be achieved through professor-student interactions, applying the knowledge being learned, and with in-class demonstrations. Hence, in class demonstrations have also been incorporated throughout this course to improve the student learning outcome.

Understanding the philosophy to a "good lecture", this project's course applies broader learning-teaching techniques over the entirety of the course as well. These techniques are Project-Based Learning, Flipped Classroom, and Simulation. Each of these techniques incorporates the individual lecture philosophy explained by Domizio, but more importantly addresses the CDIO syllabus levels introduced earlier. Table 2 below illustrates each learning technique and the syllabus level/levels it fulfills.

Table 2. Link Between Learning-Teaching Techniques and the CDIO Syllabus Levels[11]

Learning-Teaching Techniques	SL1	SL2	SL3	SL4
Project-Based Learning		●	●	●
Flipped Classroom	●	●	●	
Simulation	●	●	●	●

In detail, project-based learning is targeted in our 5th lecture topic/series, where students take their newly developed knowledge of general acoustics and develop a class lecture on another topic within it. Through a flipped classroom, students are encouraged to review lecture slides before class. This leaves opportunity for professors to assign outside work to survey the student's understanding of what concepts need additional attention and time during the class period. Simulation, the only learning technique that targets all four syllabus levels, allows students to practice the knowledge they learn in lectures, and will be used throughout this course. This is done through MATLAB simulations, that will be presented in tandem with the lecture material.

---

[10] Domizio, 284.

[11] Forcael, Garcés, and Orozco, "Relationship Between Professional Competencies Required by Engineering Students According to ABET and CDIO and Teaching–Learning Techniques," para. 30.

It should be noted that no assessment methods were mentioned in this teaching method. This allows individual professors to have adaptability in their courses, and choose their own respective method of assessment or grading.

2.3 MATLAB Modeling

The software MATLAB was chosen to be the modeling platform used within this course. After exploring different options, we chose MATLAB because of its accessibility to students as a free and WPI supported software, familiarity to the project team, and presumed knowledge students will possess as other WPI courses use this software. The specific modeling environments used when modeling were Simulink and Simscape, both integrated within MATLAB. There are some distinct differences between the Simulink and Simscape environments in MATLAB. According to MATLAB "Simulink is a block diagram environment used to design systems with multidomain models, simulate…, and deploy without writing code."[12] More specifically, Simulink is for Model-Based Design, where complex systems are broken down and sub-models are systematically used through the entire process. This provides versatility for a user to generate, test, and redevelop models early and often.[13] In the scope of this project, Simulink generally appears as electrical components and appears as blue components in the modeling window, like the example in Figure 4 below.

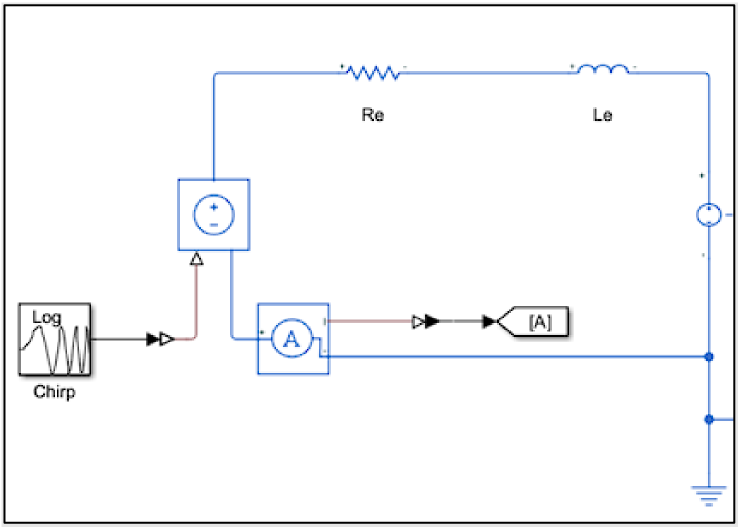


Figure 4: Example of Simulink Modeling for a Portion of a Loudspeaker

Simscape varies slightly from Simulink, however. This is because Simscape is part of the Simulink environment within MATLAB. Specifically, "Simscape enables [a user] to rapidly create models of physical systems within the Simulink environment."[14] The components within

---

[12] MATLAB, "Simulink - Simulation and Model-Based Design," n.d., para. 1, https://www.mathworks.com/products/simulink.html.
[13] MATLAB, para. 2.
[14] MATLAB, "Simscape," n.d., para. 1, https://www.mathworks.com/products/simscape.html.

Simscape can be directly integrated with the block diagrams of Simulink via converters within the modeling environment.[15] Additionally, Simscape enables a user to create a stand-alone mirror model of a Simulink block-model but using the applicable physical systems instead. An example of this is shown in Figure 5 below, where the spring-mass system was created in both the Simulink and Simscape domains.

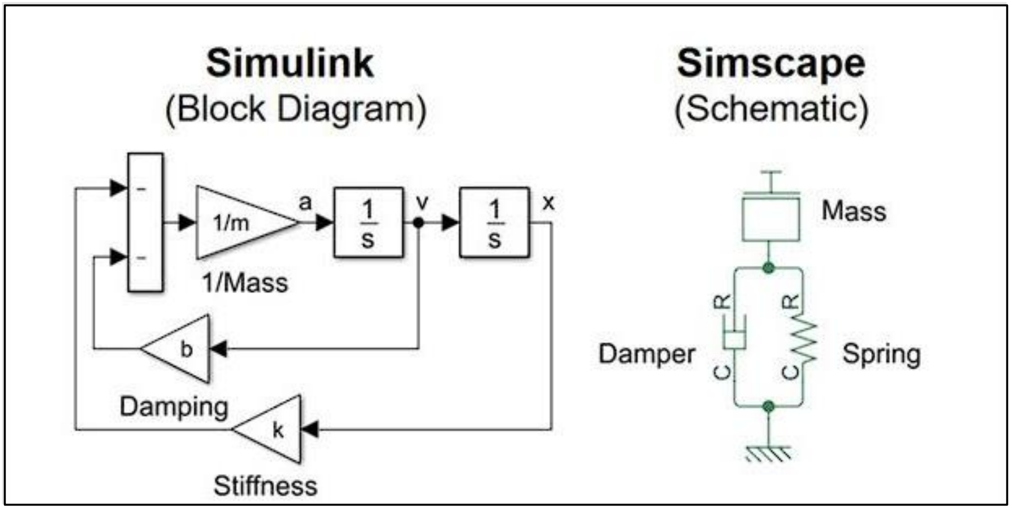


Figure 5: Spring-Mass System Example for Simscape and Simulink[16]

Through the application of MATLAB Modeling, students can create and simulate acoustic phenomena within models they create. In turn, students become familiar with an accessible and powerful free to use software. Furthermore, modeling applications within a course takes the students out of remote memorization of information and provides an opportunity for students to apply concepts learned. In turn, depth is fostered within a course structure. Finally, MATLAB modeling with Simulink and Simscape is simple to use through drag and drop components from a library of different applications. The user interface of the software provides a practical and easy to learn experience, making it worthwhile for users of all technical backgrounds.

---

[15] MATLAB, para. 1.
[16] MATLAB, para. 3.

3.0 Methods

Within this project's timeline, we went through 3 different design method iteration processes to complete it. Each design method iteration varied in the work being completed, how each team member was used, and project productivity.

3.1 Design Method - Iteration One

Design Method Iteration One began prior to the first advisor meeting of the project. Within this iteration, the student team members created a plan that we believed would optimize our working hours and help us deliver the best material we could. Specifically, Design Method Iteration One had 3 phases: Research, Compile and Review, and Finalize. Each of the phases is expanded below.

3.1.1 Research Phase

The student team initially believed in a plan that would allow for the first quarter of the project to be dedicated primarily to researching the topic of acoustics. The belief was that through a quarter of research, the team could deliver the researched material in a concrete and easy to understand fashion. Part of the reason to establish this phase of the project was due to most team members being new to the topic of acoustics. By the end of the first quarter, we believed that all team members would have obtained a working knowledge of the topics we wished to cover in detail within this course.

3.1.2 Compile and Review Phase

The Compile and Review Phase was projected to take up the middle 1/3 of the project. Within this phase, team members would have worked together to turn the researched material into digestible lecture slides to be used in class. We presumed that the information researched within the first quarter of the project would not have been the same among all team members. Therefore, the team planned to have lecture review meetings both alone and with the project advisor. During these meetings, lectures would be reviewed from beginning to end. That way any gaps and/or misleading/confusing information could be confronted, discussed, and reviewed. Ultimately, the compiling phase would have been the most labor-intensive portion of the project since team members would have been editing the current lecture in review, while drafting the future lectures simultaneously.

3.1.3 Finalize Phase

In this final phase of Design Method Iteration One, the team would have been looking over all material, constructing all complete deliverables, and with extra time, creating demonstrations to assist the lecture material. The timeline of this phase would have begun after the Compile and Review Phase and ended with project submission. The belief was that all lectures would be given a polishing period to make sure the desired flow of the course was achieved. With enough dedication and work on the front end, the finalization phase would not have weighed much on the team, we believed.

3.2 Design Method - Iteration Two

Following our first advisor meeting, the team was directed to explore an alternative design method iteration process. The new iteration idea split the project timeline into two phases: Hardware Development and Hardware Review.

3.2.1 Hardware Development

Prior to any research of material, we explored how other schools delivered their version of “Introduction to Acoustics”. We eventually came together and, as a team, combed through the information gathered from Massachusetts Institute of Technology (MIT), Georgia Tech, and Penn. State. Specifically, we discovered some textbooks used, the course level at the respective schools, and the material covered via syllabi. The conclusion from this exercise was that introductory acoustics was often a higher-level course in undergraduate studies, or a beginning level graduate course. Once we had an idea of what level information was delivered at, we decided to structure the course to meet a medium to high-level undergraduate course.

After addressing the course level, each team member began research into textbooks that could be used as the foundation for the material in this course. These textbooks came from online searching and reviewal, a folder of sources provided from the project advisor, and textbooks/sources team members had previously seen or used. Once a team member was satisfied with their sources, it was the task of each member to make a running list of all topics covered in their sources. Upon completion of each member’s topic list, we came together and devised a master topic list that encompassed similarities between books and topics that we wished to cover within this course. This process served as the ideation phase of our project. The master topic list was then broken down into the timeline the course would be taught in. This led to our proposed 7-week course plan, found below. From here the in-depth research and lecture slide (hardware) creation began.

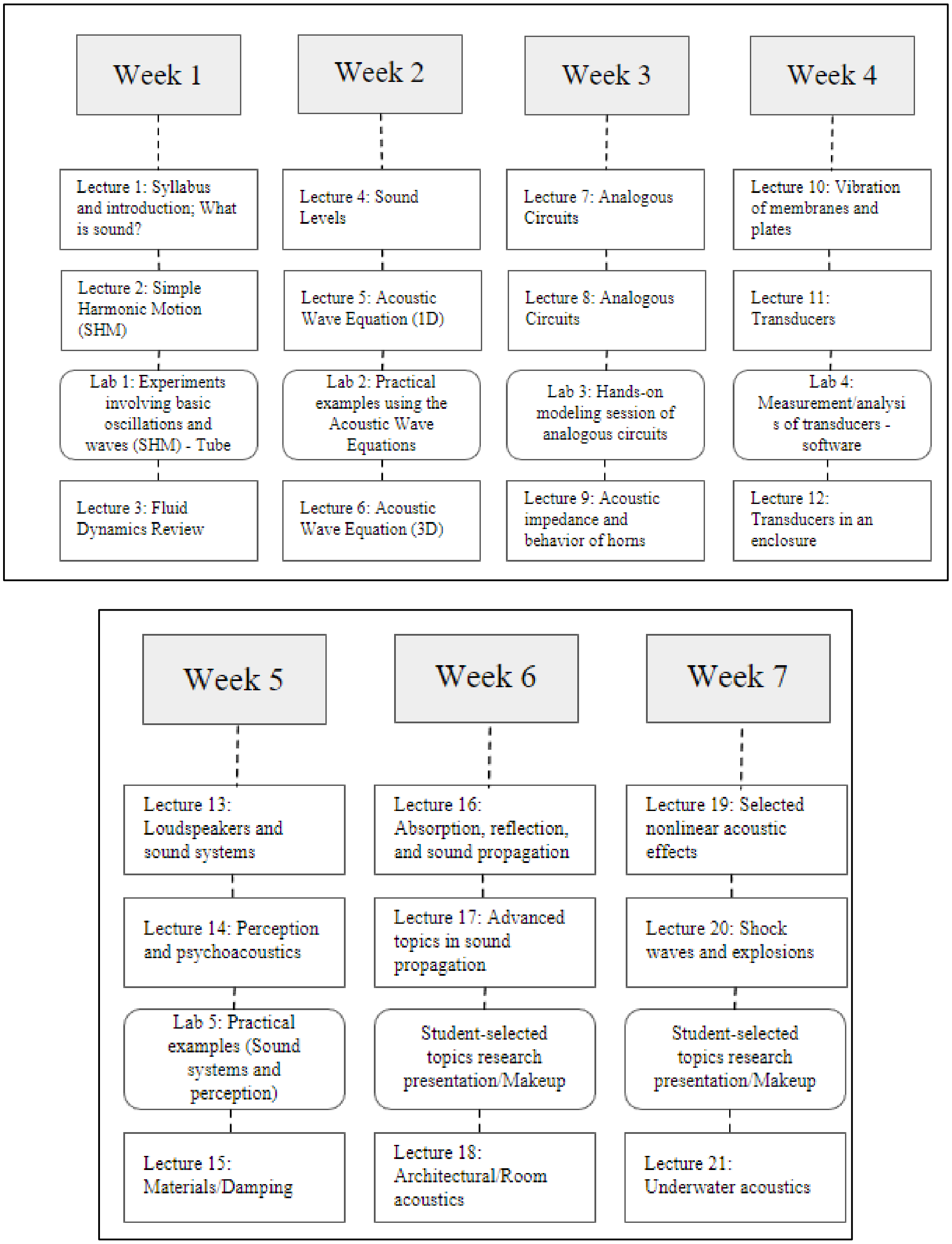


Figure 6: Design Iteration Two Ideation 7-Week Plan

Within Design Method Iteration Two, the idea was to research and draft lecture slides simultaneously. In the earliest lecture creation, all team members created their own version of the lecture topic for that week. Then, we collaborated and made a singular presentation that encompassed the work of all team members. Soon after, with the guidance of our project advisor, we realized that productivity was hindered by all team members researching the same topics.

Therefore, we devised a plan that divided the team members up into different lecture topics and live demonstration work. In this way, while others were researching topics and constructing live demonstrations, other team members would be working on the lecture in review and finalizing the work done there. Soon after this change of pace, we realized the amount of time a lecture in review physically took away from the week and meetings. This left many team members in limbo once they completed their research and initial lecture slide drafting. Furthermore, through advisory meetings and input we began to deviate heavily from the latter portion of the 7-week course plan, Figure 6, initially created and approved at the beginning of the project. Ultimately, by the end of the first half of the project, we had created the desired slide decks, and were presumed to be ready for review in the second half.

3.2.2 Hardware Review

Beginning in the spring semester, advisory meetings were spent reviewing all created material for the project. This meant beginning at Lecture 1, and going slide by slide to make sure information was understandable and accurate. Alongside this, the created live demonstrations were reviewed, and certain team members were explicitly dedicated to the creation and functioning of the live demonstrations. However, soon into the second half of the project, the student team members hit a major hurdle. Through meetings and personal work on the project, the student team members discovered that the course being created in this project had no linear direction and instead appeared to be a bundle of facts about acoustics in no order. It became apparent that a change was needed before any more progress was to be made.

3.3 Design Method - Iteration Three

Within this iteration, no structured timeline was set for the team to follow. Instead, we addressed the aspects of the course that needed to be improved upon and assigned tasks to individuals based on their knowledge and skill set. The areas of improvement were creating lecture topic flow, delivering applicable/useful information, and creating a modeling portion of the course created within this project. This led to the group being split into two parts, the lecture creators, and the modeling creators. Each group had their own specific tasks that contributed to the project.

3.3.1 Lecture Creators

Within this group, the primary task was to take the already developed lecture material and create a new course flow. In some cases, this led to researched and created material being deleted, restructured, simplified, or in the best case left the same. It became apparent to the lecture creators that the bulk of the work was on the latter portion of the material created. In this respect, during advisory meetings, the overall change in course direction was addressed, but material that was reviewed and approved was not addressed again. Furthermore, there were cases where lecture creators added information that was previously left out of the lectures created in

Design Method Iteration Two. Once implemented, this information was reviewed and approved during advisory meetings.

3.3.2 Modeling Creators

Modeling was a portion of this course that was initially introduced in an arbitrary fashion. Therefore, within Design Method Iteration Three, members of the team were tasked with taking the circuitry information delivered in the lecture slides and using software to bring the information to life. For this, the model creators chose software that was easy to use and access. From there, the modeling creators began researching how to represent the phenomenon the lecture creators believed should be modeled. Once an idea was found, it was then the task of the model creators to replicate the information they found within their own model. If able to be completed, the model creators then made modeling guides in the form of PowerPoints that would be supplied for students and professors to follow.

Within each of the Design Iterations, the team continued its path towards a final course for this project. In going through the different Design Iterations, we learned many valuable lessons and were better able to structure the final course layout of this project.

4.0 Final Course Layout and Design

The final course layout reflects the shift to Design Method Iteration Three described above. Generally, the final course layout mimics the team's initial plan of a 7-week course with 3 lectures and 1 lab period a week. The major difference is the division of the course into lecture topic blocks, in contrast to individual lecture topics. Resorting to this method allows a professor to have fluidity in their teaching design. The accessibility to speed up or slow down lecture material delivery can be vital to students' ability to retain information.

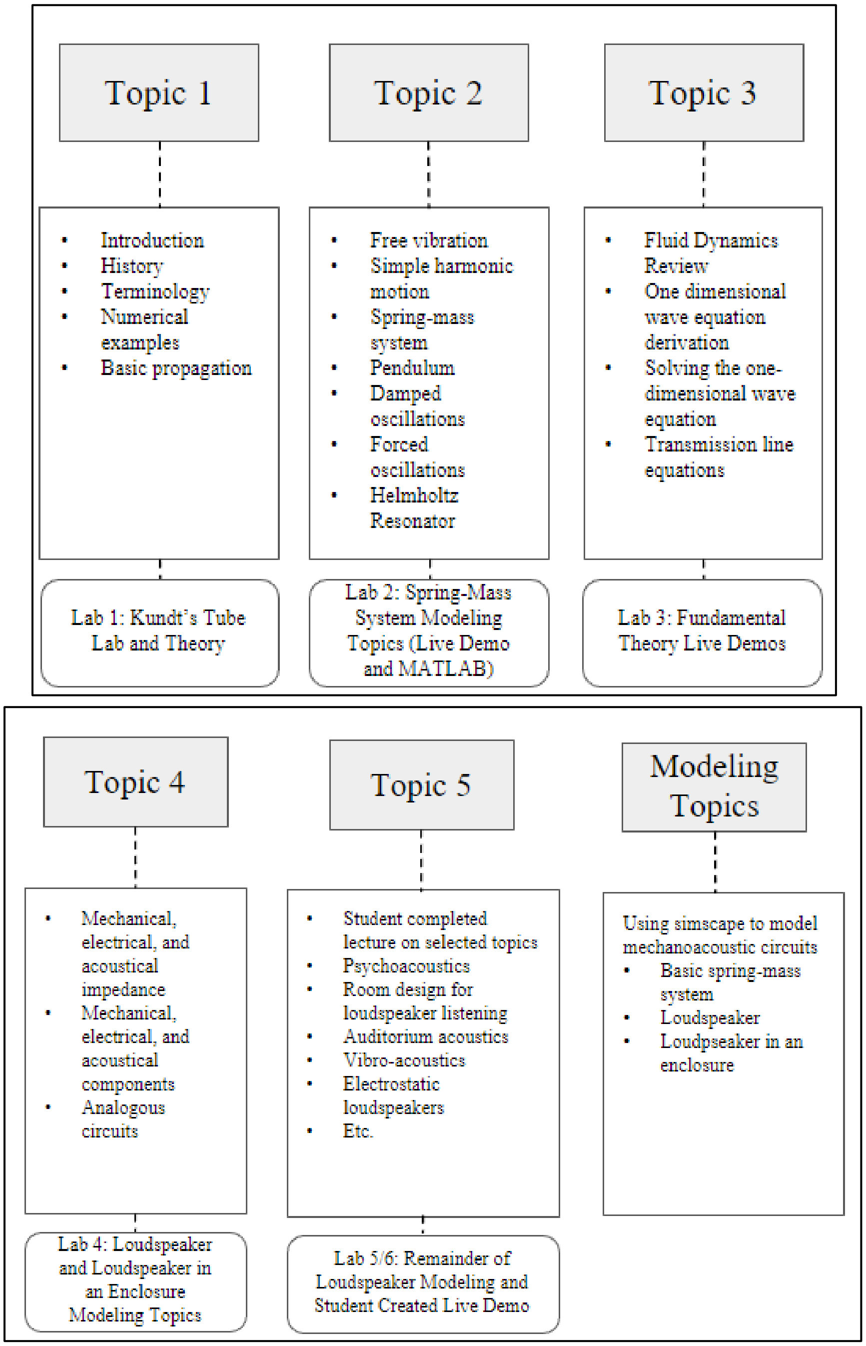
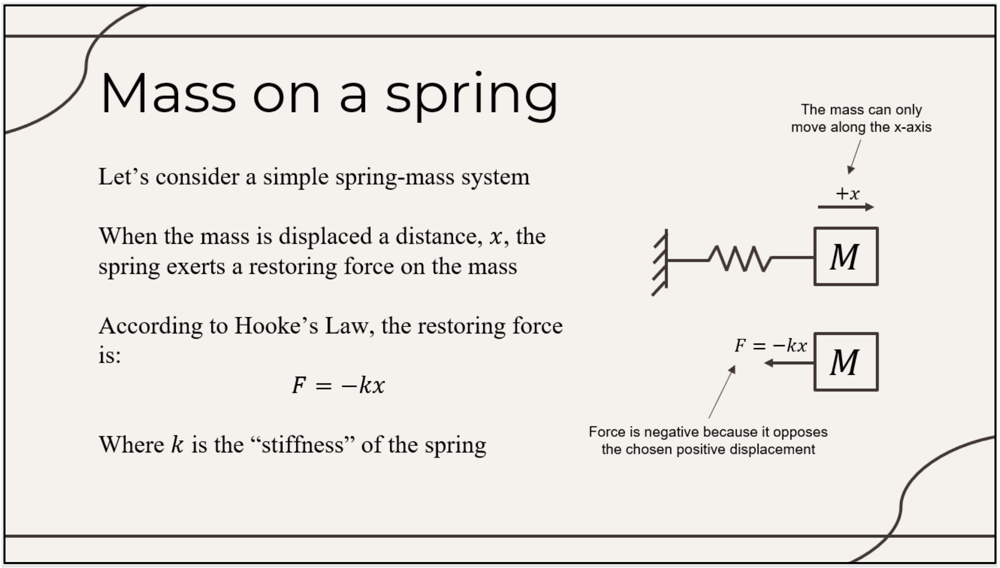


Figure 7: Final Course Layout

As portrayed in the figure above, the final course layout contains 5 lecture topic blocks, one of which is a student research topic. In creating the lecture topics blocks, the team built upon the finished material from the hardware development phase of Design Method Iteration Two. Within Design Method Iteration Three, the lecture creators collaborated to distinguish and organize the material into the 5 topics blocks seen above. Through their understanding of the material, the final course lecture topic layout maintains a flow that the course was lacking previously. Directing back to Figure 7, the right side of the figure contains the Modeling Topics Block of this course. It is separated from the other topics because the modeling topics are recommended to be introduced once the topic being modeled has been discussed within the lecture material. The discretion of when to implement the modeling exercises is therefore directed to the instructor presently teaching the course. In addition, the assessment/examination portion of this course is directed toward instructor preference and is not included as a solidified element to the final course created in this project.

4.1 Course Design – Conceptualization and Mathematics

The completed lecture slide decks contain all material this project team created to be taught within the course structure. The team wanted the lecture slides to illustrate our teaching method of conceptualization, math implementation, and then modeling application as described in Section 2.2. The first major example of this kind of teaching method is introduced in lecture topic 2, specifically with the spring-mass system.



Mass on a spring

Let’s consider a simple spring-mass system

When the mass is displaced a distance, x, the spring exerts a restoring force on the mass

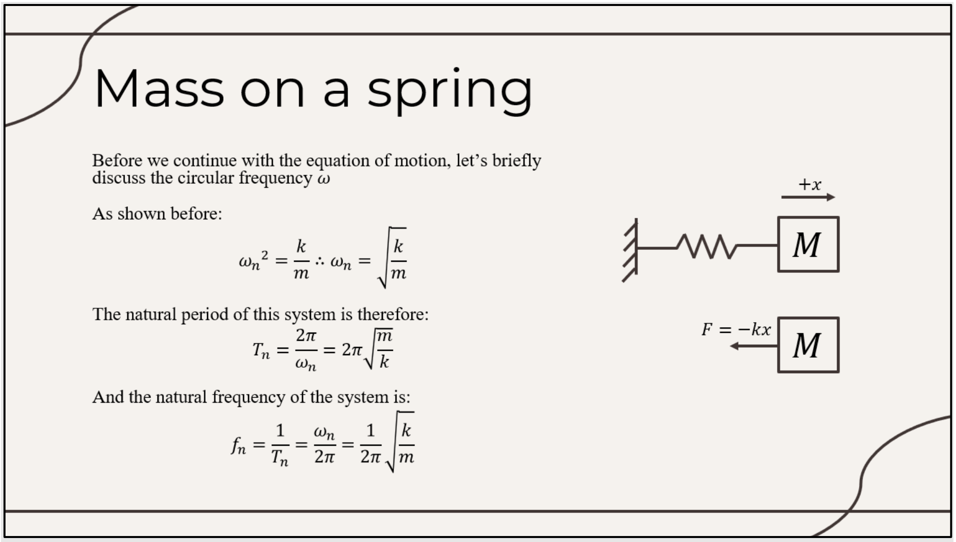
According to Hooke’s Law, the restoring force is:

$$F = -kx$$

Where k is the “stiffness” of the spring

Figure 8: Teaching Method Lecture Slide Illustration

Figure 8 alone illustrates the first two portions of our teaching method. Not withholding the fact that this course is designed for 2nd to 3rd year students, the lecture slide example above conceptually illustrates the spring-mass system through the Free-Body Diagram (FBD) and complete system diagram seen to the right of the slide. In addition, the spring-mass system phenomenon is conceptually understood through the words on the page, and animation videos (not pictured within this report) on the succeeding slide within the deck. Figure 8 upholds the second portion of our teaching method through the addition of mathematics to the conceptual information. In the figure above, the phenomenon is mathematically explained through the application of Hooke's Law, which is written out and illustrated within the slide. Figure 9 below mirrors the introduction of mathematics to conceptual topics more vividly.



Mass on a spring

Before we continue with the equation of motion, let's briefly discuss the circular frequency ω

As shown before:

$$\omega_n{}^2 = \frac{k}{m} \therefore \omega_n = \sqrt{\frac{k}{m}}$$

The natural period of this system is therefore:

$$T_n = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{m}{k}}$$

And the natural frequency of the system is:

$$f_n = \frac{1}{T_n} = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

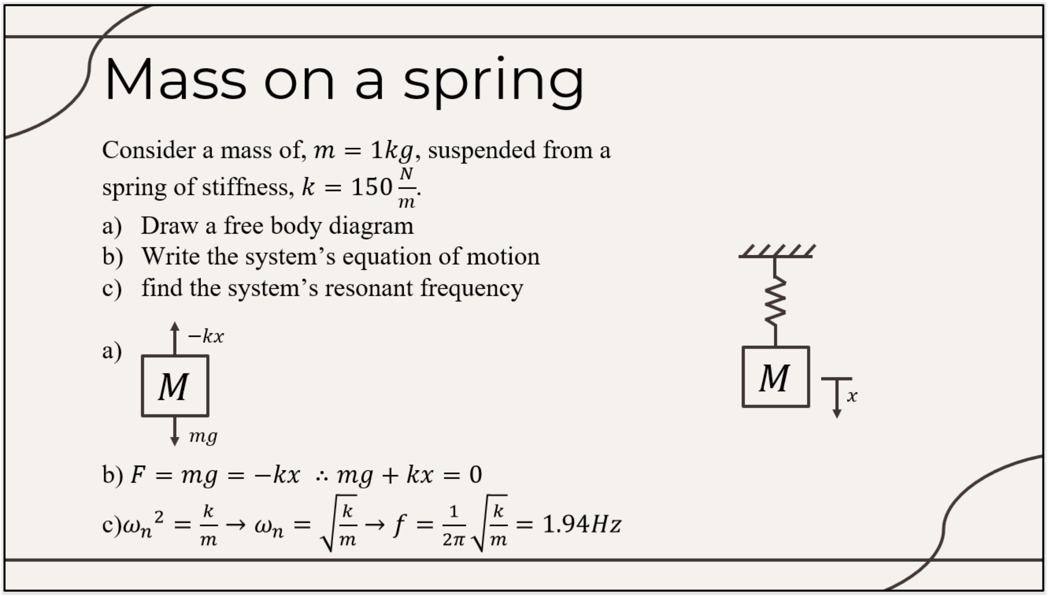
Figure 9: Mathematical Introduction to Conceptual Topics

The topics of natural period and frequency are respectively discussed within the introductory lecture for this course, lecture topic 1. Therefore, on the slide presented in Figure 9, the students fundamentally see the mathematics governing the conceptual principles they have already been introduced to. Mathematical concepts are built from the ground up within a lecture deck. That way the students engaging with the course can work through a slide deck and understand where the variables seen in Figure 9 come from, and what the individual variables mean. It is our belief that through understanding conceptually, and then applying mathematics, students will better grasp the governing principles and phenomena presented within the course.

It is important to add that student engagement with the lecture material was a goal we tried to achieve throughout all lecture topics. Many of the topics within the acoustic textbooks used in the formation of this course contained complex verbiage and descriptions of the topics we sought to cover. In turn, it was the goal of this project team to value student engagement by turning complex descriptions into digestible material mainly using pictures and animations. In this way, the team held tightly to the phrase that "a picture is worth a thousand words", attributed to Fred R. Barnard.

4.2 Course Design – Application of Material

Through being students, the team understood that the final course of this project needed a level of material application. Without it, we felt that students would read over information with no intention of retaining the material. One method to achieve this goal comes from adding questions into the lectures for students to work through.



Mass on a spring

Consider a mass of, $m = 1kg$, suspended from a spring of stiffness, $k = 150\,\frac{N}{m}$.

a) Draw a free body diagram
b) Write the system's equation of motion
c) find the system's resonant frequency

a)

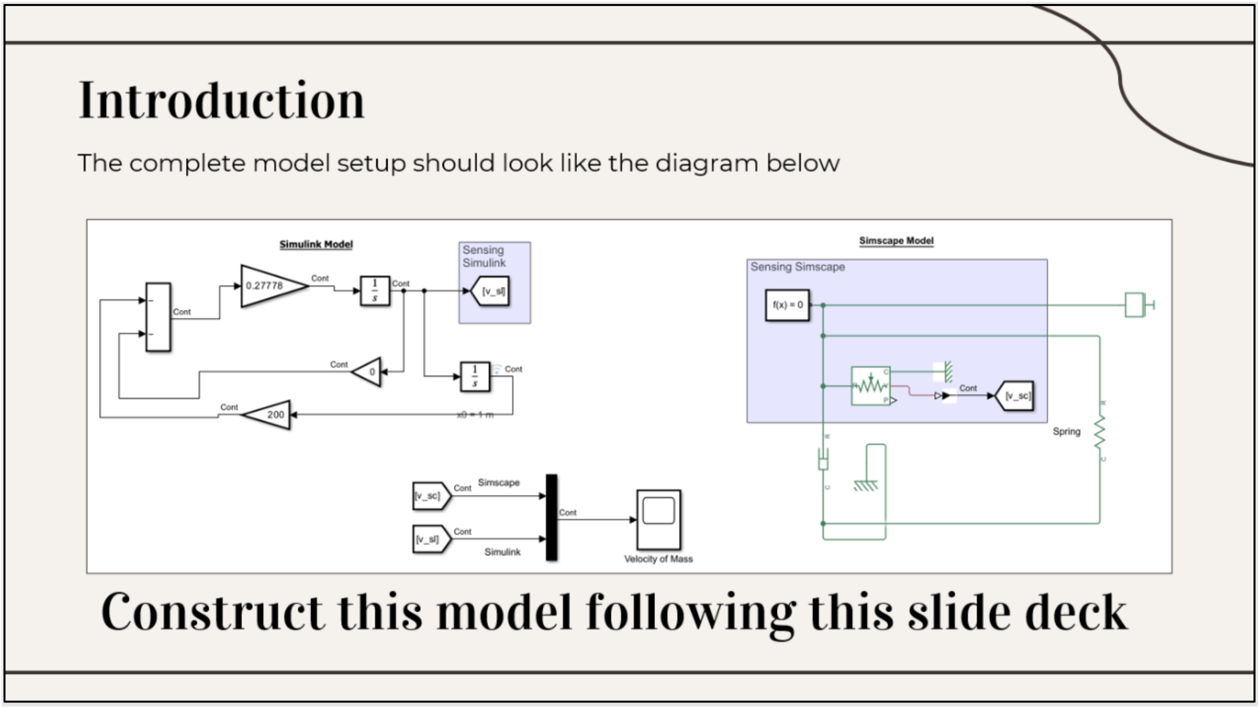
b) $F = mg = -kx \;\therefore mg + kx = 0$

c)$\omega_n{}^2 = \frac{k}{m} \rightarrow \omega_n = \sqrt{\frac{k}{m}} \rightarrow f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1.94Hz$

Figure 10: Example of an Application Lecture Slide

From Figure 10 above, students are directly involved in applying the concepts presented previously in the slide deck on the spring-mass system. Noting that the answers are shown in the figure, these would be hidden from view while presenting, and brought into view through animations. Within this example, a student finds themselves applying their knowledge of the conceptual FBD introduced earlier, the mathematical equation governing the motion of the spring-mass system, and the use of equations to solve for a characteristic of the spring-mass system. While the work presented is simple in and of itself, these kinds of exercises aid in lecture flow and information retention.

In a more complex example, all modeling done within this course is intended to be used for applying material learned in lecture. As stated previously, Simscape and Simulink additions to the MATLAB software govern where the modeling application was constructed. Each modeling topic presented on the right side of Figure 7, has its own step-by-step lecture guide to aid the students. For example, sticking with the Spring-Mass System concept, Figure 11 below illustrates an opening slide to the modeling deck.



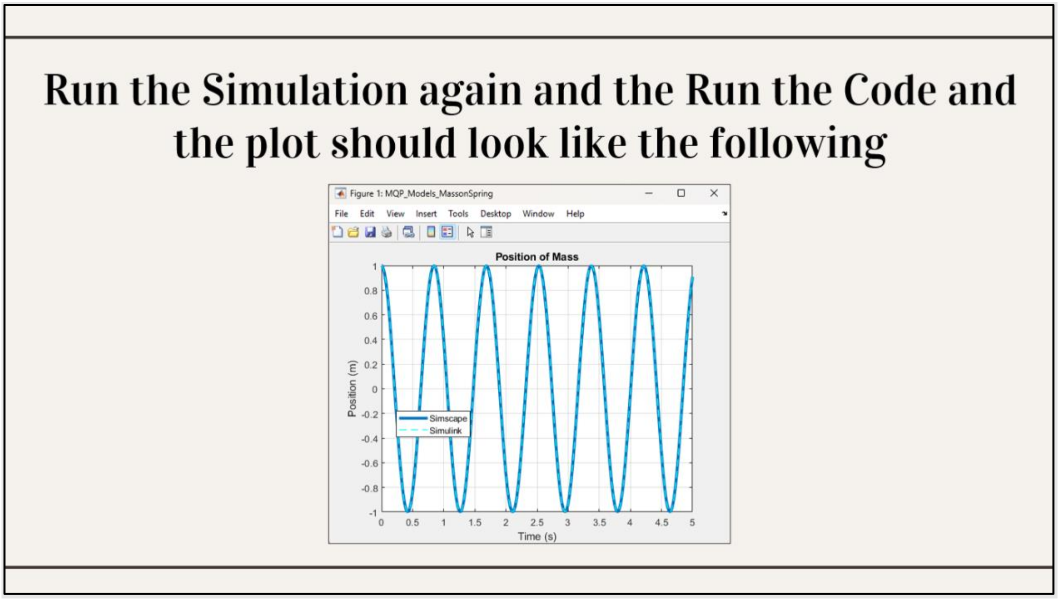
Introduction

The complete model setup should look like the diagram below

Construct this model following this slide deck

Figure 11: Spring-Mass System Introductory Slide

From the slide presented above, students can conceptually see what the entire model they will be creating will look like prior to beginning the exercise. The succeeding slides then break down each block of the diagram into the name of the block, where the block is located within the modeling library, and what the input for the respective block is. Once the model is created and the simulation is run, students can then see the application of the topics they have learned through the simulation output. For example, in an undamped spring-mass system, the position of the mass over time should be a perfect sinusoidal graph; students will have learned this concept prior to beginning the modeling exercise. In this fashion, through constructing the model prescribed in the modeling lecture, and running the simulation, a student could produce the following output.



Run the Simulation again and the Run the Code and the plot should look like the following

Figure 12: Application of Material Through Spring-Mass Modeling

At this point, the student will have conceptually learned about all parts of the Spring-Mass System, dealt with and applied the mathematics behind the system, and then observed the application of the Spring-Mass System through a modeling exercise. The application exercises were also constructed in such a way that the more complex phenomena can also be explored. For example, once the student has a working spring-mass system, they can then apply a damper to the system and observe what they believe should conceptually happen to the system. Therefore, the student is not hindered in their curiosity of the application of the topic presented and can observe what the lecture slides conceptually and mathematically attempt to portray.

In the end, the final course layout and design provides a simple and easy to use method for instructors to follow. While there is guidance on the direction information should flow in, the final course layout provides flexibility and personalization for an instructor. Overall, we believe the final course layout and design upholds the teaching methods and principles presented earlier and will foster student engagement from the beginning to the end.

4.3 Completed Lecture Topic Breakdown

The final course layout is based on 5 main lecture topics, with each lecture topic including several related subtopics. The course layout is designed to be delivered over 7 weeks, with 3 lectures and 1 lab session each week. The content of these topics are as follows:

1. Lecture Topic 1 – Introduction to Acoustics
    a. Introduction
    b. History
    c. Terminology
    d. Numerical examples

    e. Basic propagation
2. Lecture Topic 2 – Oscillations and Waves
    a. Free vibration
    b. Simple harmonic motion
    c. Spring-mass system
    d. Pendulum
    e. Damped oscillations
    f. Forced oscillations
    g. Helmholtz Resonator
3. Lecture Topic 3 – The Acoustic Wave Equation
    a. Fluid Dynamics Review
    b. One dimensional wave equation derivation
    c. Solving the one-dimensional wave equation
    d. Transmission line equations
4. Lecture Topic 4 – Analogous Circuits
    a. Mechanical, electrical, and acoustical impedance
    b. Mechanical, electrical, and acoustical components
    c. Analogous circuits
5. Lecture Topic 5 – Student Selected Topics in Acoustics
    a. Psychoacoustics
    b. Room design for loudspeaker listening
    c. Auditorium acoustics
    d. Vibro-acoustics
    e. Electrostatic loudspeakers
    f. Etc.
6. Modeling Topics - using simscape to model Mechanoacoustic circuits
    a. Basic spring-mass system
    b. Loudspeaker
    c. Loudspeaker in an enclosure

Lecture topics one through four are designed to be delivered in a standard lecture style, with modeling topics being introduced to provide application of course material. These modeling topics, along with supplementary live demonstrations, are meant to be introduced during the lab sessions. The last topic, lecture topic 5, is designed to be a student designed topic. In this section of the course, students will form small groups which are meant to research a related topic to something they’ve learned in class. This will allow students to expand their understanding of previous course material and give them the opportunity to learn a topic in acoustics they want to explore.

The choice of lecture topic structure was made to provide fluidity in course delivery. In the previous iteration of the course structure, the lectures were based on subtopics meant to be delivered at a specific time during the 7-week period (Day 5 being X subtopic for example). Removing when the subtopics should be delivered and instead creating a generalized timing for larger topic groups allows the instructor to move the course at the pace of the class (week 1 should cover X, Y, and Z subtopics for example). This way if the class is struggling with a certain topic, more time can be allotted to that topic.

5.0 Improvements for Future Iterations

In the creation of this course, the team understood that not all material related to the introductory topic of acoustics would be possible to cover. Therefore, it is our hope that in future iterations of this project, students will build on this framework with new and more in-depth information. Specifically, there are subtopics to the phenomena presented in this course that a future project team could add to their version of the course presented in this project. In this way, the course would gain another layer of depth within the topic of acoustics, providing students with a more comprehensive introduction to acoustics. Additionally, the team proposes expanding the lab/live demonstration portion of our final course within future iterations. We believe that the labs and live demonstrations make learning inherently more fun and provide a direct application of topics learned in class for the students. Building from the current course, a future team may seek to add more complex examples or simulations within the modeling software. Furthermore, there are acoustic topics not presently demonstrated in a live model within this course. Introducing more live demonstrations of acoustic phenomena would better help students to visualize the versatility the topic of acoustics has. This could be completed in tandem with the addition of the deeper fundamental acoustic topics not covered within this course, therefore maintaining the idea of presenting a topic conceptually before application of the topic is completed.

In conjunction with improving the depth of the course, a future iteration could pertain to the mathematic concepts not deeply covered within this course. Mathematics is integrated into the final course design of this project to explain and/or illustrate the more fundamental topics related to acoustics. However, the team discovered early on and began working through more complex math not included in our project's final scope. One such example of math improvements could be the use of complex variables related to the governing properties of waves and acoustic characteristics. Most textbooks used in this course's formation introduced complex variables to simplify the mathematical concepts and application. From there, the textbooks were better able to present and work through mathematical concepts and examples. In a future project, through introducing complex math from the ground up like the textbooks used, a student team may discover an additional mathematical layer this course is not currently supporting.

These improvements for future iterations are a starting point for a future team. It is our hope that a future team will be inspired by the foundation we have built and will use their own creativity and ingenuity to improve what is presented within this project.

6.0 Broader Impacts

As with all project work, the creation of this course has a broader impact than just this project. Each topic of broader impact is expanded on below. It is important to note, however, that broader impacts related to Societal and Global, Environmental, and Economic Impact were not considered in the formation of this project.

6.1 Engineering Ethics/Codes and Standards

Ethically, the information within the final course design presented in this project were not fundamental ideas created by the team or the advisor. All information was taken from textbooks and online academic sources. Furthermore, the modeling and live demonstrations presented within this course were created from other examples found online and through video searches. Because of this, the team does not claim that any academic information from the course is of our own discovery. Instead, the team formulated the researched information into our own structural design, upholding the codes and standards to give credit where credit is due for the information presented. In addition, the final course developed within this project reflects the level of understanding the project team had throughout the course development. With this, certain topics introduced within research readings were not included in the final course as the team did not have a grasp on the material. Therefore, the course reflects the team members’ competence within the topic of interest.

Creating this course ethically also required that the team members looked beyond ourselves. Throughout this project, we strived to create course material that would benefit the greater student body. In doing so, we believed in eliminating busy work within the course. In this way, we designed the course with the mindset of how a student 3 years down the road would perceive the course when taking it. Furthermore, the live demonstrations and modeling within this course were created to maintain the safety of both instructors and students. Because of this, we believe that live demonstrations and modeling can be completed by all students and instructors within the course. Furthermore, there is no inherent risk when seeking to replicate the live demonstrations or modeling exercises.

7.0 Conclusion

Throughout the development of the final course presented in this project, we learned many valuable lessons on curriculum development, and a bountiful amount of information pertaining to the topic of acoustics. Curriculum development is a lengthy process, and we believe the course developed within this project is a solid foundation. By no means is the course developed within this project incomplete, however. It is instead our hope that future students will see the work completed here and take it further than we were capable of.

In the end, this project was rooted in teaching principles and lecture design theory. The project brought material down to a digestible level that should be accessible for all student knowledge levels within the course. Furthermore, the course materials created help to eliminate students from being discouraged to learn if this course would be a stretch for them. This was done by breaking material down to a step-by-step basis, for both lecture slides and modeling/live demonstration exercises. Standing alone, this project can provide a lot of information and launch a student into a desire to learn more about acoustics.

8.0 Works Cited

“Acoustics | Definition, Physics, & Facts | Britannica,” n.d. https://www.britannica.com/science/acoustics.

Domizio, Paola. “Giving a Good Lecture.” *Diagnostic Histopathology* 14, no. 6 (June 1, 2008): 284–88. https://doi.org/10.1016/j.mpdhp.2008.04.004.

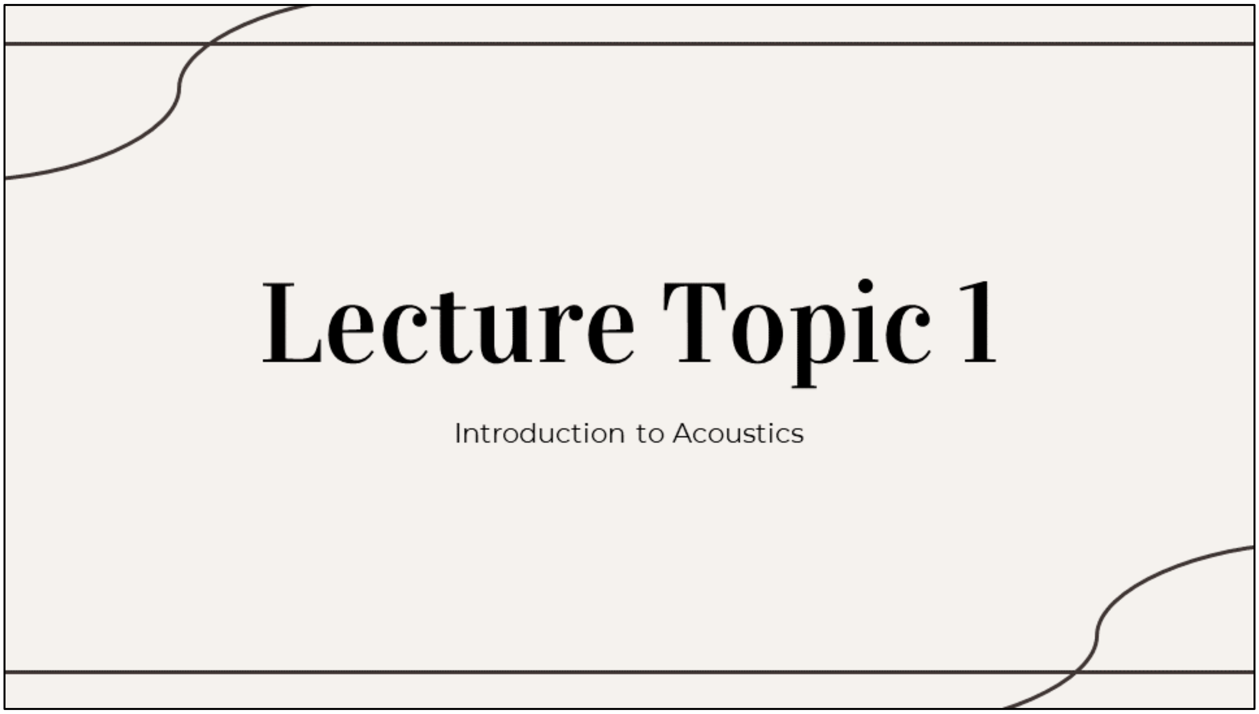
Forcael, Eric, Gonzalo Garcés, and Francisco Orozco. “Relationship Between Professional Competencies Required by Engineering Students According to ABET and CDIO and Teaching–Learning Techniques.” *IEEE Transactions on Education* 65, no. 1 (February 2022): 46–55. https://doi.org/10.1109/TE.2021.3086766.

MATLAB. “Simscape,” n.d. https://www.mathworks.com/products/simscape.html.

MATLAB. “Simulink - Simulation and Model-Based Design,” n.d. https://www.mathworks.com/products/simulink.html.

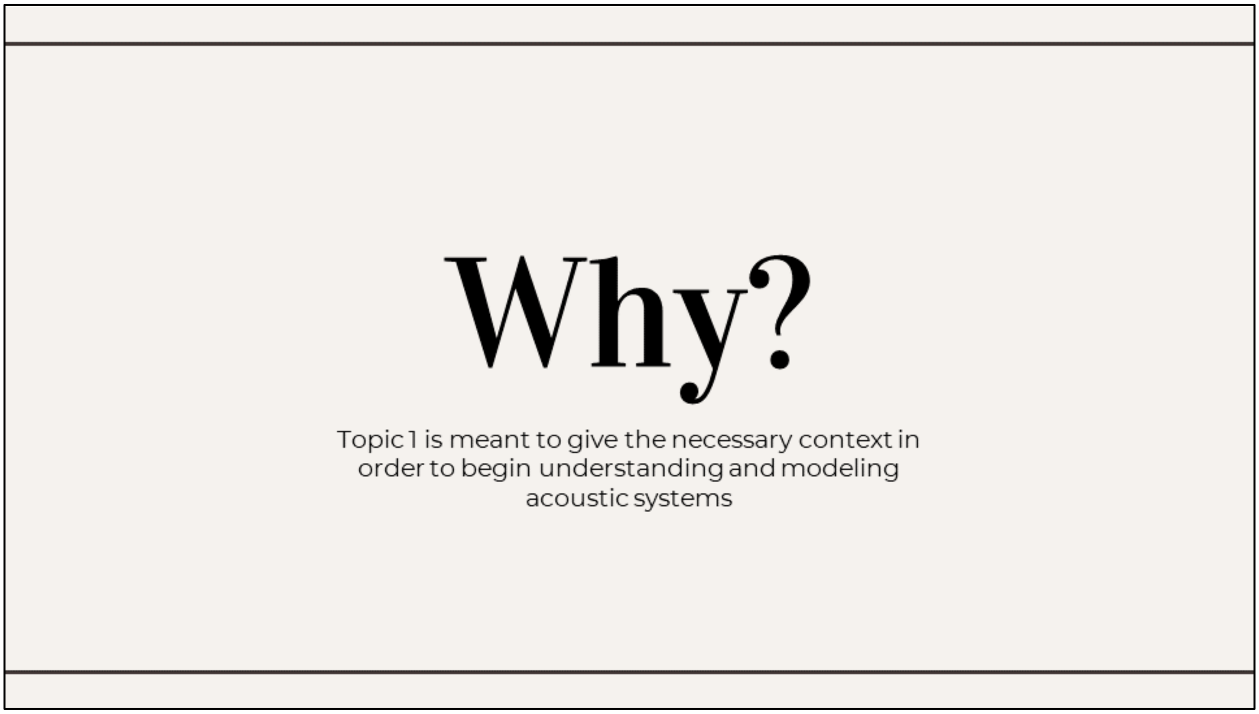
“Sound - Overtones, Frequency, Wavelength | Britannica,” n.d. https://www.britannica.com/science/sound-physics/Overtones.

Appendix A – Lecture Topic 1 Slides



Lecture Topic 1

Introduction to Acoustics



Why?

Topic 1 is meant to give the necessary context in order to begin understanding and modeling acoustic systems

What are we learning?

01

Introduction

What is acoustics?

02

Some history

A little background for context

03

Terminology

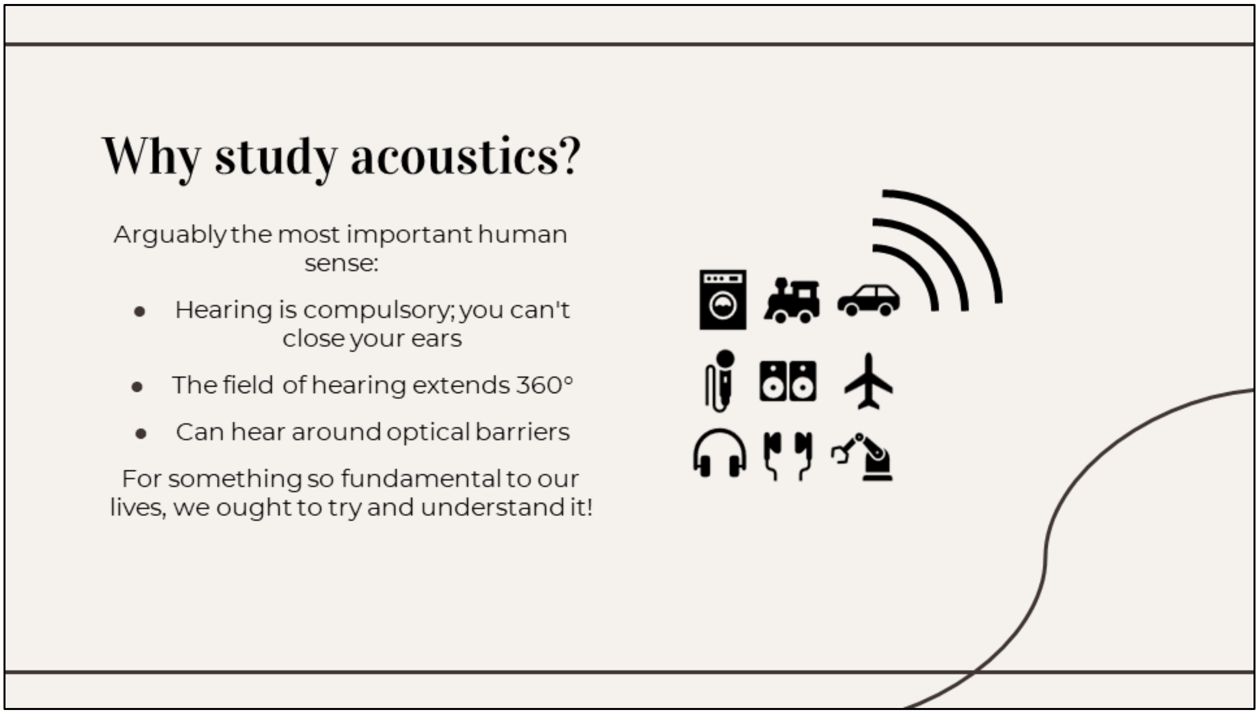
Important quantities and how to use them

04

Propagation

Conceptual overview of the propagation of sound

> "It is audition and not vision that is the most relevant social sense of human beings. The auditory system is their most prominent communication organ, particularly in speech communication. Take as proof that it is much easier to educate blind people than deaf ones."
>
> **–Ning Xiang and Jens Blaubert**

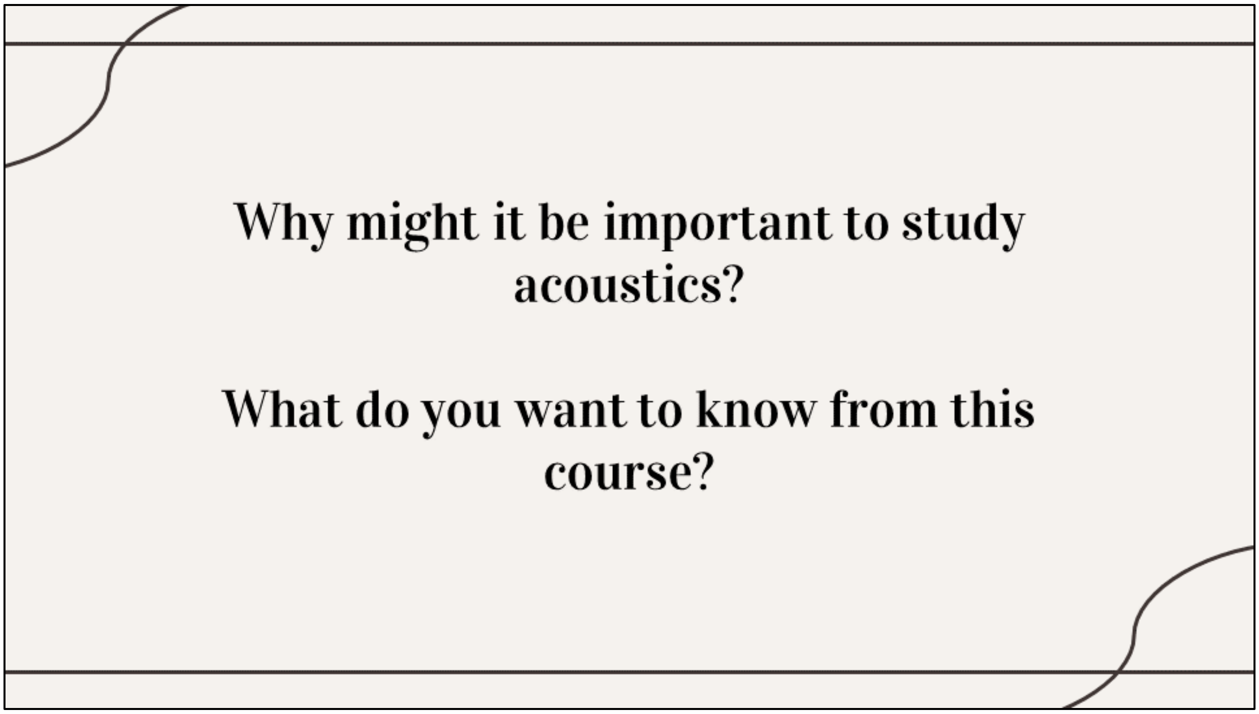


Why study acoustics?

Arguably the most important human sense:

- Hearing is compulsory; you can't close your ears
- The field of hearing extends 360°
- Can hear around optical barriers

For something so fundamental to our lives, we ought to try and understand it!



Why might it be important to study acoustics?

What do you want to know from this course?

01

Introduction

What is acoustics?

Three Important Terms

Sound

Mechanic vibration caused by a disturbance in an elastic medium

Auditory Event

An event that exists as heard, only becoming actual after the act of hearing

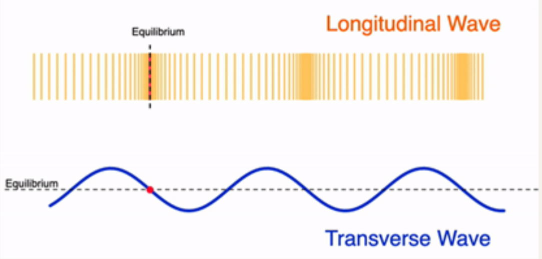
Acoustics

The study and science of sound and its accompanying auditory events

Acoustics

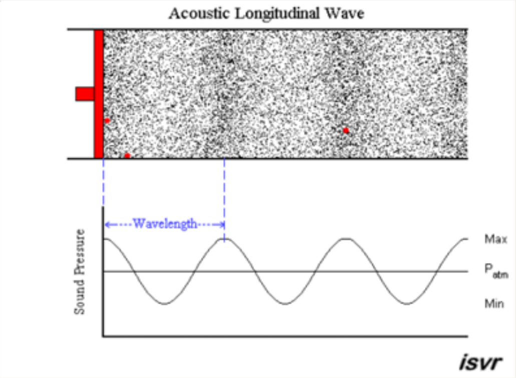
Formally, acoustics is the study of the **generation**, **transmission**, and **reception** of energy as vibrational waves in matter

When molecules of a fluid or solid are displaced, an **internal elastic restoring force** arises.



Essential concept which governs all vibrating systems

Sound

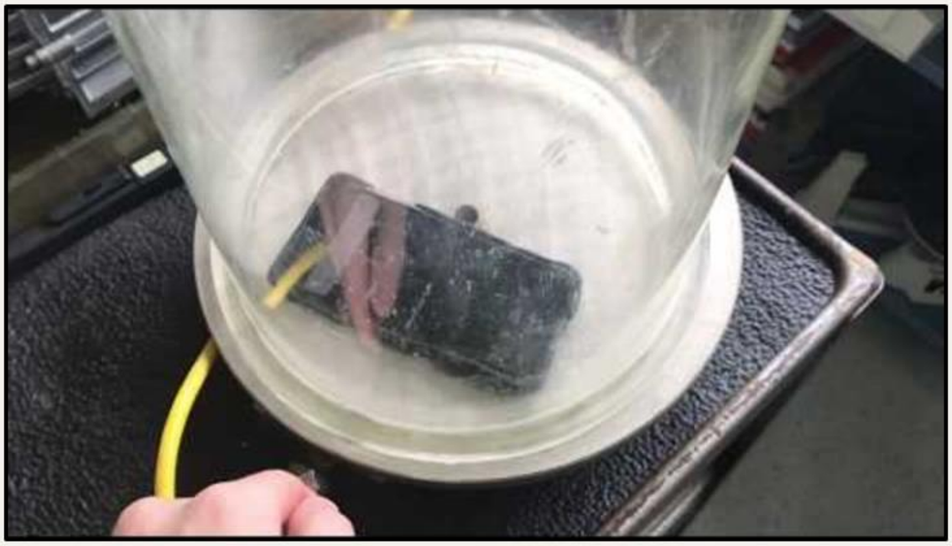


Sound Waves

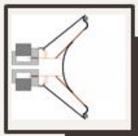
- Needs an elastic medium to propagate
- Particles oscillate about an equilibrium position
- What happens when there's no medium?

Propagation

- Sound is a pressure wave
- Sound propagates in all directions
- We will describe more properties in depth in further sections



Common Sources of Sound

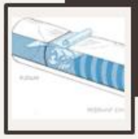


Vibration

Vibrating Bodies –
focus of this course

Explosion

Shockwaves and
nonlinear acoustics



Air Stream

Air across a bottle top
or whistle

Thermal

pressure variation
from temperature
change

02

A Little History

Where did acoustics
come from?

History of Acoustics

500 BC

Pythagoras find
the relationship
between string
length and pitch

1643

Toricelli demonstrated
that sound
cannot propagate in
a vacuum

1842

Chladni
Introduced acoustics as
a separate branch of physics

1896

Acoustics had now matured
and Rayleigh writes a
full text on acoustics

1910

von Lieben and de Forest
invent the vacuum tube
triode allowing for
amplification of weak
signals

Notable Inventions

Telephone
1867

Radio
1920

PA Loudspeakers
1920s

Consumer Audio
1930s

DSP
1970s

Smartphones
1990s

So many applications!

Acoustics

Mechanical Engineering
Vibroacoustics, Noise Control, Architectural, Seismology, Underwater Acoustics

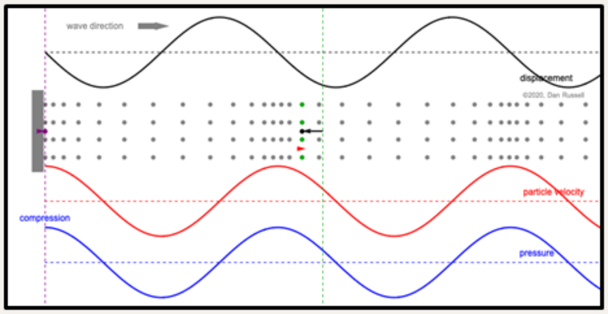
Electrical Engineering
Signal Processing, Amplifier Design, Bio-Sensing, Sound Reproduction, Sound Recording

Psychoacoustics
Human Perception of sound; Loudness, Sharpness, Tonality, Roughness, Pitch

03

Terminology

The vocabulary to describe acoustics

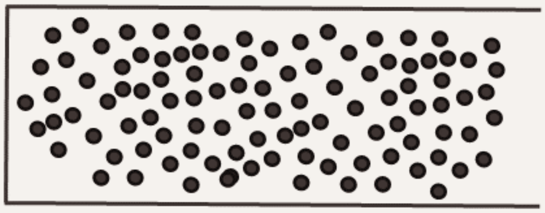


Measurable aspects of sound

Let us first consider what measurements might be made on a medium

Let's take a gas at rest for example, what can be said about:

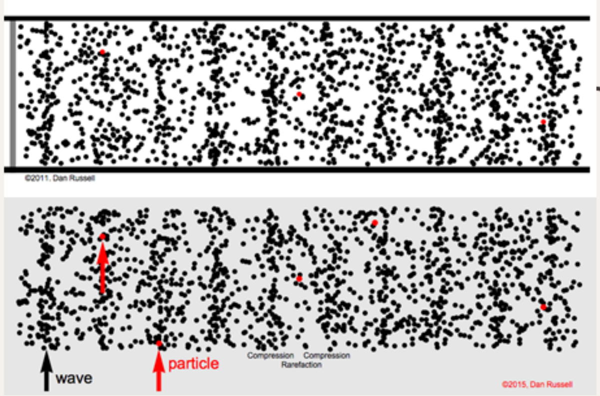
- Particle movement?
- Pressure?
- Density?



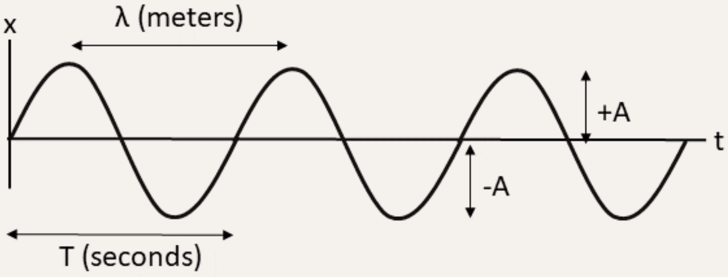
Measurable aspects of sound

When a sound wave propogates through that medium what can be said about:

- Particle position?
- Particle velocity/acceleration?
- Pressure?
- Temperature?
- Density?



Properties on Waves



Wavelength(λ)
Amplitude (A)
Frequency (ν)
Period (T)
Displacement (x)

Properties of a Waves

Frequency (ν)

- The number of complete cycles in each amount of time

Period (T)

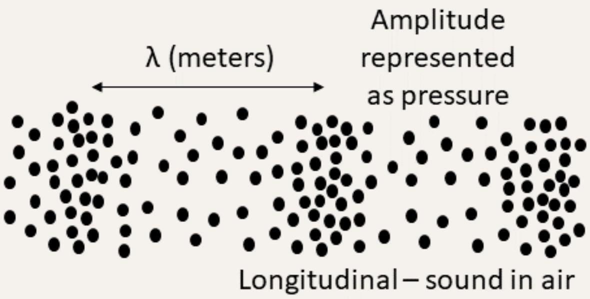
- The amount of time taken to complete one full cycle

Wavelength(λ)

- The linear distance between two wavefronts

Amplitude (A)

- The maximum displacement from equilibrium position



Relevant Terminology

Particle Velocity

Alternating velocity of an oscillating particle/element;

Meter/Second [m/s]

Sound Pressure

Alternating pressure caused by particle oscillation;

Newton/Meter^2 = Pascal
[N/m^2 = Pa]

Frequency Band

Interval in the frequency domain, usually defined by an upper and lower frequency;

Hertz [Hz]

Relevant Terminology

Acoustic Impedance

the opposition a system has to the acoustic flow resulting from an acoustic pressure;

Newton * Second/Meter^2
[Ns/m^5]

Sound Pressure Level

Level on a logarithmic scale of the effective sound intensity to a reference;

SPL = 10log(I/I_ref) [dB]

Sound Intensity

Sound power per unit area, where area is only the area component perpendicular to propagation;

Watts/Meter^2 [W/m^2]

Numerical Context: Frequency Bands

Human hearing extends from 20 – 20kHz, here are some common examples of frequency bands

	Frequency Band
Kick Drum, Bass, Organ	20 – 40 Hz
Vocals	100 – 3 kHz
Guitar	2.5 – 5 kHz
Hi-hat, cymbals, snare	8+ kHz

Let’s See for Ourselves

Spectrogram

Numerical Context: Particle Velocity

- Particle velocity can be solved for using the following relationship:
  - $Pressure = Acoustic\ Impedance\ * Particle\ Velocity$
- Using the above relationship and an acoustic impedance of air of $\approx 412\,\frac{Ns}{m^3}$, solve for the following particle velocities:
  - Measured pressure of $20\mu Pa$
    - $v = \frac{P}{Z_{air}} = \frac{0.000002}{412} = 5 * 10^{-8}\frac{m}{s}$
  - Measured pressure of $100mPa$
    - $v = \frac{P}{Z_{air}} = \frac{0.1}{412} = 25 * 10^{-5}\frac{m}{s}$
  - Measured pressure of $100Pa$
    - $v = \frac{P}{Z_{air}} = \frac{100}{412} = 0.25\frac{m}{s}$

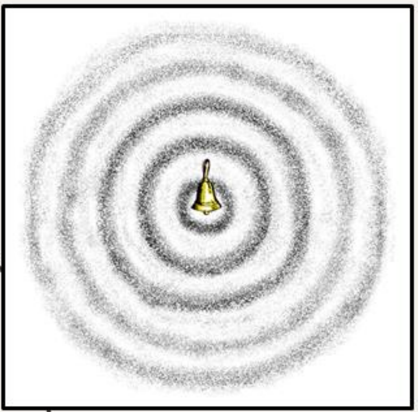
Numerical Context: Sound Pressure and SPL

- The quantity sound pressure level (SPL) is a common quantity for measuring the sound pressure with respect to a reference pressure
    - This reference is commonly the threshold of human hearing at $20\mu Pa$
    - As shown before: $\mathrm{SPL} - dB = 10\log\left(\frac{Intensity_{measured}}{Intensity_{reference}}\right)$
    - $Acoustic\ Intensity = \frac{pressure^2}{density * speed\ of\ sound}$
    - How would you express SPL in terms of sound pressure?

- $SPL - dB = 10\log\left(\frac{I_{meas}}{I_{ref}}\right) = 10log\left(\frac{\frac{{p_{meas}}^2}{\rho c}}{\frac{{p_{ref}}^2}{\rho c}}\right) = 10log\left(\frac{{p_{meas}}^2}{{p_{ref}}^2}\right) = 10log\left(\frac{p_{meas}}{p_{ref}}\right)^2$
- $SPL - dB = 20log\left(\frac{p_{meas}}{p_{ref}}\right)$ ← This equation is frequently used!

Numerical Context: Sound Pressure and SPL

- The lowest pressure change a human can hear is $20\mu Pa$, called the threshold of hearing – This commonly serves as p_{ref}
- A noise just above the threshold of hearing was recorded at $30\mu Pa$, express that in terms of dB-SPL
    - $SPL_{low} = 20log\left(\frac{30\mu Pa}{20\mu Pa}\right) \approx 3.5dB$
- A person talking generates a sound pressure change of $100mPa$, express that in terms of dB-SPL
    - $SPL_{norm} = 20log\left(\frac{100mPa}{20\mu Pa}\right) \approx 34dB$
- The loudest a human can hear before pain is $100Pa$, express that in terms of dB-SPL
    - $SPL_{norm} = 20log\left(\frac{100Pa}{20\mu Pa}\right) \approx 130dB$



04

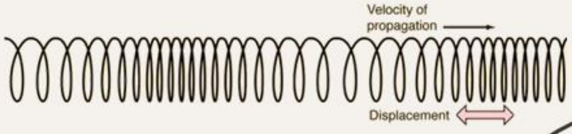
Sound Propagation

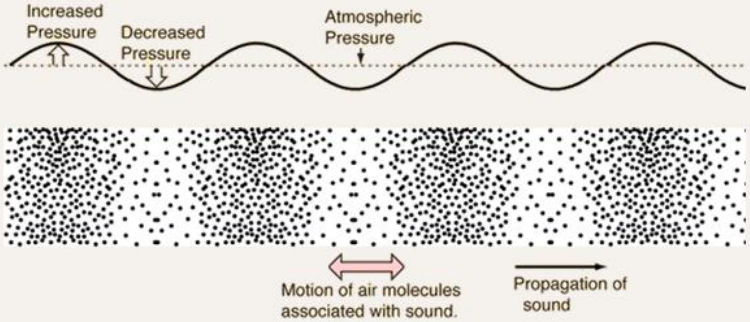
How does sound travel?

Review on Waves

- **Transverse Waves**
  - Displacement perpendicular to the direction of propagation
  - Ex) Wave on a string

- **Longitudinal Waves**
  - Displacement in the direction of propagation
  - Ex) Slinky or **Sound in air**





Sound waves in air

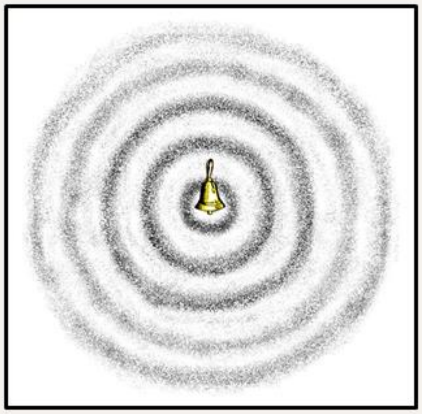
Oftentimes, transverse waves are used graphically to describe the motion of sound in air

---

Sound Propagation

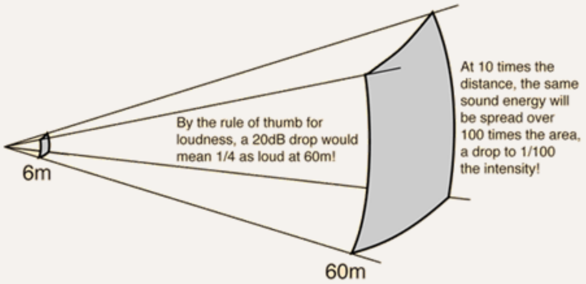
Sound propagates in all directions around the sound source

As such, sound tends to propagate spherically unless interfered with



Sound Propagation in Air

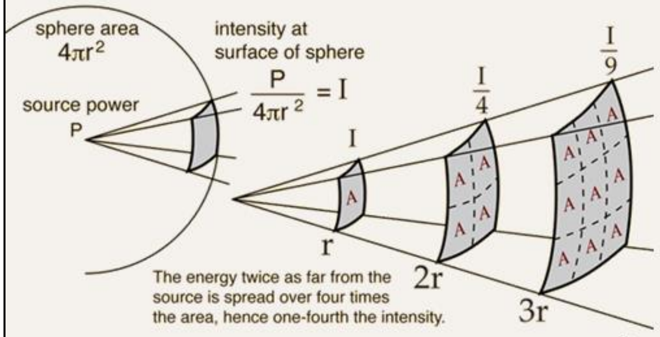
Due to this spherical propagation, sound energy distributes following the inverse square law



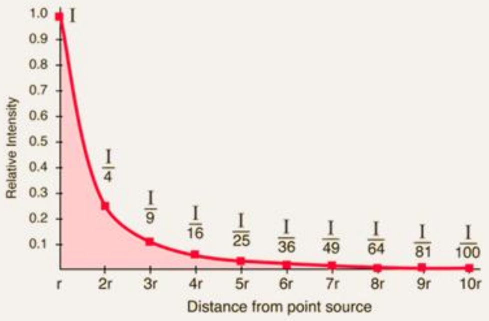
By the rule of thumb for loudness, a 20dB drop would mean 1/4 as loud at 60m!

At 10 times the distance, the same sound energy will be spread over 100 times the area, a drop to 1/100 the intensity!

The Inverse Square Law



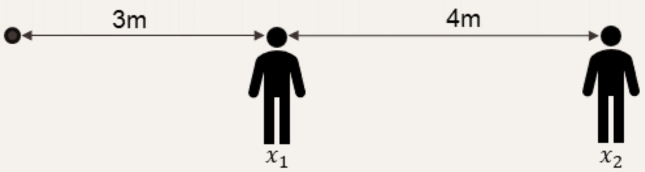
The energy twice as far from the source is spread over four times the area, hence one-fourth the intensity.



Click for simulation!

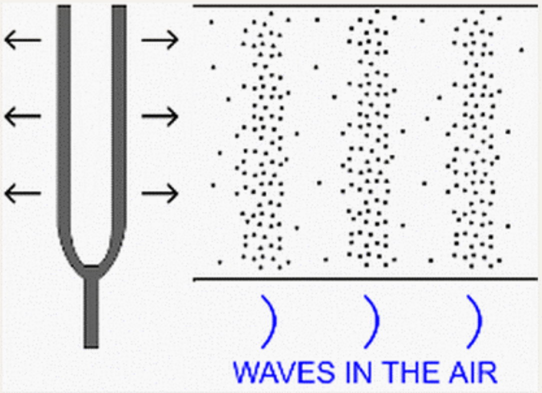
Numerical Context: Sound Propagation

- Consider a point source producing an acoustic power of 1 Watt in free air;



  - What is the difference in intensity heard by listener 1 when compared to listener 2?
    - $I_1 = \frac{P}{4\pi {r_1}^2} = \frac{1}{4\pi(3)^2} = 0.0088\frac{W}{m^2}, I_2 = \frac{P}{4\pi {r_2}^2} = \frac{1}{4\pi(7)^2} = 0.0016\frac{W}{m^2}, I_1 - I_2 = 0.0072\frac{W}{m^2}$
  - What would this difference be in decibels?
    - $I_1 - I_2 = 0.0072\frac{W}{m^2}, 20\log\left(\frac{0.0072}{0.000002}\right) \approx 50dB$

Speed of Sound



Dependent on Medium and Temperature

- Generally, the speed of sound is not dependent on:
- Frequency
- Period
- Amplitude
- Etc.

Variables Affecting Speed

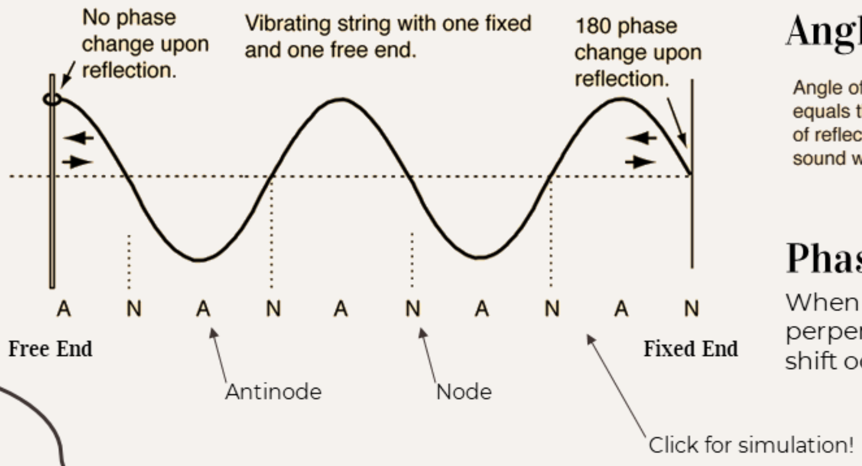
- Elastic property; Bulk Modulus, B
- Inertial Property; Density, ρ
- Related by:

$$v = \sqrt{\frac{B}{\rho}}$$

Table of Propagation Speeds

Gases	
Hydrogen (0°C)	1286
Helium (0°C)	972
Air (0°C)	343
Air (20°C)	331
Liquids at 25°C	
Water	1493
Sea Water	1533
Mercury	1450
Solids	
Diamond	12000
Iron	5130
Aluminum	5100
Brass	4700

Reflection



Angle of reflection

Angle of incidence equals the angle of reflection for sound waves too.

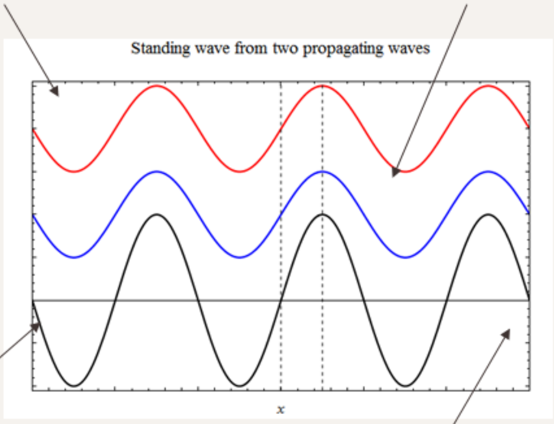
Phase Change

When sound reflects off a perpendicular wall a 180° phase shift occurs

Standing Waves

Standing wave appear to stand still, oscillating around nodes and antinodes

Standing wave amplitude is twice the incident wave amplitude



Incident Wave

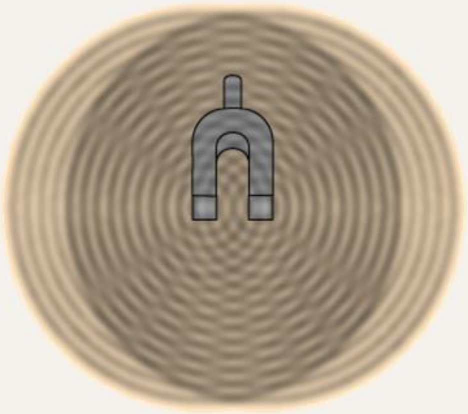
Reflection

Click for simulation!

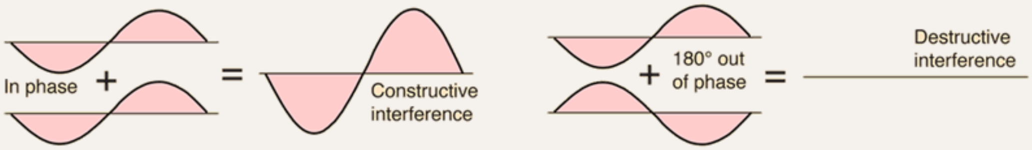
Incident + Reflection = Standing Wave

Interference

Two travelling waves that exist in the same medium will interfere with each other



Interference



Constructive

If phases align and amplitudes add, it is considered constructive

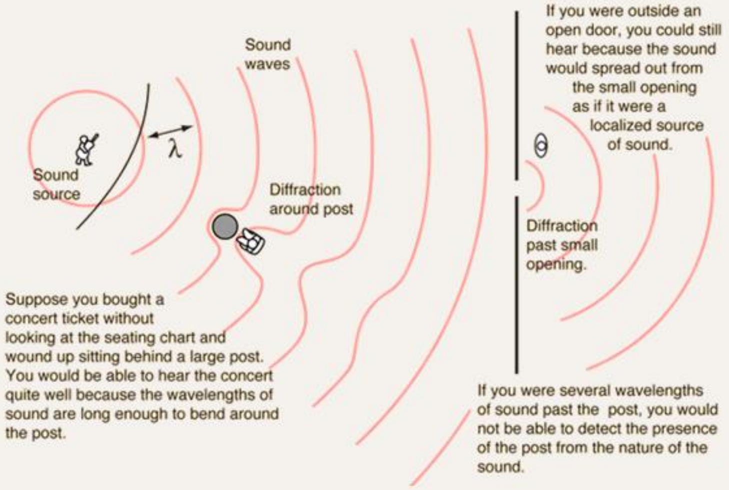
Destructive

If phases misalign and amplitudes subtract, it is considered destructive

Simulation

Diffraction

If an obstacle is sufficiently smaller than the wavelength of an incident wave, it will bend around that object



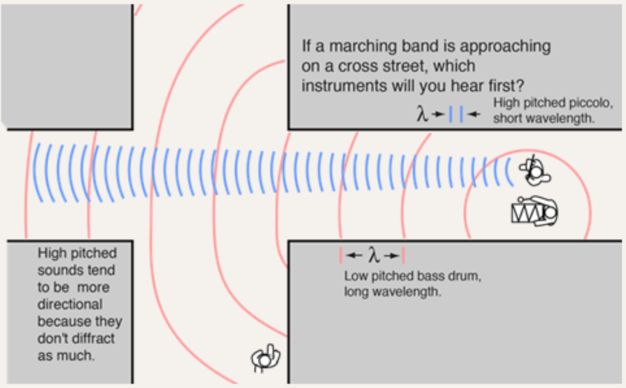
If you were outside an open door, you could still hear because the sound would spread out from the small opening as if it were a localized source of sound.

Suppose you bought a concert ticket without looking at the seating chart and wound up sitting behind a large post. You would be able to hear the concert quite well because the wavelengths of sound are long enough to bend around the post.

If you were several wavelengths of sound past the post, you would not be able to detect the presence of the post from the nature of the sound.

Click for simulation!

Diffraction



If a marching band is approaching on a cross street, which instruments will you hear first?

High pitched piccolo, short wavelength.

Low pitched bass drum, long wavelength.

High pitched sounds tend to be more directional because they don't diffract as much.

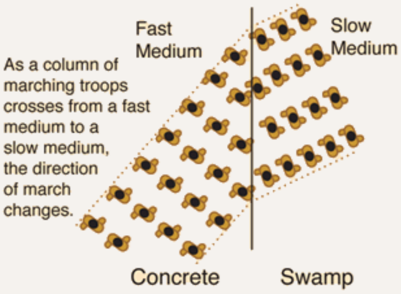
Directionality

Because higher frequencies have shorter wavelengths, they "spread out" less than lower frequencies

This makes higher frequencies more directional than lower frequencies

---

Refraction

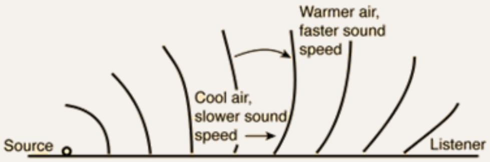


As a column of marching troops crosses from a fast medium to a slow medium, the direction of march changes.

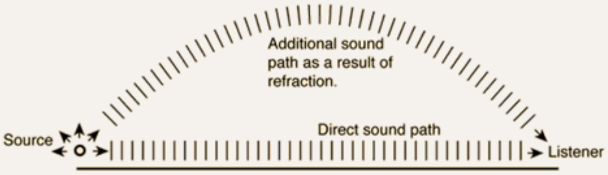
As previously discussed, when sound travels through two different mediums its speed will change

As its speed changes, the travelling wave changes direction in the new medium

The change in the medium changes the speed of propagation, and due to $v = f\lambda$, the wavelength changes as well



If the air above the ground is warmer than the air on the ground, it acts as a medium change



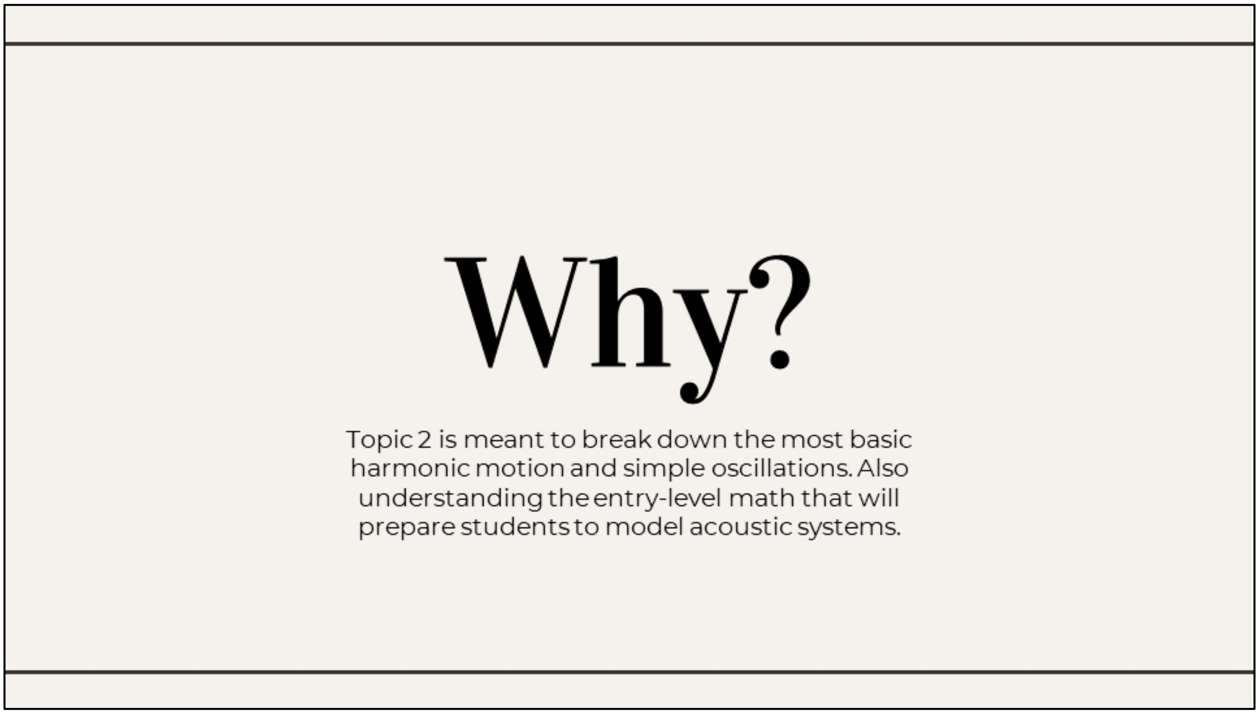
This causes an effect where not only the direct sound from the source can be heard, but additional sound that is bent down

Fisherman have often described being able to hear much greater distances on water than on land due to the water cooling the surface air, allowing more sound to be refracted down

Appendix B – Lecture Topic 2 Slides

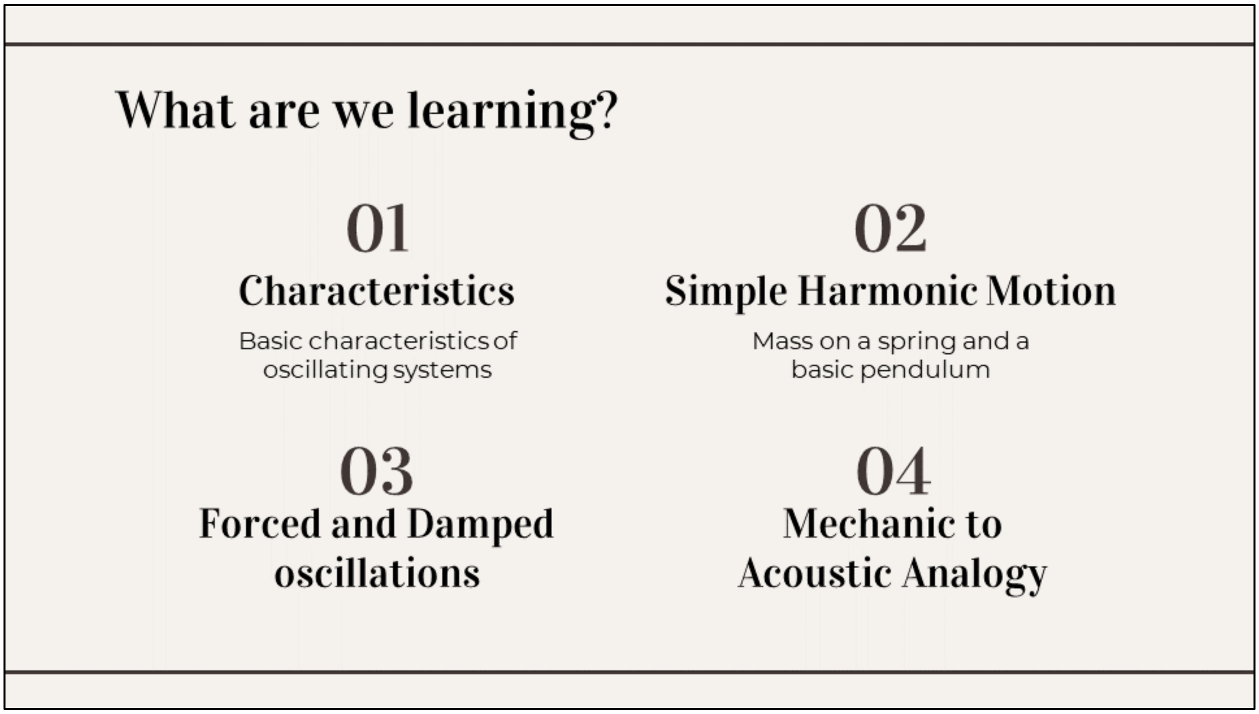
Lecture Topic 2

Waves and Oscillations



Why?

Topic 2 is meant to break down the most basic harmonic motion and simple oscillations. Also understanding the entry-level math that will prepare students to model acoustic systems.



What are we learning?

01

Characteristics

Basic characteristics of oscillating systems

02

Simple Harmonic Motion

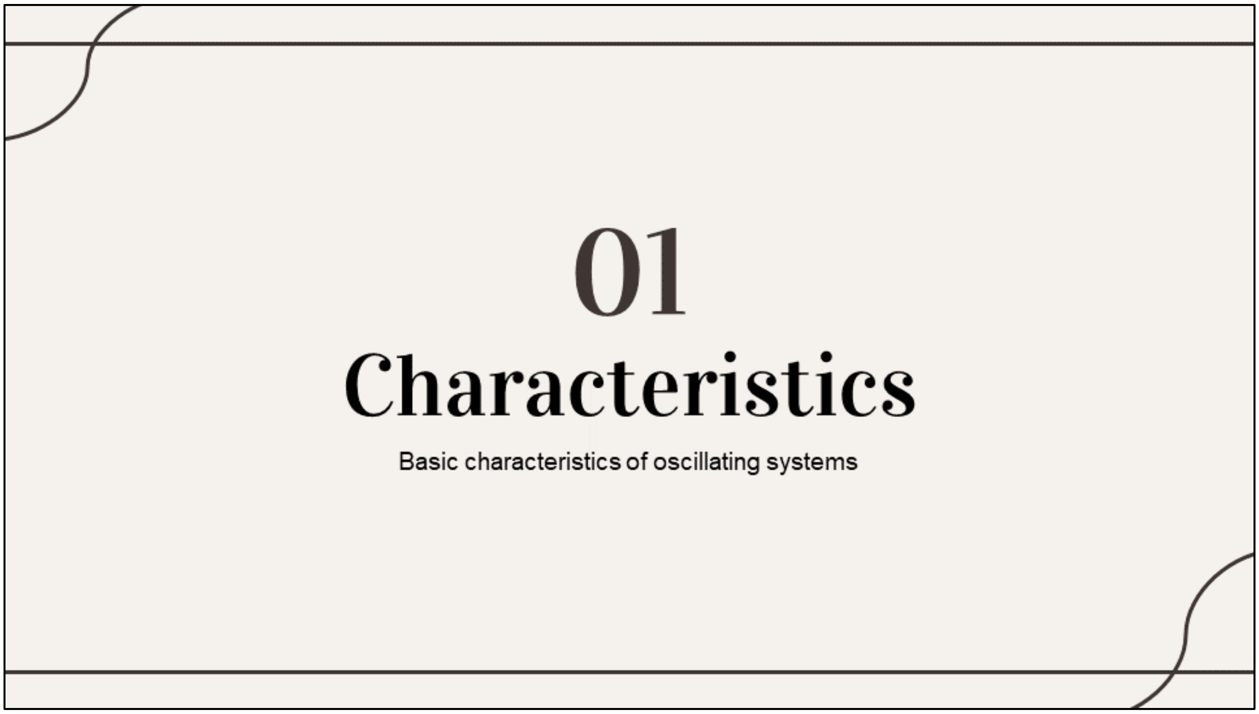
Mass on a spring and a basic pendulum

03

Forced and Damped oscillations

04

Mechanic to Acoustic Analogy



01

Characteristics

Basic characteristics of oscillating systems

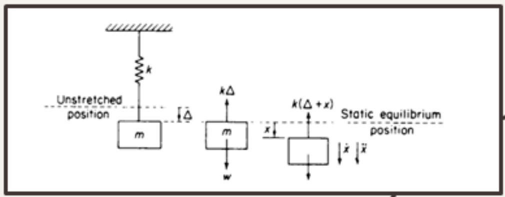
Mechanical Oscillations

All bodies with some mass and elasticity are capable of vibration

Oscillating systems can be categorized as follows:

- Linear or Nonlinear
- Free or Forced
- Damped or Undamped

Let's get into what some of that means!



Linear vs. Nonlinear

Linear

Systems which follow the principles of superposition and homogeneity

Double the input → Double Output

Combining inputs yields an additive effect on output

Non-Linear

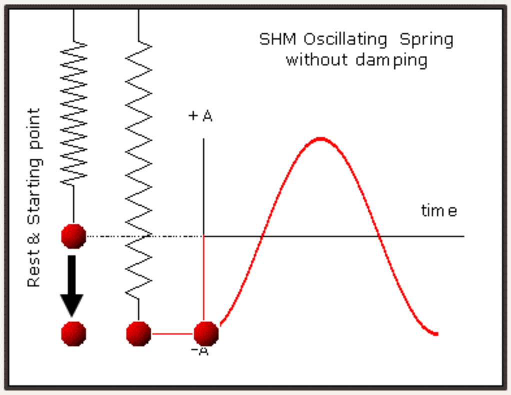
Systems which do not follow the principles of superposition and homogeneity

Double the input ≠ Double Output

Often results in complex behavior like chaotic motion or frequency-dependent responses

Focus of this course!

Free vs. Forced



Free Vibration

Every system possessing mass and elasticity can vibrate

When disturbed from equilibrium position, a system has a frequency it tends to vibrate at

- This is called its **natural frequency**

Free vibration refers to a system's oscillation when left to vibrate at its natural frequency

Assuming no damping, this is referred to as **simple harmonic motion (SHM)

Free vs. Forced

Force Vibration

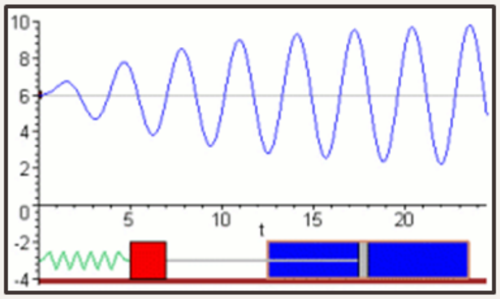
Forced vibration is simply when a system is driven by an external force

This system is forced to oscillate at the **excitation frequency**

When the excitation frequency aligns with the natural frequency, a phenomenon called **resonance** occurs

- The resultant amplitude grows larger than the free vibration amplitude

In fields like noise and vibration control, avoiding resonance is often desired



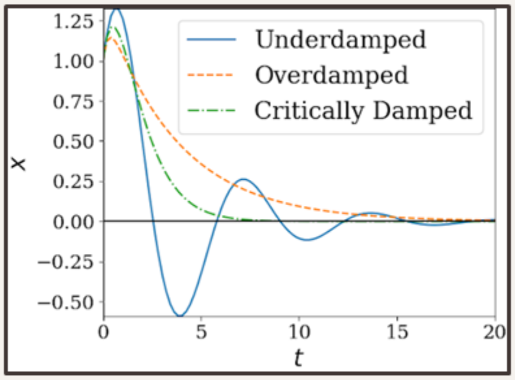
Damping

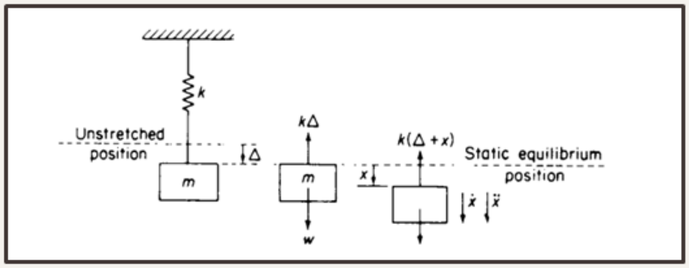
Damped vibrations decrease in amplitude over time

Damping dissipates energy in the system through friction, air resistance, or other damping methods

Three main kinds of damping:

- **Underdamping**: Oscillation descreases over time
- **Overdamping**: No oscillation, slowly returning to equilibrium
- **Critically damping**: No oscillation, fastest return to equilibrium





Mass on a spring

A mass on a spring serves as a simple model to demonstrate and apply these concepts

Let's dive into how to use what we've discussed!

02

Simple Harmonic Motion

Mass on a spring

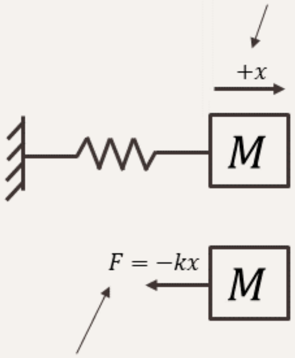
Let’s consider a simple spring-mass system

When the mass is displaced a distance, x, the spring exerts a restoring force on the mass

According to Hooke’s Law, the restoring force is:

$$F = -kx$$

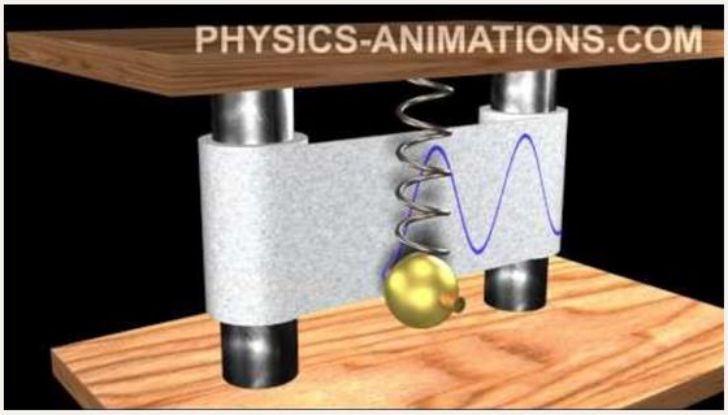
Where k is the “stiffness” of the spring



The mass can only move along the x-axis

Force is negative because it opposes the chosen positive displacement

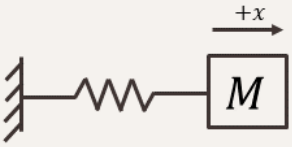
Mass on a spring - animation

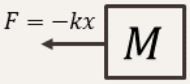


Mass on a spring

According to Newton's second law:

$$F = ma = m\frac{d^2x}{dt^2} = m\ddot{x}$$





Therefore,

$$m\ddot{x} = -kx$$
$$m\ddot{x} + kx = 0$$
$$\ddot{x} + \frac{k}{m}x = 0$$

This is the differential **equation of motion** for the spring mass system

Mass on a spring

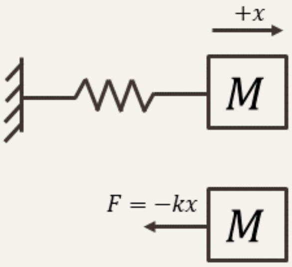
$$\ddot{x} + \frac{k}{m}x = 0$$

$\frac{k}{m}$ can be defined as the circular frequency of the system, ${\omega_n}^2$

The top eq now becomes:

$$\ddot{x} + {\omega_n}^2 x = 0$$

Intuitively, how would you describe the motion of the spring-mass system?



Mass on a spring

Before we continue with the equation of motion, let's briefly discuss the circular frequency ω

As shown before:

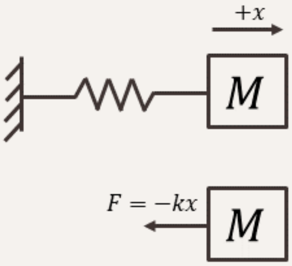
$${\omega_n}^2 = \frac{k}{m} \therefore \omega_n = \sqrt{\frac{k}{m}}$$

The natural period of this system is therefore:

$$T_n = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{m}{k}}$$

And the natural frequency of the system is:

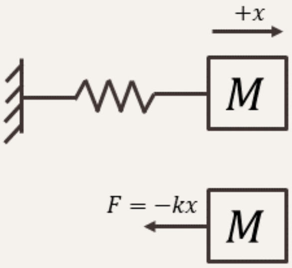
$$f_n = \frac{1}{T_n} = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$



Mass on a spring

With these equations in mind:

$$T_n = 2\pi\sqrt{\frac{m}{k}} \qquad f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$



- What happens if we increase mass?
- What happens if we increase the spring stiffness?

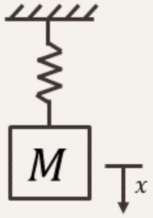
SHM Demonstration

$$T = 2\pi\sqrt{\frac{m}{k}}$$

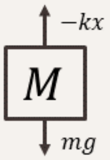
Mass on a spring

Consider a mass of, $m = 1kg$, suspended from a spring of stiffness, $k = 150\,\frac{N}{m}$.

a) Draw a free body diagram
b) Write the system's equation of motion
c) find the system's resonant frequency



a)



b) $F = mg = -kx \;\therefore\; mg + kx = 0$

c) $\omega_n{}^2 = \frac{k}{m} \rightarrow \omega_n = \sqrt{\frac{k}{m}} \rightarrow f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1.94Hz$

Mass on a spring

Recall:

$$\ddot{x} + \omega_n{}^2 x = 0$$

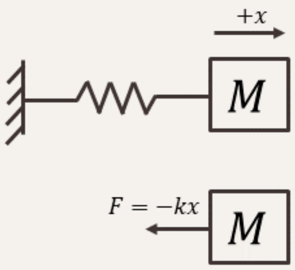
Assuming motion to be sinusoidal in nature, the **general solution** for the equation of motion is:

$$x = A_1\cos(\omega t) + A_2\sin(\omega t)$$

The constants A_1 and A_2 are found using the system's initial conditions:

$$x(t = 0) = x_0$$

$$v(t = 0) = \frac{dx(t = 0)}{dt} = v_0$$



Mass on a spring

$$x(t=0) = x_0$$

$$v(t=0) = \frac{dx(t=0)}{dt} = v_0$$

Substituting this into our solution equation:

$$x = A_1\cos(\omega t) + A_2\sin(\omega t)$$

$$x_0 = A_1\cos(0) + A_2\sin(0)$$

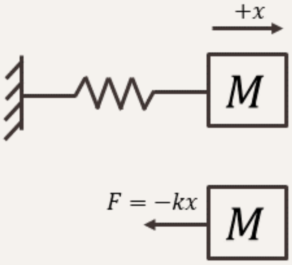
$$A_1 = x_0$$

$$v = \frac{dx}{dt} = -A_1\omega\sin(\omega t) + A_2\omega\cos(\omega t)$$

$$v_0 = -A_1\omega_n\sin(0) + A_2\omega_n\cos(0)$$

$$v_0 = A_2\omega_n$$

$$A_2 = \frac{v_0}{\omega_n}$$



Mass on a spring

$$A_1 = x_0 \qquad A_2 = \frac{v_0}{\omega_n}$$

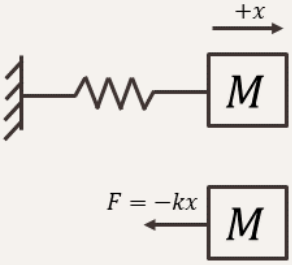
Substituting this into our general solution:

$$x = A_1\cos(\omega t) + A_2\sin(\omega t)$$

$$x = x_0\cos(\omega_n t) + \left(\frac{v_0}{\omega_n}\right)\sin(\omega_n t)$$

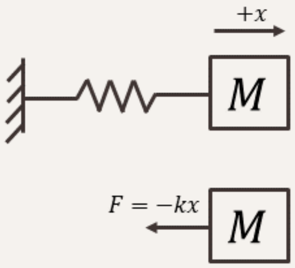
Looking at this equation, what would happen if the mass is never given an initial displacement?

What if it was given an initial displacement **and** velocity?



Mass on a spring

$$x = x_0 \cos(\omega_n t) + \left(\frac{v_0}{\omega_n}\right) \sin(\omega_n t)$$



If given an initial velocity, the motion equation gets **scaled,** and a **phase change** occurs

This can be demonstrated here:

Desmos Link

Phase

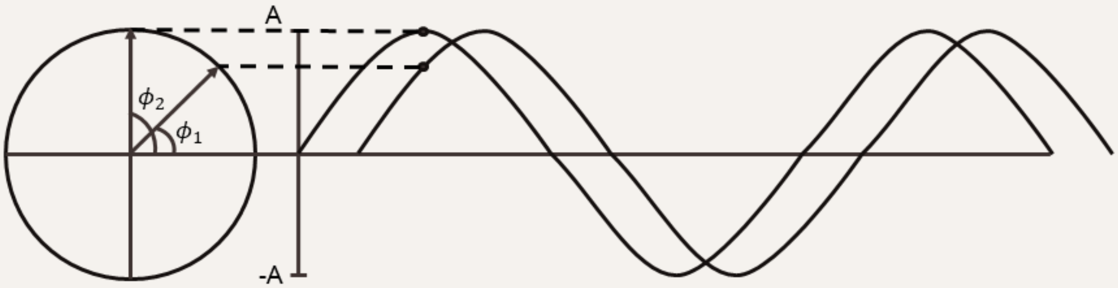
The concept of phase is important to understanding the graphs of signals, including displacement, velocity, and acceleration

Phase, ϕ, describes a waveform's position in a cycle – Take a standard sine wave as an example:

Phase

If two waves have the same frequency, but a different position in time, they are said to be "out of phase"

An example of this is shown below:



Mass on a spring

Recall:

$$x = A_1 \cos(\omega_n t) + A_2 \sin(\omega_n t)$$

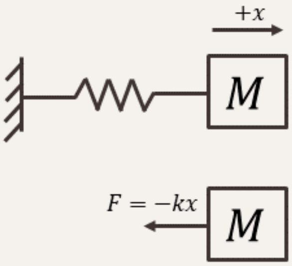
If we let:

$$A_1 = x_0 \cos(-\phi) = x_0 \cos(-\phi)$$

$$A_2 = \frac{v_0}{\omega_n} sin(-\phi) = -\frac{v_0}{\omega_n} sin(\phi)$$

Then:

$$x = x_0 \cos(\omega_n t) \cos(\phi) - \frac{v_0}{\omega_n} \sin(\omega_n t)\, sin(\phi)$$



Mass on a spring

Using the identity:

$$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

Where,

$$\alpha = \omega_n t, \qquad \beta = -\phi$$

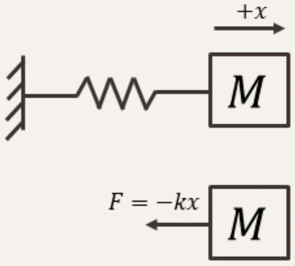
Then,

$$x = Acos(\omega_n - \phi)$$

Where,

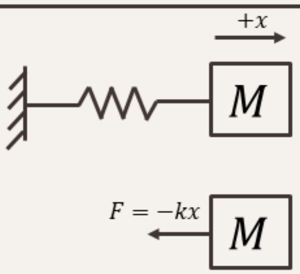
$$A = \sqrt{(A_1{}^2) + (A_2{}^2)}$$

$$\phi = tan^{-1}\left(\frac{A_2}{A_1}\right)$$



Desmos Link

Mass on a spring



With this equation:

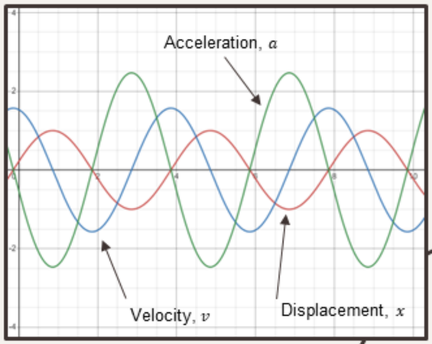
$$x = Acos(\omega_n t - \phi)$$

The displacement of a system undergoing SHM can be described

Using the equation of displacement, how would you find an equation to describe velocity and acceleration?

$$velocity = \frac{d}{dt}(x) = \dot{x} = -A\omega sin(\omega_n t - \phi)$$

$$acceleration = \frac{d}{dt}(v) = \ddot{x} = -A\omega_n{}^2\cos(\omega_n t - \phi)$$



Desmos Link

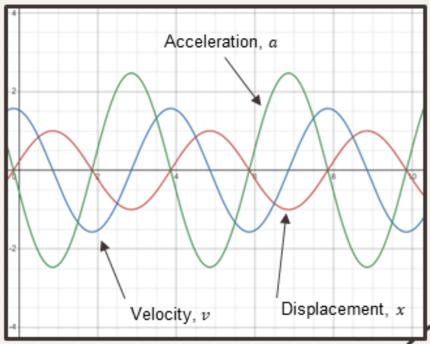
Mass on a spring

Looking at the graph of displacement, velocity, and acceleration;

- What can you say about their phase relationship?

Velocity lags displacement by 90°
Acceleration lags velocity by 90°

Therefore,
Acceleration lags displacement by 180°



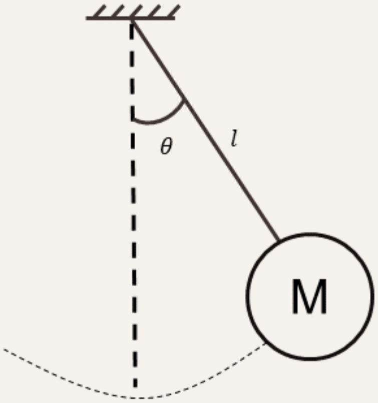
Pendulum

Another example of SHM is a **simple pendulum**

Consider the system to the right which consists of a mass, m, suspended a distance, l, from its pivot

This mass is free to swing at an angle, θ, in either direction

Let's find the equation of motion for this system!

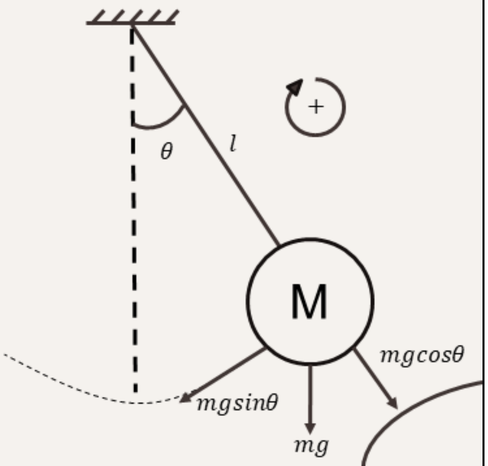


Pendulum

The process for finding the equation of motion and the natural frequency is the same

1. Draw a free body diagram
2. Write out the **sum of forces** (Newton's second law)
3. Use the coefficient for the **restoring force** to solve for **natural frequency**

Given the free body diagram to the right, try and complete steps 2 and 3 (Hint, instead of x think in terms of θ)



Pendulum

2.

$$\sum F = ml\ddot{\theta} = -mgsin\theta$$

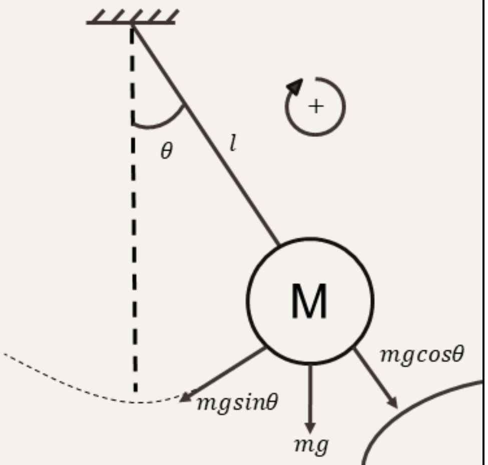
$$0 = ml\ddot{\theta} + mgsin\theta$$

$$0 = \ddot{\theta} + \frac{g}{l}sin\theta$$

For small angles, $sin\theta \approx \theta$

$$0 = \ddot{\theta} + \frac{g}{l}\theta$$

$$\therefore\ \omega_n{}^2 = \frac{g}{l}$$



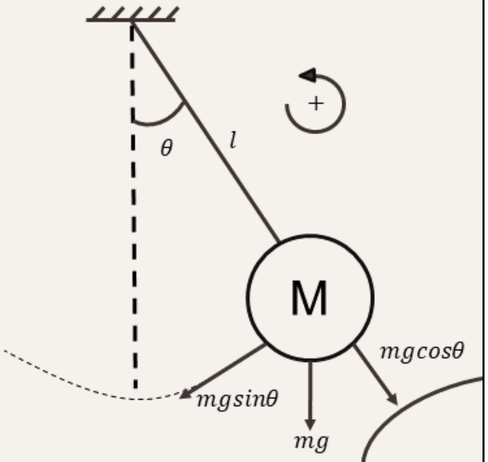
Pendulum

3.

$$\omega_n{}^2 = \frac{g}{l}$$

$$\omega_n = \sqrt{\frac{g}{l}}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

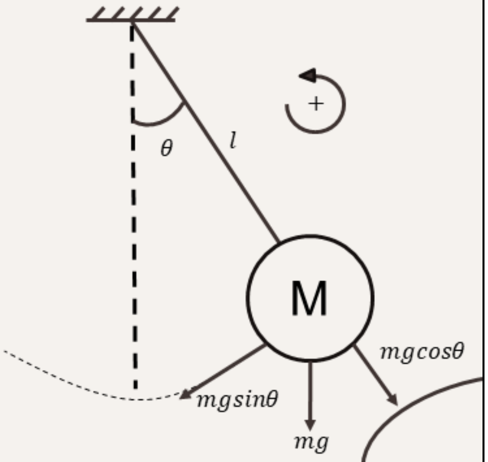


Pendulum

Now that we've found the equation of motion and natural frequency of the system, find the **natural period** of the system

$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

$$T_n = \frac{1}{f_n} = 2\pi\sqrt{\frac{l}{g}}$$



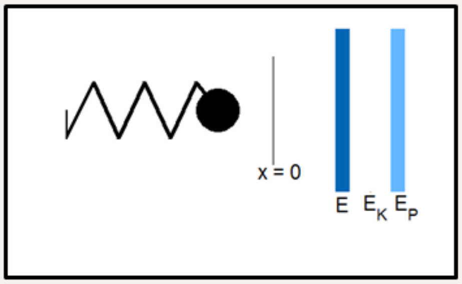
Energy Method

Another way to view a system's oscillation is through the conservation of energy

The big idea: In an ideal system, the total energy remains constant

As a system oscillates, its potential energy is transferred to kinetic, then back to potential, and so on

Let's derive the equations for potential and kinetic energy



Energy Method

Potential Energy $\boldsymbol{(E_p)}$**:** Related to the initial displacement of the system

$$\boldsymbol{E_p = \frac{1}{2}kx^2}$$

Substitute: $x = A\cos(\omega_o t - \phi)$

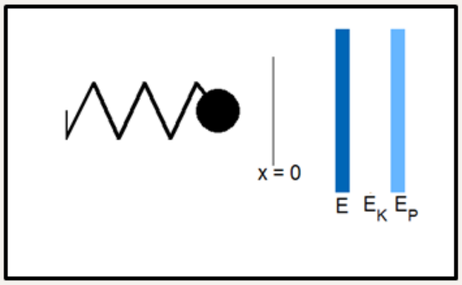
$$\boldsymbol{E_p = \frac{1}{2}kA^2\cos^2(\omega_o t + \phi)}$$

Kinetic Energy $\boldsymbol{(E_k)}$**:** Related to the velocity of the system

$$\boldsymbol{E_k = \frac{1}{2}mv^2}$$

Substitute: $v = -A\omega\sin(\omega_n t - \phi)$

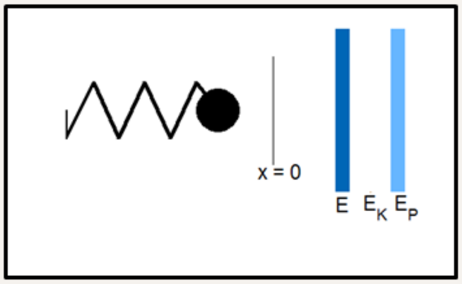
$$\boldsymbol{E_k = -\frac{1}{2}mA^2\omega^2 sin^2(\omega_n t - \phi)}$$



Energy Method

$$E_p = \frac{1}{2} kA^2 \cos^2(\omega_o t + \phi)$$

$$E_k = -\frac{1}{2} mA^2 \omega^2 sin^2(\omega_n t - \phi)$$



Total energy of the system:

$$E_{total} = E_p + E_k$$

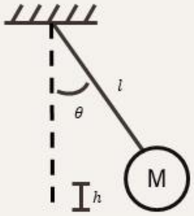
$$= \frac{1}{2} kA^2 \cos^2(\omega_n t + \phi) - \frac{1}{2} mA^2 \omega^2 sin^2(\omega_n t - \phi)$$

Since:

$$E_{total} = \frac{1}{2} m{v_{max}}^2 = \frac{1}{2} k{x_{max}}^2$$

$$\boldsymbol{E_{total} = \frac{1}{2} mA^2 = -\frac{1}{2} k(A\omega)^2}$$

Energy Method



Similarly for a pendulum:

$$E_p = mgl(1 - \cos(\theta))$$

$$E_k = \frac{1}{2} mv^2$$

Therefore

$$E_{total} = \frac{1}{2} mv^2 - mgl(1 - \cos(\theta))$$

And

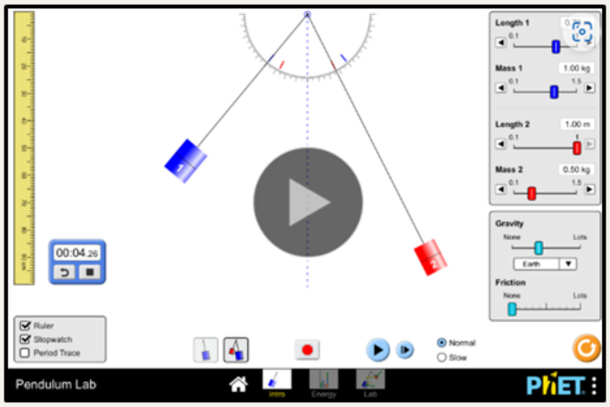
$$E_{total} = \frac{1}{2} m{v_{max}}^2 = mgh$$

Pendulum Simulation

Let's try a simulation!

[Pendulum Interactive Demo]

Press the play Button and go to "intro"



Length 1
Mass 1 1.00 kg
Length 2 1.00 m
Mass 2 0.50 kg
Gravity None Lots Earth
Friction None Lots
☑ Ruler
☑ Stopwatch
☐ Period Trace
● Normal
○ Slow

Simple Pendulum

In the Simulation, you can model:

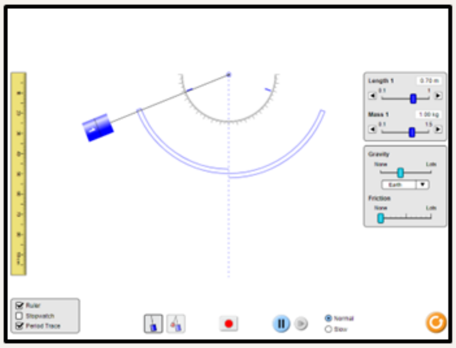
Free oscillation:

- Length - .7m
- Mass -1kg
- Gravity - Earth
- Friction - none
- Click period trace

What happens to the period and amplitude when…

- You increase the length?
- You increase the mass?
- You increase the gravity?
- You add friction?

Why does the pendulum slow down with friction? Let's find out…



Length 1 0.70 m
Mass 1 1.00 kg
Gravity None Lots Earth
Friction None Lots
☑ Ruler
☐ Stopwatch
☑ Period Trace
● Normal
○ Slow

03

Forced and Damped Oscillations

Dampers

As previously mentioned, Damped vibrations decrease in amplitude over time

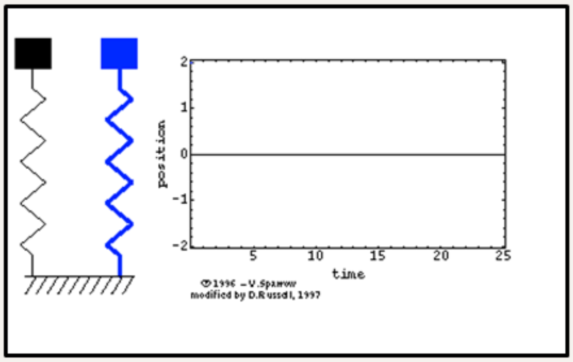
The damping force is defined:

$$F_d = c\dot{x}$$

To account for damping, this term is added to the equation of motion:

$$m\ddot{x} + c\dot{x} + kx = 0$$

Where c is the damping coefficient



Dampers

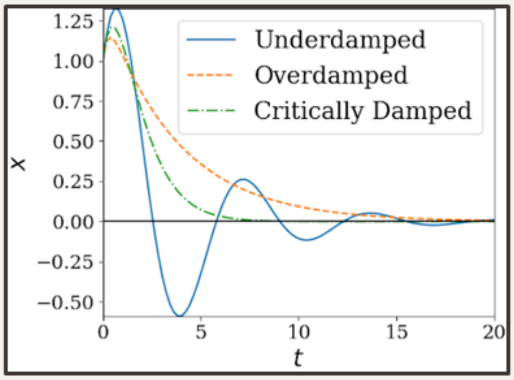
Recall, there are three main kinds of damping:

- **Underdamping**: Oscillation descreases over time
- **Overdamping**: No oscillation, slowly returning to equilibrium
- **Critically damping**: No oscillation, fastest return to equilibrium

We'll label the coefficient of damping related to critical dampening as c_c

The critical damping ratio is defined as:

$$c_c = 2m\sqrt{\frac{k}{m}} = 2m\omega_n$$



Dampers

To express the damping in a given system, we'll use the damping ration, ζ

$$\zeta = \frac{c}{c_c}$$

The conditions for the three main forms of damping are as follows:

Critical damping:

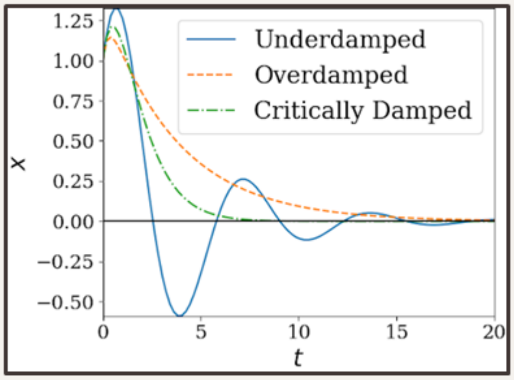
$$\zeta = \frac{c_c}{c_c} = 1$$

Underdamping:

$$\zeta < 1; c < c_c$$

Overdamping:

$$\zeta > 1; c > c_c$$



Dampers

$$c_c = 2m\sqrt{\frac{k}{m}} = 2m\omega_n$$

$$\zeta = \frac{c}{c_c}$$

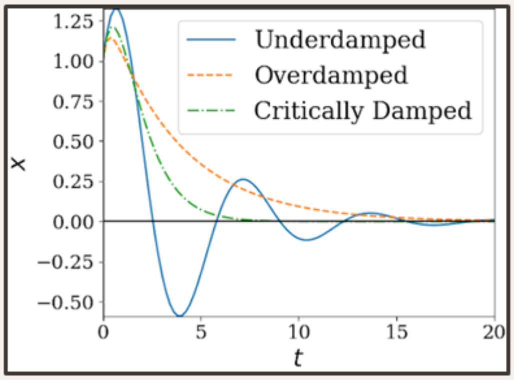
There are many kinds of damping which could change this calculation, but for this course we're assuming viscous damping

The damping ratio is therefore equal to:

$$c = \zeta c_c = 2\zeta m\omega_n$$

Knowing the damping coefficient, you can successfully describe the motion of a damped oscillatory system:

$$m\ddot{x} + c\dot{x} + kx = 0$$



Forced Oscillation

Forced vibration is simply when a system is driven by an external force

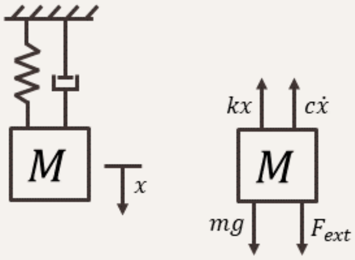
This system is forced to oscillate at the **excitation frequency**

Recall, a system undergoing free vibration:

$$m\ddot{x} + c\dot{x} + kx = 0$$

To describe forced oscillation, simply have the sum of forces equal to the external force:

$$m\ddot{x} + c\dot{x} + kx = F_{ext}$$



Forced Oscillation

Take, for example, a **harmonically excited** system:

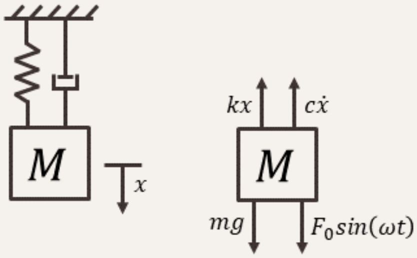
$$m\ddot{x} + c\dot{x} + kx = F_0 sin(\omega t)$$

From before, we can assume a particular solution of the form:

$$x = Xsin(\omega t - \phi)$$

Here, X, is the amplitude of oscillation

Amplitude, X, and phase, ϕ, can be solved by substituting Eq [] into Eq []



Forced Oscillation

We'll save some time with the algebra…

$$X = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$$

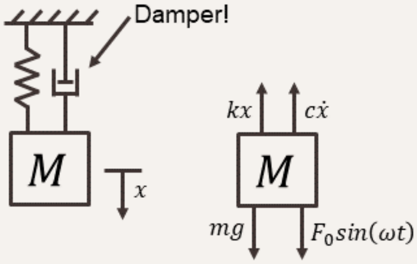
And

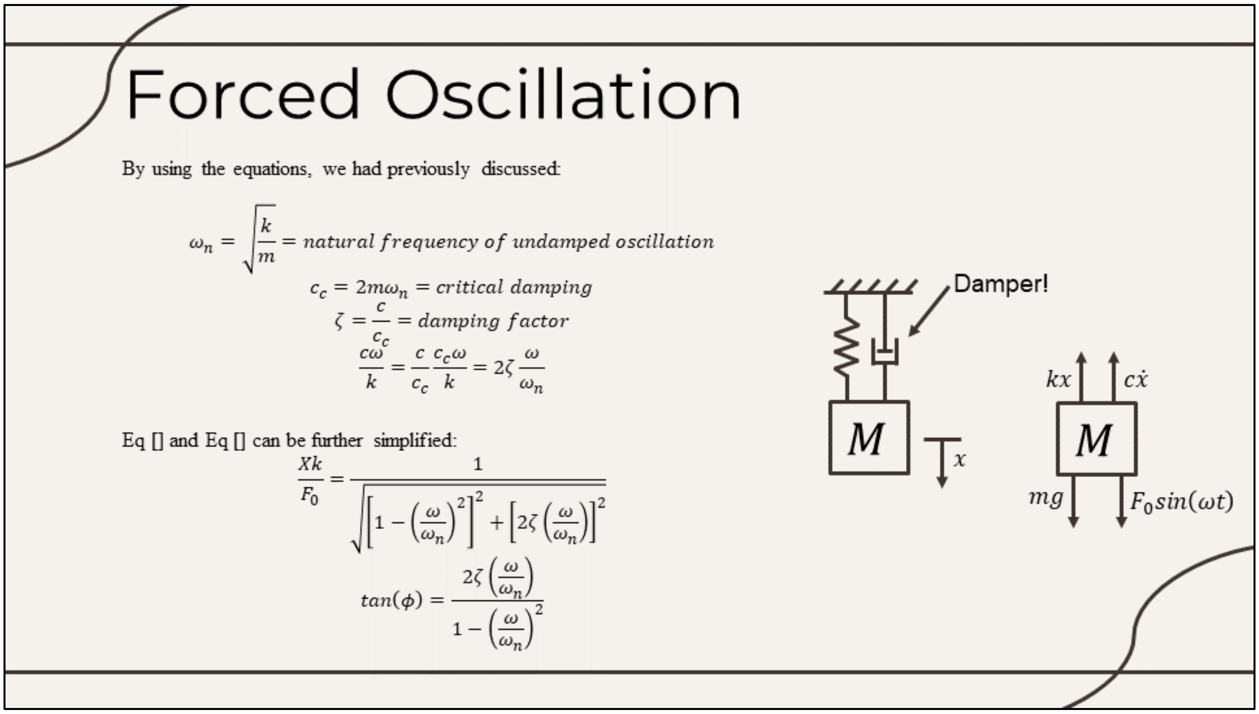
$$\phi = tan^{-1}\frac{c\omega}{k - m\omega^2}$$

This can be simplified to:

$$X = \frac{\frac{F_0}{k}}{\sqrt{\left(1 - \frac{m\omega^2}{k}\right)^2 + \left(\frac{c\omega}{k}\right)^2}}$$

$$tan(\phi) = \frac{\frac{c\omega}{k}}{1 - \frac{m\omega^2}{k}}$$





Forced Oscillation

By using the equations, we had previously discussed:

$$\omega_n = \sqrt{\frac{k}{m}} = natural\ frequency\ of\ undamped\ oscillation$$

$$c_c = 2m\omega_n = critical\ damping$$

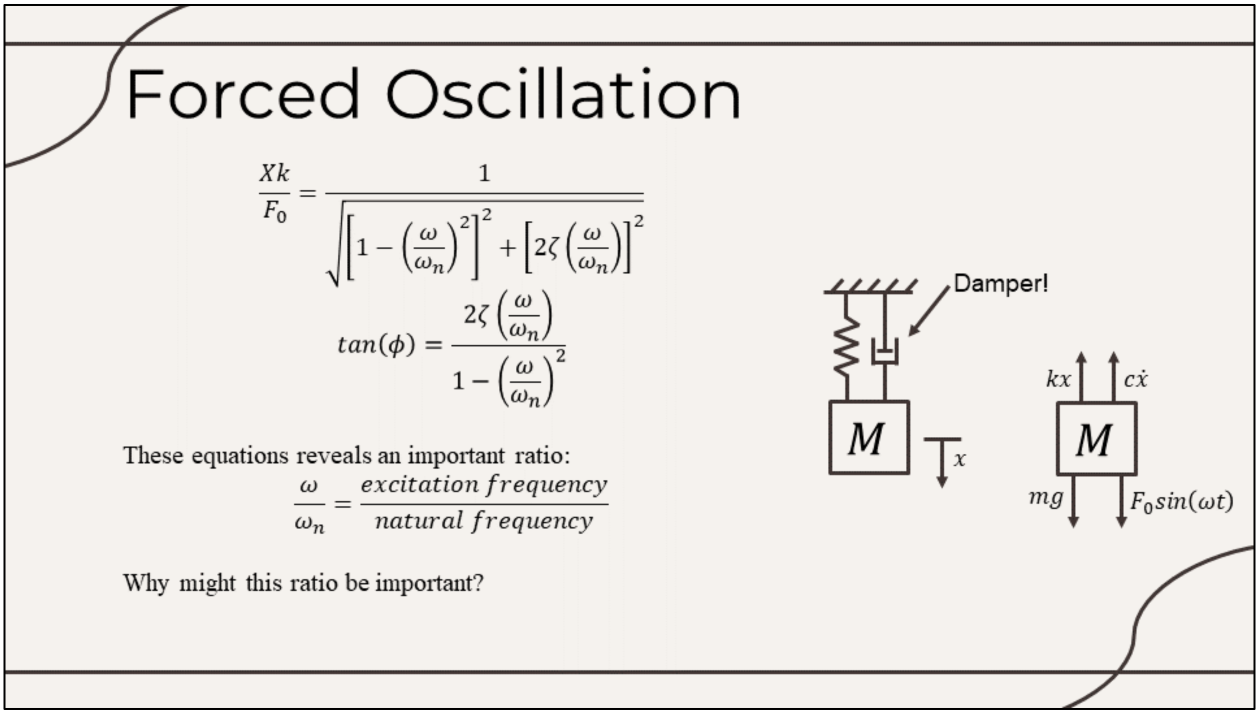
$$\zeta = \frac{c}{c_c} = damping\ factor$$

$$\frac{c\omega}{k} = \frac{c}{c_c}\frac{c_c\omega}{k} = 2\zeta\frac{\omega}{\omega_n}$$

Eq [] and Eq [] can be further simplified:

$$\frac{Xk}{F_0} = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\left(\frac{\omega}{\omega_n}\right)\right]^2}}$$

$$tan(\phi) = \frac{2\zeta\left(\frac{\omega}{\omega_n}\right)}{1-\left(\frac{\omega}{\omega_n}\right)^2}$$



Forced Oscillation

$$\frac{Xk}{F_0} = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\left(\frac{\omega}{\omega_n}\right)\right]^2}}$$

$$tan(\phi) = \frac{2\zeta\left(\frac{\omega}{\omega_n}\right)}{1-\left(\frac{\omega}{\omega_n}\right)^2}$$

These equations reveals an important ratio:

$$\frac{\omega}{\omega_n} = \frac{excitation\ frequency}{natural\ frequency}$$

Why might this ratio be important?

Forced Oscillation

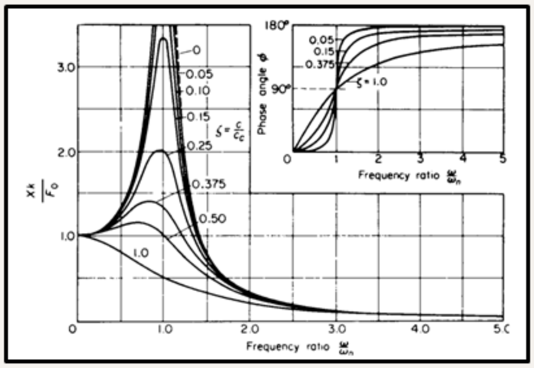
This ratio is what describes the condition for **resonance**

When $\omega = \omega_n$,

$$\frac{\omega}{\omega_n} = 1$$

This means that the **excitation frequency** and **natural frequency** of the system are aligned and there will be an **increased amplitude response**

Take a moment to look at the graph to the right, discuss some of the things you notice relating to the concept of resonance



Forced Oscillations

Transitioning from harmonically excited systems, we encounter a spectrum of loading conditions

Excitation isn't always harmonic; various loading scenarios shape system response differently:

Harmonic Loading

When applied load varies as a sine (or cosine function).

Ex. Periodic Loading

Transient Loading

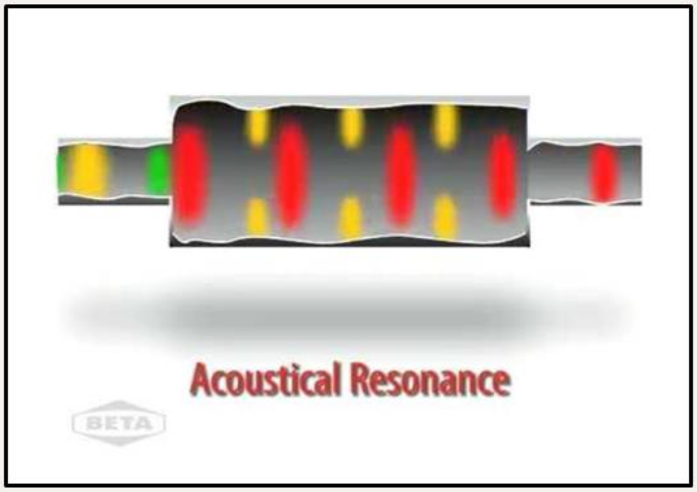
Sudden changes in the magnitude and/or the direction of a torque load (sudden applied nonperiodic excitations)

Random Continuous Loading

When applied load that only has continuous values. Loading can happen over a specific range of time ex. [0,60] seconds instead of discrete times Ex. 5 seconds

For this course we won't introduce the more complex loading conditions, but its important to conceptually understand there is more than harmonic loading

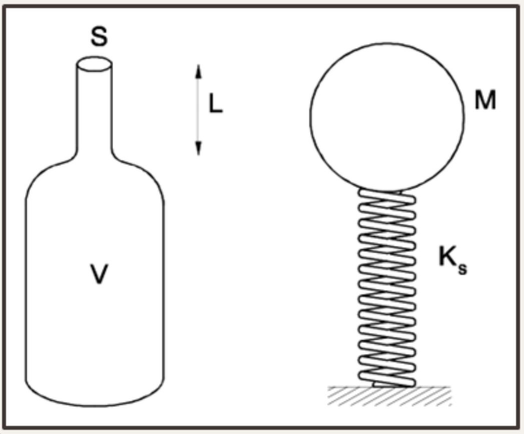
Acoustic Resonance



04

Mechanical to acoustic analogies

Helmholtz Resonator



To start thinking about acoustic systems in terms of **mechanical system analogies**, let's consider the **Helmholtz resonator**

The volume of air within the neck of the bottle has a certain mass

When that mass travels some distance downwards, it compresses the volume of air within the bottle

The compressed air applies a force back upwards, thus causing oscillation

---

Helmholtz Resonator

Do the following:

1. Write an equation to describe the volume of air within the neck
2. Using that equation, find the mass of air within the neck

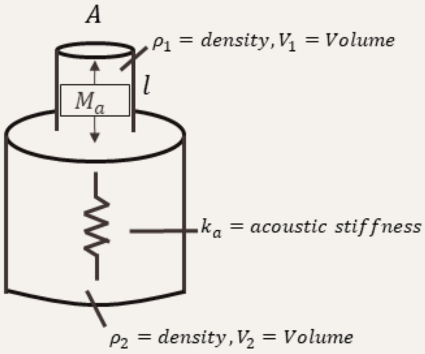
Let the volume of air within the neck be

$$Volume = Area * Length \rightarrow V_1 = A * L$$

Since

$$density = \frac{mass}{Volume} \rightarrow \rho = \frac{M}{V_1}$$

$$M = \rho A l$$



Helmholtz Resonator

This mass, $M = \rho A l$, is the mechanical mass of this system

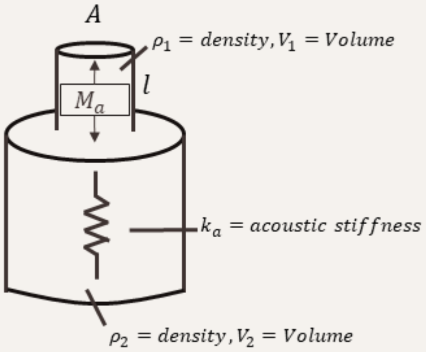
$$M_{mech} = \rho A l$$

We now need a way to transfer from the mechanical to acoustic domain

Given:

$$Z_{mech} = \frac{F}{v} = \frac{Force}{Velocity} \text{ and } Z_a = \frac{P}{U} = \frac{Pressure}{Volume\ Velocity}$$

Can you solve for what the equivalent acoustic mass would be? (Hint: Compare units!)



Helmholtz Resonator

$$Z_{mech} = \frac{F}{v} = \frac{Force}{Velocity} \text{ and } Z_a = \frac{P}{U} = \frac{Pressure}{Volume\ Velocity}$$

Write in terms of units:

$$Z_{mech} = \frac{N}{\frac{m}{s}} = \frac{Ns}{m}$$

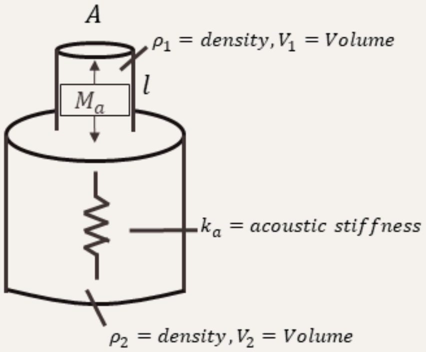
$$Z_a = \frac{\frac{N}{m^2}}{\frac{m^3}{s}} = \frac{Ns}{m^5}$$

Therefore:

$$Z_a = \frac{1}{m^4} Z_{mech}$$

Now,

$$M_a = \frac{1}{m^4} M_{mech} = \frac{1}{A^2} \rho A l = \frac{\rho l}{A}$$



Helmholtz Resonator

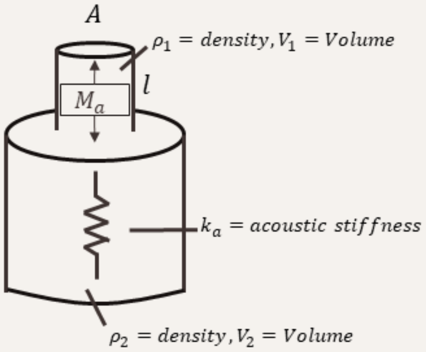
Using the equation for the acoustic stiffness of the larger volume of air below:

$$k_a = \frac{\rho c^2}{V_2}$$

Write the resonant frequency of the system

Recall,

$$M_a = \frac{\rho l}{A}$$



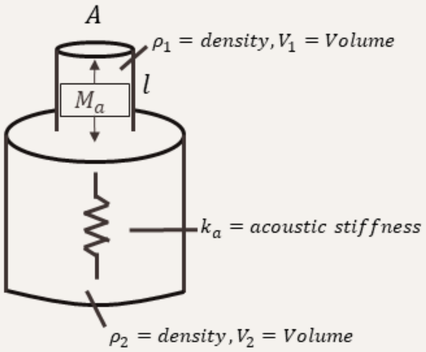
Helmholtz Resonator

$$k_a = \frac{\rho c^2}{V_2}\ and\ M_a = \frac{\rho l}{A}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{k_a}{M_a}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{\rho c^2}{V_2} * \frac{A}{\rho l}}$$

$$= \frac{c}{2\pi}\sqrt{\frac{A}{V_2 l}}$$



Just like that, we were able to model this acoustic system with the analogous mechanical system!

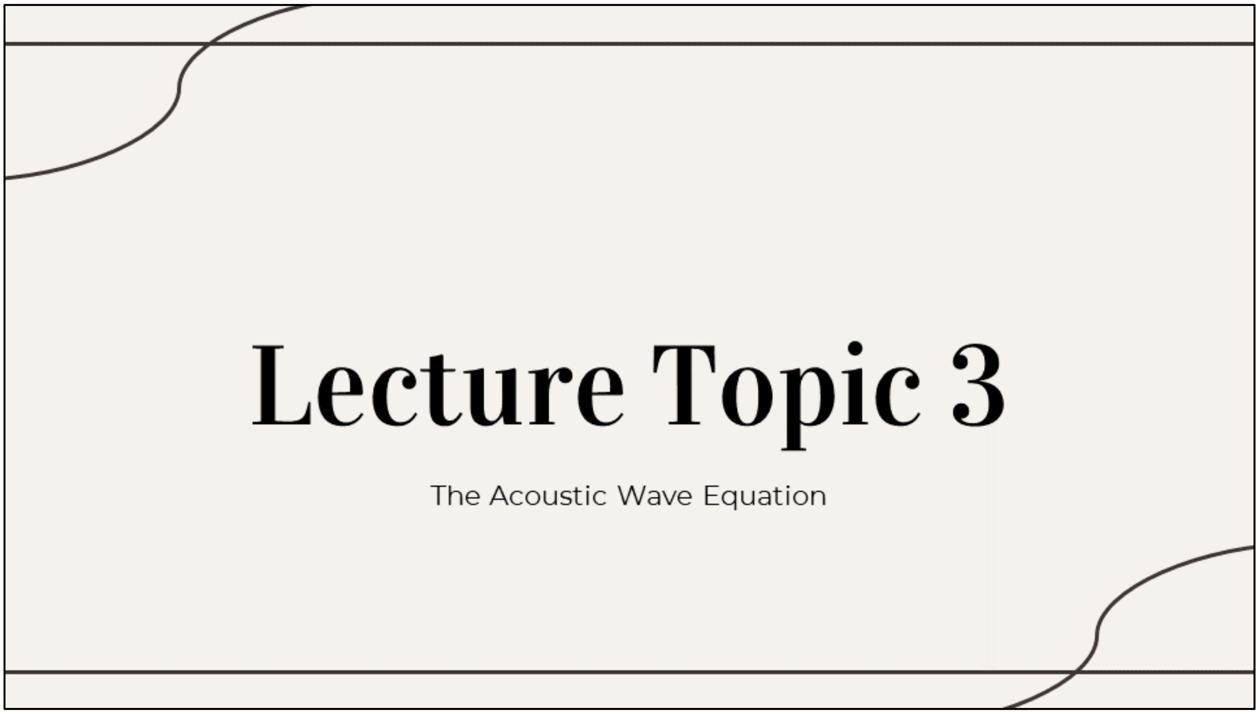
Expanding The Idea

This method of analogous modeling is often used to effectively model systems in the mechanical, electrical, and acoustic domains

This has proven to be a powerful tool in the analysis of complex systems

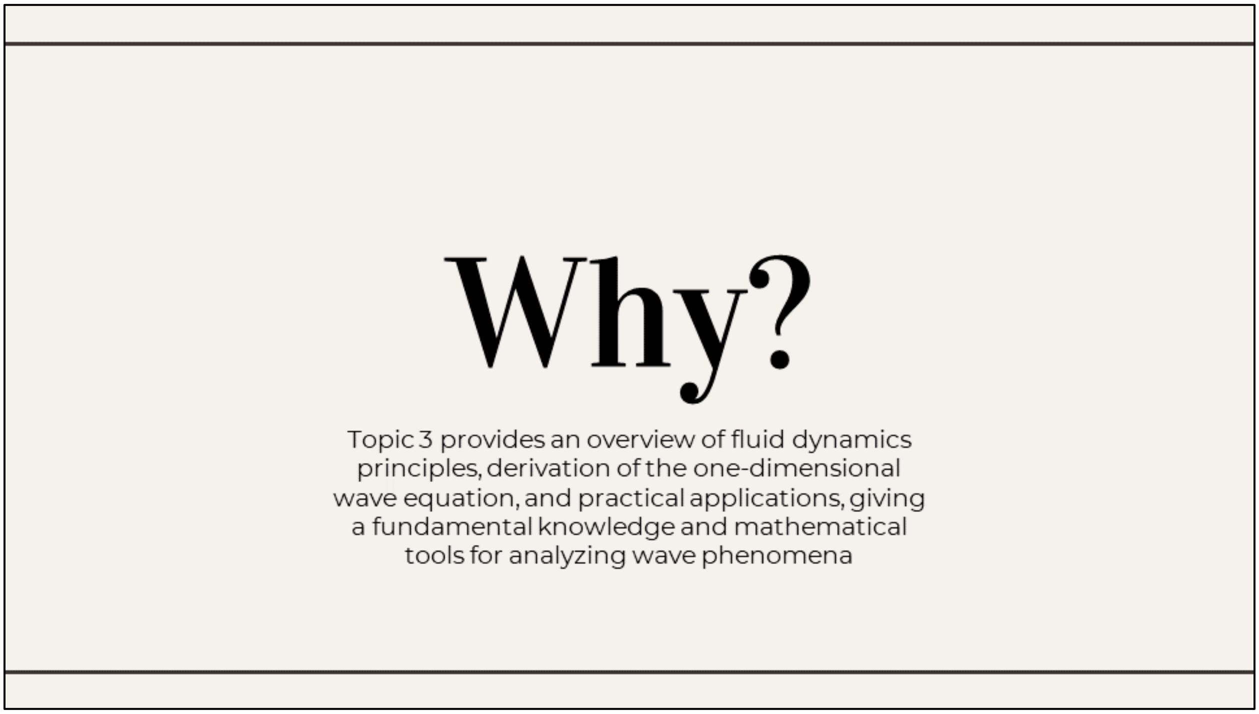
We will begin discussing this modeling method in terms of **analogous circuits** in a later lecture topic!

Appendix C – Lecture Topic 3 Slides



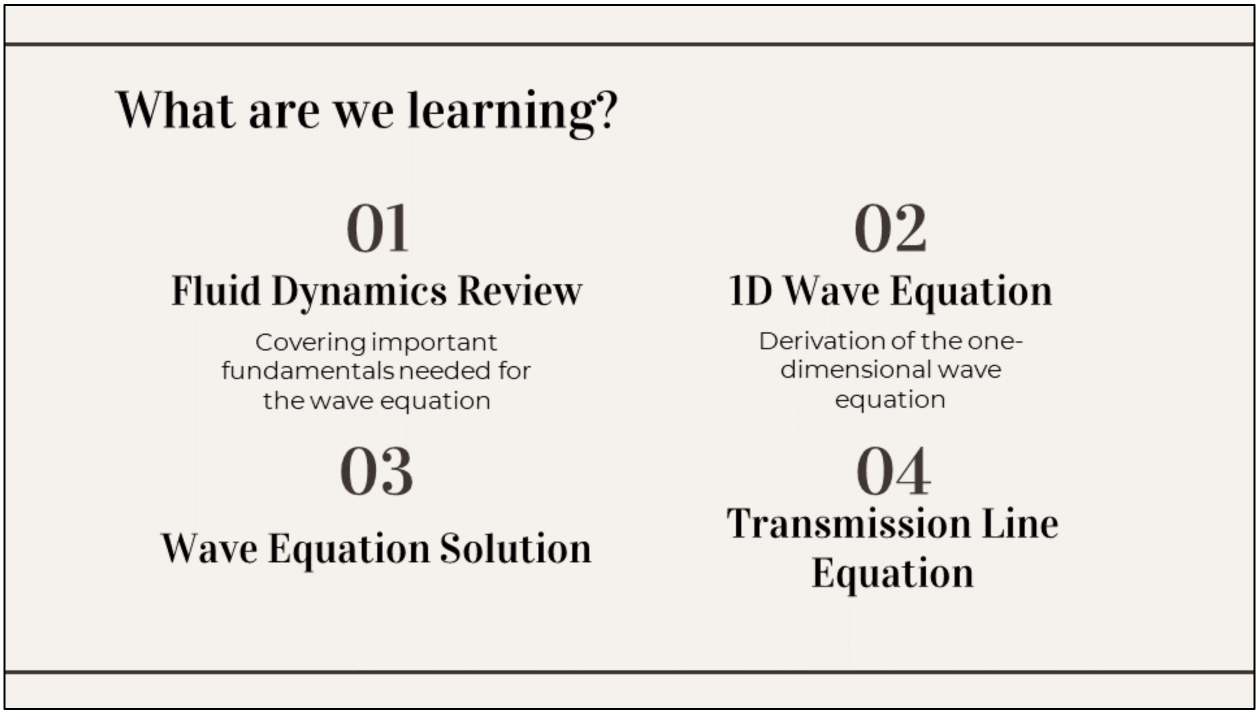
Lecture Topic 3

The Acoustic Wave Equation



Why?

Topic 3 provides an overview of fluid dynamics principles, derivation of the one-dimensional wave equation, and practical applications, giving a fundamental knowledge and mathematical tools for analyzing wave phenomena



What are we learning?

01

Fluid Dynamics Review

Covering important fundamentals needed for the wave equation

02

1D Wave Equation

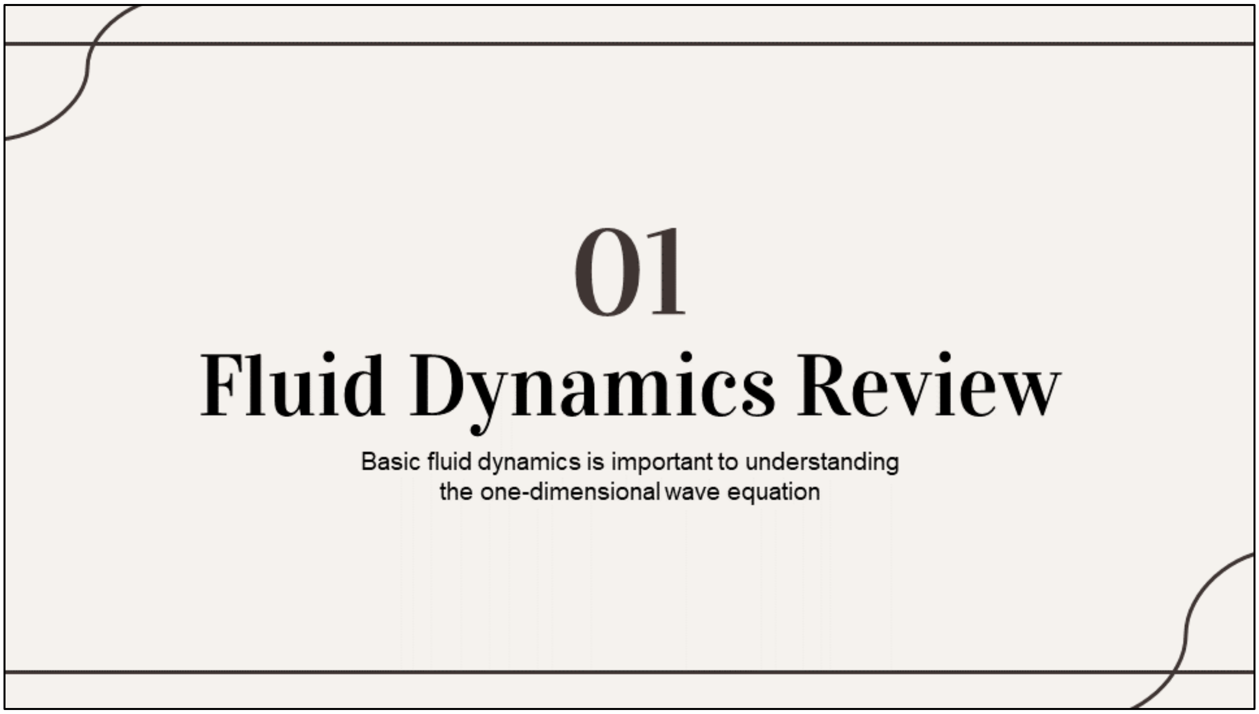
Derivation of the one-dimensional wave equation

03

Wave Equation Solution

04

Transmission Line Equation



01

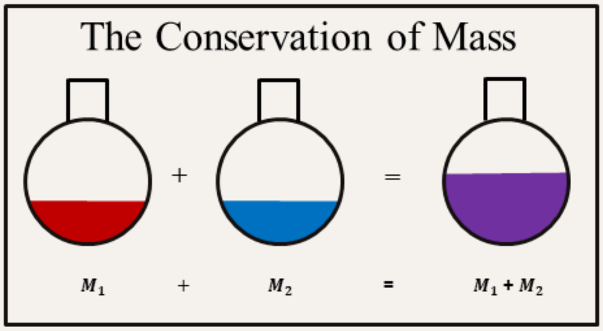
Fluid Dynamics Review

Basic fluid dynamics is important to understanding the one-dimensional wave equation

Fluid Dynamics Review

Why do we need fluid dynamics in acoustics?

Conservation Equations
- Mass
- Momentum
- Energy

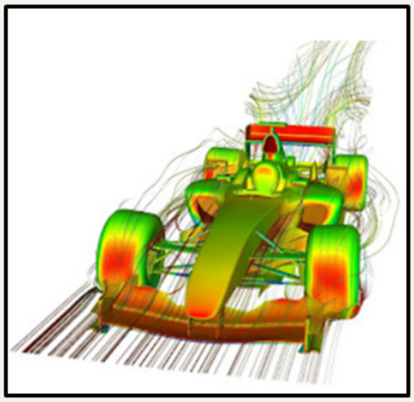

The Conservation of Mass

Fluid Dynamics Review

What is described in conservation equations?

Physical quantities
in incompressible and compressible fluids of:
- Density
- Pressure
- Velocity

Shows linearization methods that are applicable to the wave equation

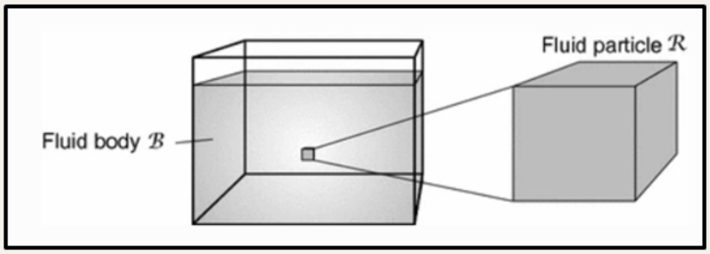


Fluid Dynamics Review

Physical quantities density, ρ, pressure, p, and velocity, v, are functions of space and time

$Density$: $\rho(x, y, z, t)$
$Pressure$: $p(x, y, z, t)$
$Velocity$: $v(x, y, z, t)$

3 Dimensions!



Fluid Dynamics Review

Using density as an example, the change in ρ in scalar quantities looks like:

$$d\rho = \left(\frac{\delta\rho}{\delta t}\right)dt + \left(\frac{\delta\rho}{\delta x}\right)dx + \left(\frac{\delta\rho}{\delta y}\right)dy + \left(\frac{\delta\rho}{\delta z}\right)dz$$

If change in time dt is divided into each term:

$$\frac{d\rho}{dt} = \frac{\delta\rho}{\delta t} + \frac{\delta\rho}{\delta x}\left(\frac{dx}{dt}\right) + \frac{\delta\rho}{\delta y}\left(\frac{dy}{dt}\right) + \frac{\delta\rho}{\delta z}\left(\frac{dz}{dt}\right)$$

This equation can be simplified using:

$$\frac{D}{Dt} = \frac{\delta}{\delta t} + v \cdot \nabla A$$

Let's discuss what this means

Fluid Dynamics Review

$$\frac{D}{Dt} = \frac{\delta}{\delta t} + v \cdot \nabla A$$

$\boldsymbol{\frac{D}{Dt}}$: Material derivative - the rate of change of a quantity with respect to time as it moves through a fluid

$\boldsymbol{v}$: Velocity vector of an individual fluid particle

$\boldsymbol{\nabla A}$: Gradient of some scalar quantity A

$\boldsymbol{\frac{\delta}{\delta t}}$: Partial derivative with respect to time

---

Fluid Dynamics Review

$$\frac{D}{Dt} = \frac{\delta}{\delta t} + v \cdot \nabla A$$

Here, the velocity vector, v, is a 3-Dimensional term:

$$v(x, y, z) = \frac{dx}{dt} + \frac{dy}{dt} + \frac{dz}{dt}$$

The gradient of A, ∇A, is defined as:

$$\nabla \cdot A = \frac{\delta A_x}{\delta x} + \frac{\delta A_y}{\delta y} + \frac{\delta A_z}{\delta z}$$

Fluid Dynamics Review

$$v(x,y,z) = \frac{dx}{dt} + \frac{dy}{dt} + \frac{dz}{dt} \qquad \nabla A = \frac{\delta A_x}{\delta x} + \frac{\delta A_y}{\delta y} + \frac{\delta A_z}{\delta z}$$

Therefore:

$$v \cdot \nabla A = \frac{\delta A_x}{\delta x}\left(\frac{dx}{dt}\right) + \frac{\delta A_y}{\delta y}\left(\frac{dy}{dt}\right) + \frac{\delta A_z}{\delta z}\left(\frac{dz}{dt}\right)$$

Looking back at Eq [] and Eq []

$$\frac{d\rho}{dt} = \frac{\delta\rho}{\delta t} \boxed{+ \frac{\delta\rho}{\delta x}\left(\frac{dx}{dt}\right) + \frac{\delta\rho}{\delta y}\left(\frac{dy}{dt}\right) + \frac{\delta\rho}{\delta z}\left(\frac{dz}{dt}\right)}$$

$$\frac{DA}{Dt} = \boxed{\frac{\delta A}{\delta t}} + \boxed{v \cdot \nabla A}$$

Fluid Dynamics Review

$$\frac{D}{Dt} = \frac{\delta}{\delta t} + v \cdot \nabla A$$

This equation can be used to describe the material derivative of any of the previously mentioned physical quantities (Density, ρ, pressure, p, and :

$$\frac{D\rho}{Dt} = \frac{\delta\rho}{\delta t} + v \cdot \nabla\rho$$

$$\frac{Dp}{Dt} = \frac{\delta p}{\delta t} + v \cdot \nabla p$$

$$\frac{DV}{Dt} = \frac{\delta V}{\delta t} + v \cdot \nabla V$$

In this equation, V represents the velocity field of the fluid. This describes how the velocity of the fluid varies from point to point.

Fluid Dynamics Review

$$\frac{D\rho}{Dt} = \frac{\delta\rho}{\delta t} + v \cdot \nabla\rho \qquad \frac{Dp}{Dt} = \frac{\delta p}{\delta t} + v \cdot \nabla p \qquad \frac{DV}{Dt} = \frac{\delta V}{\delta t} + v \cdot \nabla V$$

Write out the second term on the left-side of the equation for each of the equations above

$$v(x, y, z) = \frac{dx}{dt} + \frac{dy}{dt} + \frac{dz}{dt} \ and \ \nabla \cdot A = \frac{\delta A_x}{\delta x} + \frac{\delta A_y}{\delta y} + \frac{\delta A_z}{\delta z}$$

Therefore:

$$\frac{D\rho}{Dt} = \frac{\delta\rho}{\delta t} + \frac{\delta\rho}{\delta x}\left(\frac{dx}{dt}\right) + \frac{\delta\rho}{\delta y}\left(\frac{dy}{dt}\right) + \frac{\delta\rho}{\delta z}\left(\frac{dz}{dt}\right)$$

$$\frac{Dp}{Dt} = \frac{\delta p}{\delta t} + \frac{\delta p}{\delta x}\left(\frac{dx}{dt}\right) + \frac{\delta p}{\delta y}\left(\frac{dy}{dt}\right) + \frac{\delta p}{\delta z}\left(\frac{dz}{dt}\right)$$

$$\frac{DV}{Dt} = \frac{\delta V}{\delta t} + \frac{\delta V}{\delta x}\left(\frac{dx}{dt}\right) + \frac{\delta V}{\delta y}\left(\frac{dy}{dt}\right) + \frac{\delta V}{\delta z}\left(\frac{dz}{dt}\right)$$

Fluid Dynamics Review

Just discussed…

How properties in fluids (Ex. density) can be described in a fluid body

Now…

Discuss how continuity equations are used in fluid dynamics/fluid bodies

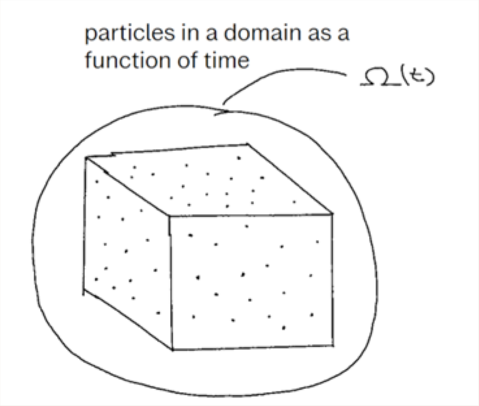
Fluid Dynamics Review

Spatial reference systems, or coordinate systems, serve to define positions within a fluid medium

Oftentimes in fluids, the geometry or boundaries of the fluid domain changes over time

This introduces a new variable, $\Omega(t)$, which serves to describe how the coordinate system changes over time

- We will be using $\Omega(t)$ as a volume term in the following slides



Fluid Dynamics Review

Fluid dynamics considers gas and liquids as a continuum

When sound disturbs a gaseous medium, mass is conserved

- No change in mass relative to space and time

As we know, $\rho = \frac{M}{V}$ and $\mathrm{M} = \rho V$

$$M = \int_{\Omega} \rho(x,t)\, dx$$

Fluid Dynamics Review

$$M = \int_{\Omega} \rho(x,t)\, dx$$

Recall, $\frac{DA}{Dt} = \frac{\delta A}{\delta t} + v \cdot \nabla A$

$$\frac{DM}{Dt} = \int_{\Omega} \left(\frac{\delta \rho}{\delta t} + v \cdot \nabla \rho \right) dx = 0$$

The above equation is the mass conservation equation and by using assumptions like this, we can greatly simplify the mathematics

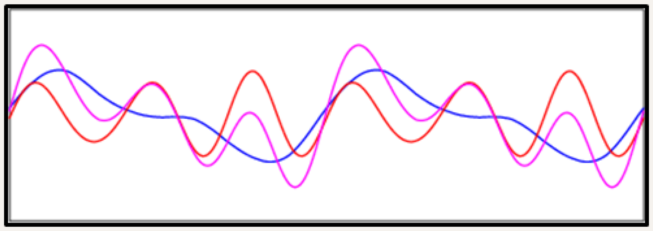
We will see very similar operations in the derivation of the wave equation

---

02

Acoustic Wave Equation

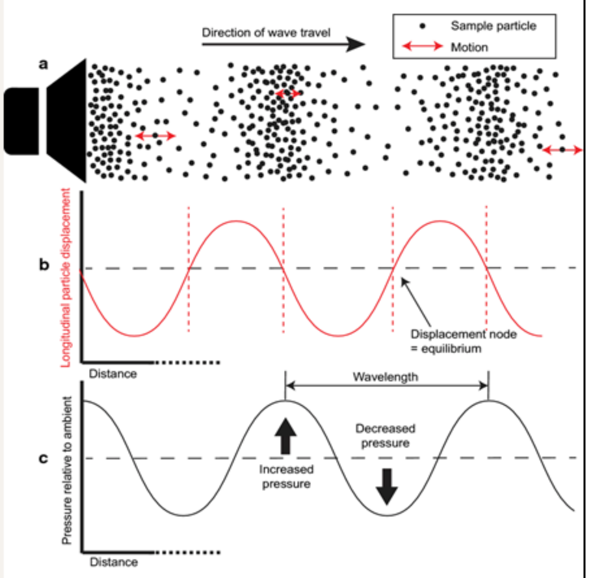
We'll derive the one-dimensional wave equation and practice your understanding with some applications



Three Fundamental Variables

We will begin by describing three key variables:

- Pressure, p
- Density, ρ
- Particle velocity, u



Pressure

$p_T(x,t)$ = Total pressure
p_0 = atmospheric pressure
$p(x,t)$ = incremental change in pressure

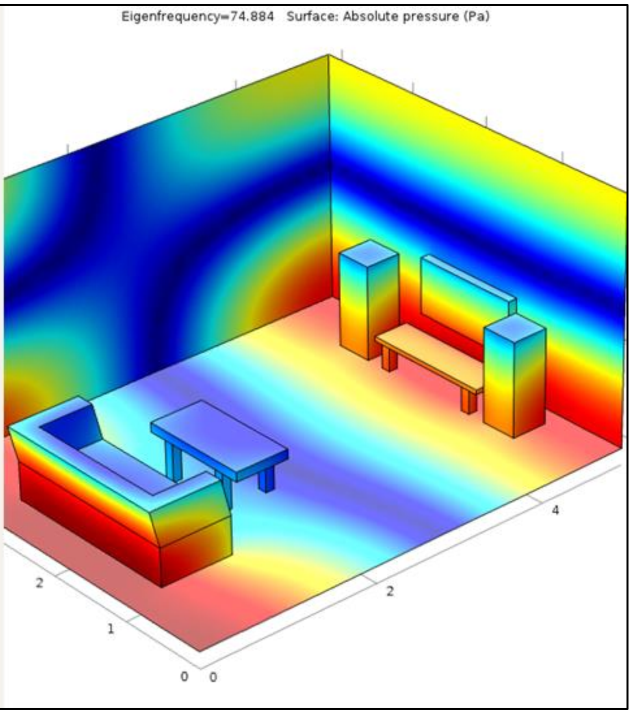
Averaged value (Constant term)

Time-varying term

$$\mathbf{p_T(x,t) = p_0 + p(x,t)}$$

Measured in $\frac{N}{m^2}$ = Pascals (Pa)

★★★ *Most quantities will take this form*



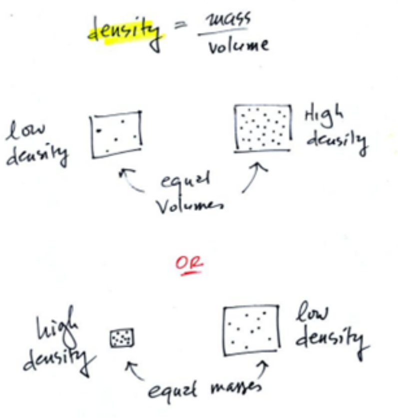
Density

$\rho_T(\mathrm{x,t})$ = Total density
ρ_0 = density under normal conditions
$\rho(\mathrm{x,t})$ = incremental change in density

$$\boldsymbol{\rho_T(x,t) = \rho_0 + \rho(x,t)}$$

Measured in $\frac{\mathrm{kg}}{\mathrm{m}^3}$

★★ *Same Form!*



Particle Velocity

externalities which affect particle velocity in a fluid

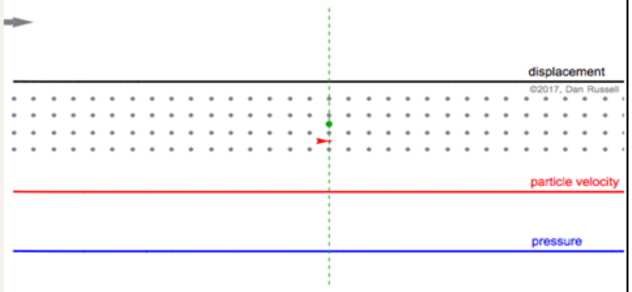
$\mathrm{u_T(x,t)}$ = Total particle velocity
$\mathrm{u_0}$ = bias term
$\mathrm{u(x,t)}$ = incremental change in particle velocity

For our purposes, this can be assumed to be negligible

$$\mathbf{u_T(x,t)} = \overset{\mathbf{0}}{\cancel{\mathbf{u_0}}} + \mathbf{u(x,t)}$$

$$\mathbf{u_T(x,t) = u(x,t)}$$

Measured in $\frac{\mathrm{m}}{\mathrm{s}}$



Where does the 1D Wave Equation Come From?

Pressure: $\mathbf{p_T(x,t) = p_0 + p(x,t)}$
Density: $\boldsymbol{\rho_T(x,t) = \rho_0 + \rho(x,t)}$
Particle Velocity: $\mathbf{u_T(x,t) = u(x,t)}$

The 1D wave equation relates these quantities using 3 fundamental equations:

Newton's Second Law of Motion
The Gas Law
The Continuity Equation

Let's dive into each individually…

Newton's Second Law of Motion

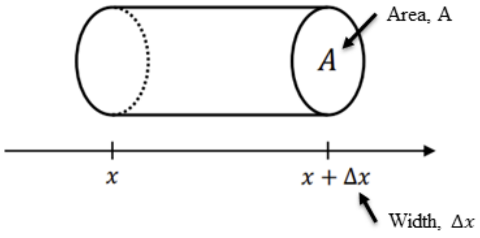
Consider a small volume of air depicted to the right

Apply Newton's Second Law ($F = Ma$); the external forces, F_{ext}, act to accelerate this volume in the positive x-direction:

$u_T(x,t) = u(x,t)$

$$\sum \mathbf{F_{ext}} = \boldsymbol{Ma} = \boldsymbol{M}\frac{\mathbf{Du_T}}{\mathbf{Dt}} = \boldsymbol{M}\frac{\mathbf{Du}}{\mathbf{Dt}}$$

Recall, $\frac{Du}{Dt}$ is the material derivative



$$\sum \mathbf{F_{ext}} = \boldsymbol{M}\frac{\mathbf{Du}}{\mathbf{Dt}}$$

The sum of the external forces, f, is equal to the difference of the two pressures, p, pushing on each side of an air particle, multiplied by the area, A:

$$\mathbf{M} = \boldsymbol{\rho V \; and \; V = A\Delta x}$$

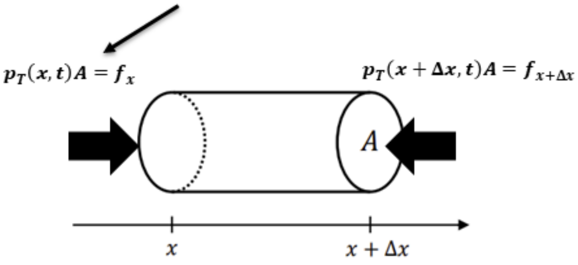
Therefore:

$$\boldsymbol{M}\frac{\mathbf{Du}}{\mathbf{Dt}} = \boldsymbol{A\Delta x \rho_T}\frac{\boldsymbol{Du}}{\boldsymbol{Dt}}$$

$$\boldsymbol{A\Delta x \rho_T}\frac{\boldsymbol{Du}}{\boldsymbol{Dt}} = \mathbf{p_T(x,t)A - p_T(x+\Delta x,t)}\boldsymbol{A}$$

$$\mathbf{p_T(x,t) - p_T(x+\Delta x,t) \; = \; \Delta x \rho_T \frac{Du}{Dt}}$$

$$pressure = \frac{force}{Area} \rightarrow f = p * A$$



$$\mathbf{p_T(x,t) - p_T(x+\Delta x,t) \; = \; \Delta x \rho_T \frac{Du}{Dt}}$$

$$\mathbf{p_T(x+\Delta x,t) - p_T(x,t) \; = \; -\Delta x \rho_T \frac{Du}{Dt}}$$

$$\mathbf{\frac{p_T(x+\Delta x,t) - p_T(x,t)}{\Delta x} \; = -\rho_T \frac{Du}{Dt}}$$

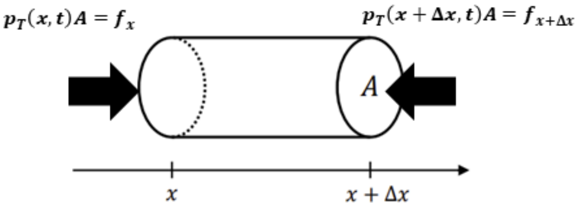
$$\mathbf{\frac{p_T(x+\Delta x,t) - p_T(x,t)}{\Delta x} \; = -\rho_T \frac{Du}{Dt}}$$

To take this discrete representation and model the pressure gradient of the system, we shrink the Δx to zero which gives us the partial derivative in space:

$$\mathbf{\lim_{\Delta x \to 0}\left(\frac{p_T(x+\Delta x,t) - p_T(x,t)}{\Delta x}\right) = \frac{\delta p_T}{\delta x} = -\rho_T \frac{Du}{Dt}}$$

Now that Δx is infinitesimally small, $p_T(x,t) - p_T(x+\Delta x, t)$ is negligible and the equation can be written as:

$$\mathbf{\frac{\delta p_T(x,t)}{\delta x} = -\rho_T(x,t)\frac{Du(x,t)}{Dt}}$$



$$\frac{\boldsymbol{\delta p_T(x,t)}}{\boldsymbol{\delta x}} = -\boldsymbol{\rho_T(x,t)}\frac{\mathbf{Du(x,t)}}{\mathbf{Dt}}$$

As mentioned previously, $\rho_T(x,t) = \rho_0 + \rho(x,t)$

For disturbances that are less than 110dB, or $20\mu Pa$, it is generally accepted that $\rho(x,t) \ll \rho_0$ such that $\rho_T(x,t) \approx \rho_0$

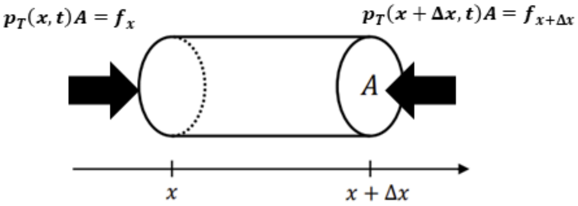
$$\frac{\boldsymbol{\delta p_T}}{\boldsymbol{\delta x}} = -\boldsymbol{\rho_0}\frac{\mathbf{Du}}{\mathbf{Dt}}$$

Let us now take a closer look at the material derivative, $\frac{Du}{Dt}$

$$\frac{\mathbf{Du}}{\mathbf{Dt}} = \frac{\boldsymbol{\delta u}}{\boldsymbol{\delta t}} + \frac{\boldsymbol{\delta u}}{\boldsymbol{\delta x}}\mathbf{u}$$

This is called the "convection term" of the material derivative

Describes the velocity of a particle as it moves through its surrounding fluid



$$\frac{\mathbf{Du}}{\mathbf{Dt}} = \frac{\boldsymbol{\delta u}}{\boldsymbol{\delta t}} + \frac{\boldsymbol{\delta u}}{\boldsymbol{\delta x}}\mathbf{u}$$

Using the small disturbance approximation mentioned before (Less than 110dB), it turns out that $\left|\frac{\delta u}{\delta t}\right|_{max} \gg \left|\frac{\delta u}{\delta x}u\right|_{max}$ and therefore the convection term can be considered negligible.

With this, Newton's 2nd Law can finally be written as:

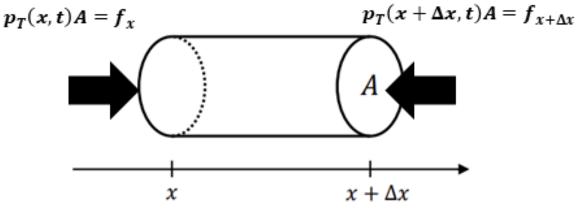
$$\boxed{\frac{\boldsymbol{\delta p}}{\boldsymbol{\delta x}} = -\boldsymbol{\rho_0}\frac{\boldsymbol{\delta u}}{\boldsymbol{\delta t}}}$$

Thinking intuitively, this equation makes sense!

In terms of units:

$$\frac{F}{A}\cdot\frac{1}{x} = -\frac{M}{V}\cdot\frac{\frac{m}{s}}{s} = -M\cdot\frac{m}{s^2} \leftrightarrow f = -M * a$$

Newton's Second Law!



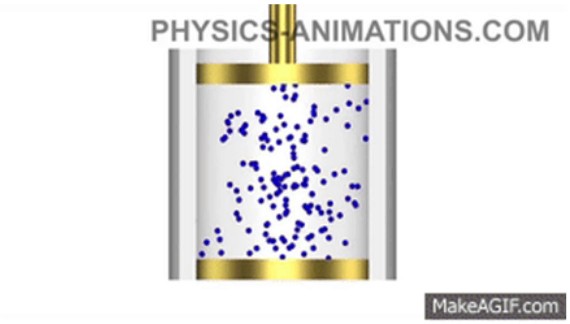
The Adiabatic Gas Law

p_T is the total pressure within the volume
V is the total volume
n is the amount of subtance in mol
R is the universal gas constant $= 8.314 \frac{J}{mol * K}$
T is temperature measured in °K

Ideal Gas Law : $\mathbf{p_T V = nRT}$

Before moving on, we must examine how temperature varies with changes to pressure and volume in a gas:

Compressed gas → temperature rise
Expanded gas → temperature drop



Acoustically, compressions and rarefactions from a sonic disturbance will cause temperature variations

In the case of sound waves, the changes in temperature that occur remain within the system; this characterizes the system as adiabatic

For an adiabatic system:

$$\mathbf{p_T V^\gamma = constant}$$

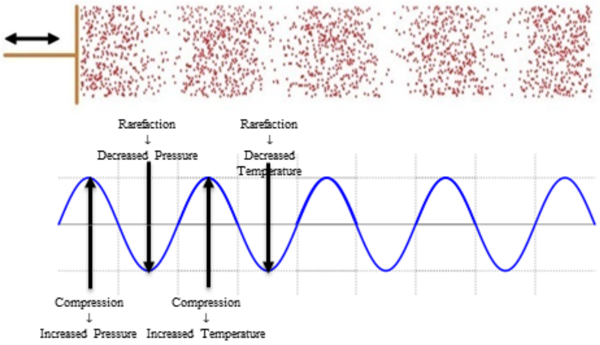
Where γ is the ratio of specific heats; $\gamma = 1.4$ for air

We'll introduce density and rewrite the equation as:

$$\mathbf{p_T \left(\frac{m}{\rho_T}\right)^\gamma = constant}$$

$$\mathbf{\frac{p_T}{{\rho_T}^\gamma} = \frac{constant}{M^\gamma}}$$

Longitudinal or compression wave



$$\frac{\mathbf{p_T}}{\boldsymbol{\rho_T}^{\gamma}} = \frac{\textbf{constant}}{M^{\gamma}} = \boldsymbol{constant}$$

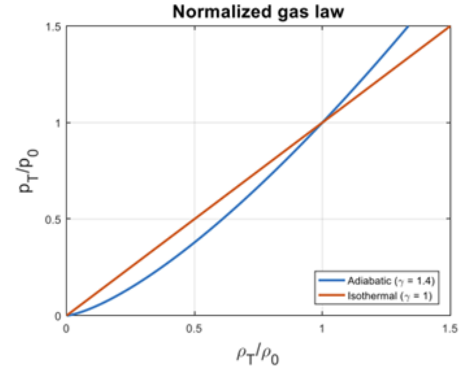
Since a constant mass is assumed, the mass can be lumped into the constant in the numerator

This constant is estimated by substituting atmospheric values for pressure and density:

$$\frac{\mathbf{p_T}}{\boldsymbol{\rho_T}^{\gamma}} = \frac{\mathbf{p_0}}{\boldsymbol{\rho_0}^{\gamma}}$$

$$\frac{\mathbf{p_T}}{\mathbf{p_0}} = \frac{\boldsymbol{\rho_T}^{\gamma}}{\boldsymbol{\rho_0}^{\gamma}}$$

$$\frac{\mathbf{p_T}}{\mathbf{p_0}} = \left(\frac{\rho_T}{\rho_0}\right)^{\gamma}$$



$$\frac{\mathbf{p_T}}{\mathbf{p_0}} = \left(\frac{\rho_T}{\rho_0}\right)^{\gamma} \leftrightarrow \boldsymbol{Y = X^{\gamma}}$$

If this system were isothermal, γ would have a value of 1 making the above expression linear

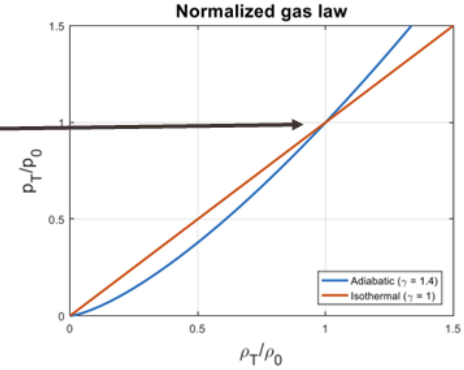
By graphing these ratios against one another we can see that both p_T and ρ_T are equivalent to p_0 and ρ_0 respectively, with slight variations around an operating point

To construct the wave equation, it's viable to approximate or linearize this equation around the established operating point

To find this linear relationship, we calculate the slope and evaluate at the operating point $X = 1$ and $Y = 1$:

$$\frac{dY}{dX} = \gamma X^{\gamma - 1}$$

$$\left.\frac{dY}{dX}\right|_{X=1} = \gamma$$



$$\left.\frac{\mathbf{dY}}{\mathbf{dX}}\right|_{\mathbf{X=1}} = \gamma$$

Here, we are interested in the ΔY as a function of ΔX, where $Y = 1 + \mathrm{d}Y$ and $X = 1 + \mathrm{d}X$

We will simply relate them by slope

$$\mathbf{\Delta Y \approx \gamma \Delta X}$$

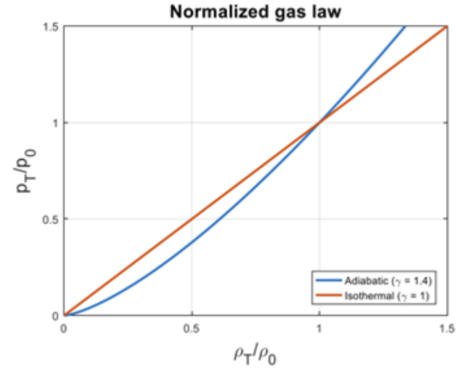
Recall,

$$Y = \frac{p_T}{p_0} \;\; and \;\; X = \frac{\rho_T}{\rho_0}$$

$$\Delta Y = \frac{1}{p_0}\Delta p_T \;\; and \;\; \Delta X = \frac{1}{\rho_0}\Delta \rho_T$$

According to the small disturbance approximation, $|p| \ll p_0$ and $|\rho| \ll \rho_0$ and make the following approximation:

$$\frac{\mathbf{p}}{\mathbf{p_0}} \approx \gamma \frac{\boldsymbol{\rho}}{\boldsymbol{\rho_0}}$$



$$\frac{\mathbf{p}}{\mathbf{p_0}} \approx \gamma \frac{\boldsymbol{\rho}}{\boldsymbol{\rho_0}}$$

Taking the partial derivative of this yields the final form of the equation we will use:

$$\boxed{\frac{\delta}{\delta t}\left(\frac{\mathbf{p}}{\mathbf{p_0}}\right) = \gamma \frac{\delta}{\delta t}\left(\frac{\boldsymbol{\rho}}{\boldsymbol{\rho_0}}\right)}$$

The Continuity Equation

The final equation comes from the conservation of mass equation, also known as the continuity equation

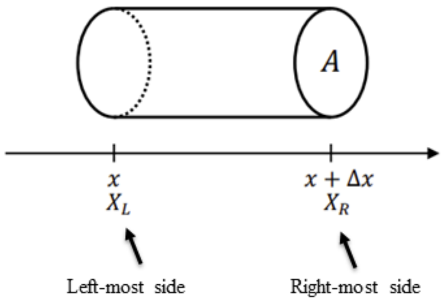
While the air particle alters its shape and position, its mass can be calculated by multiplying density with volume:

$$\mathbf{M} = \boldsymbol{\rho V} = \boldsymbol{\rho}_{\mathbf{T}}\mathbf{A}(\mathbf{X}_{\mathbf{R}} - \mathbf{X}_{\mathbf{L}})$$

We now take the partial derivative of the above equation:

Mass is constant $\longrightarrow$ $\frac{\boldsymbol{\delta M}}{\boldsymbol{\delta}\mathbf{t}} = \mathbf{0} = \frac{\boldsymbol{\delta}}{\boldsymbol{\delta}\mathbf{t}}(\boldsymbol{\rho}_{\mathbf{T}}\mathbf{A}(\mathbf{X}_{\mathbf{R}} - \mathbf{X}_{\mathbf{L}}))$

$$\mathbf{0} = \frac{\boldsymbol{\delta\rho}_{\mathbf{T}}}{\boldsymbol{\delta}\mathbf{t}}(\mathbf{X}_{\mathbf{R}} - \mathbf{X}_{\mathbf{L}}) + \boldsymbol{\rho}_{\mathbf{T}}\left(\frac{\boldsymbol{\delta}\mathbf{X}_{\mathbf{R}}}{\boldsymbol{\delta}\mathbf{t}} - \frac{\boldsymbol{\delta}\mathbf{X}_{\mathbf{L}}}{\boldsymbol{\delta}\mathbf{t}}\right)$$



$$\mathbf{0} = \frac{\boldsymbol{\delta\rho}_{\mathbf{T}}}{\boldsymbol{\delta}\mathbf{t}}(\mathbf{X}_{\mathbf{R}} - \mathbf{X}_{\mathbf{L}}) + \boldsymbol{\rho}_{\mathbf{T}}\left(\frac{\boldsymbol{\delta}\mathbf{X}_{\mathbf{R}}}{\boldsymbol{\delta}\mathbf{t}} - \frac{\boldsymbol{\delta}\mathbf{X}_{\mathbf{L}}}{\boldsymbol{\delta}\mathbf{t}}\right)$$

Notice that for X_R and X_L the partial derivatives $\frac{\delta X_R}{\delta t}$ and $\frac{\delta X_L}{\delta t}$ are simply the normal derivative:

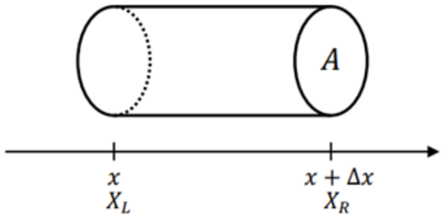
$$\frac{\boldsymbol{\delta}\mathbf{X}_{\mathbf{R}}}{\boldsymbol{\delta}\mathbf{t}} = \frac{\mathbf{dX}_{\mathbf{R}}}{\mathbf{dt}} = \mathbf{U}_{\mathbf{R}}$$

$$\frac{\boldsymbol{\delta}\mathbf{X}_{\mathbf{L}}}{\boldsymbol{\delta}\mathbf{t}} = \frac{\mathbf{dX}_{\mathbf{L}}}{\mathbf{dt}} = \mathbf{U}_{\mathbf{L}}$$

Using the $\rho_T \approx \rho_0$ approximation once again, the top-most equation can be rewritten:

$$\mathbf{0} = \frac{\boldsymbol{\delta\rho}}{\boldsymbol{\delta}\mathbf{t}}(\mathbf{X}_{\mathbf{R}} - \mathbf{X}_{\mathbf{L}}) + \boldsymbol{\rho}_{\mathbf{0}}(\mathbf{U}_{\mathbf{R}} - \mathbf{U}_{\mathbf{L}})$$

$$\frac{\boldsymbol{\delta\rho}}{\boldsymbol{\delta}\mathbf{t}}(\mathbf{X}_{\mathbf{R}} - \mathbf{X}_{\mathbf{L}}) = -\boldsymbol{\rho}_{\mathbf{0}}(\mathbf{U}_{\mathbf{R}} - \mathbf{U}_{\mathbf{L}})$$



$$\frac{\delta\rho}{\delta t}(X_R - X_L) = -\rho_0(U_R - U_L)$$

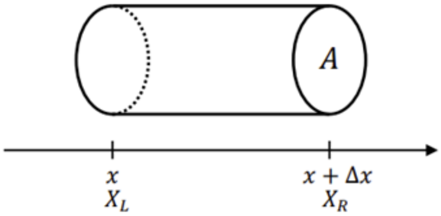
$$\frac{1}{\rho_0}\frac{\delta\rho}{\delta t} = -\frac{U_R - U_L}{X_R - X_L}$$

$$\frac{\delta}{\delta t}\left(\frac{\rho}{\rho_0}\right) = -\frac{u(x+\Delta x, t) - u(x,t)}{\Delta X}$$

By taking the limit as $\Delta x \to 0$, we shrink the Δx to zero gives us the partial derivative in space:

$$\frac{\delta}{\delta t}\left(\frac{\rho}{\rho_0}\right) = -\lim_{\Delta x \to 0}\frac{u(x+\Delta x, t) - u(x,t)}{\Delta X}$$

$$\boxed{\frac{\delta}{\delta t}\left(\frac{\rho}{\rho_0}\right) = -\frac{\delta u}{\delta x}}$$



Bringing it Together

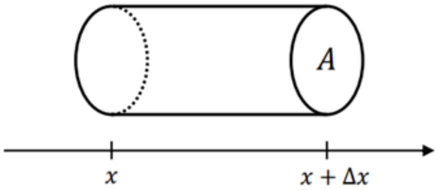
Newton's 2nd Law: $\frac{\delta p}{\delta x} = -\rho_0 \frac{\delta u}{\delta t}$

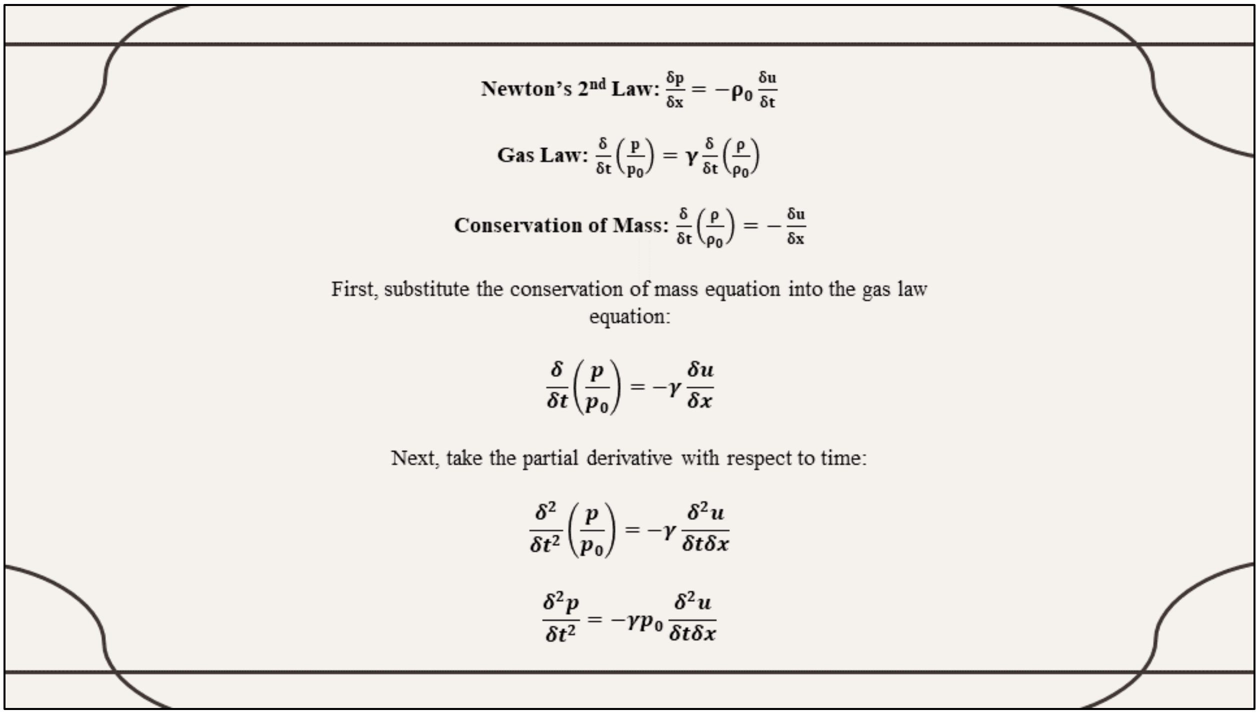
Gas Law: $\frac{\delta}{\delta t}\left(\frac{p}{p_0}\right) = \gamma \frac{\delta}{\delta t}\left(\frac{\rho}{\rho_0}\right)$

Conservation of Mass: $\frac{\delta}{\delta t}\left(\frac{\rho}{\rho_0}\right) = -\frac{\delta u}{\delta x}$

We now must combine these equations to extract the wave equation in terms of 1 of the 3 variables (pressure, density, and particle velocity)

We will find the wave equation in terms of pressure





Newton's 2nd Law: $\frac{\delta p}{\delta x} = -\rho_0 \frac{\delta u}{\delta t}$

Gas Law: $\frac{\delta}{\delta t}\left(\frac{p}{p_0}\right) = \gamma \frac{\delta}{\delta t}\left(\frac{\rho}{\rho_0}\right)$

Conservation of Mass: $\frac{\delta}{\delta t}\left(\frac{\rho}{\rho_0}\right) = -\frac{\delta u}{\delta x}$

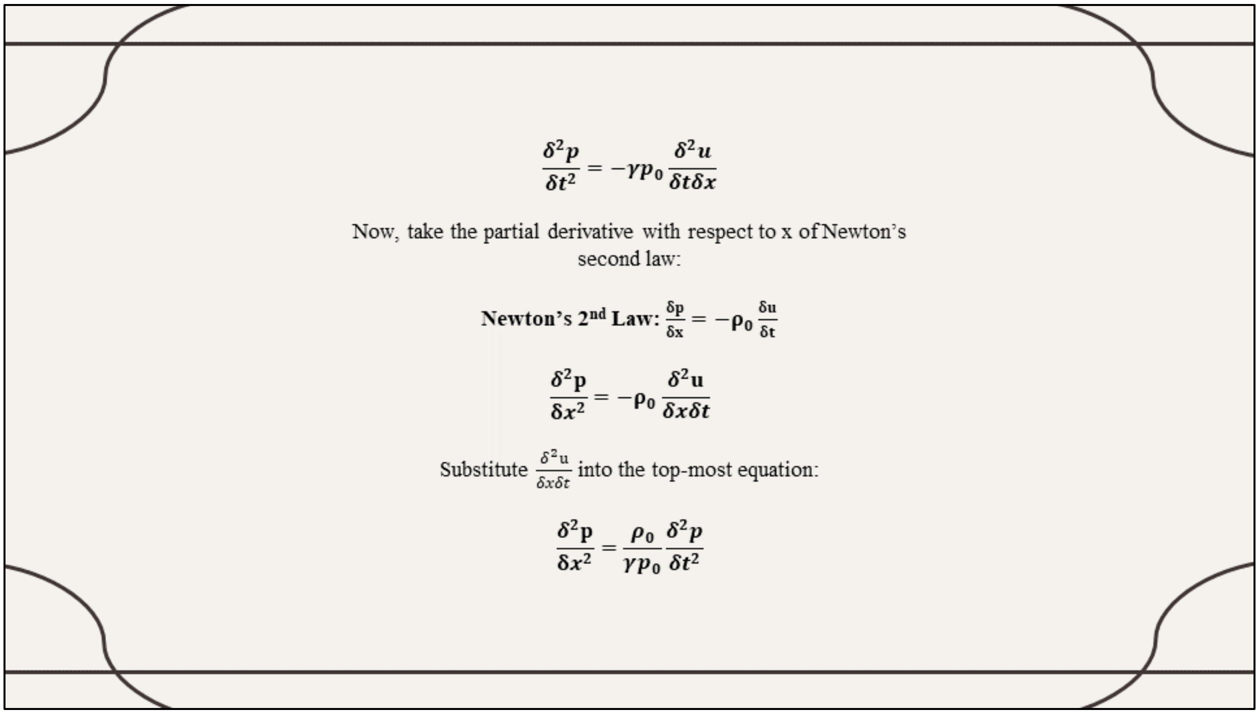
First, substitute the conservation of mass equation into the gas law equation:

$$\frac{\delta}{\delta t}\left(\frac{p}{p_0}\right) = -\gamma \frac{\delta u}{\delta x}$$

Next, take the partial derivative with respect to time:

$$\frac{\delta^2}{\delta t^2}\left(\frac{p}{p_0}\right) = -\gamma \frac{\delta^2 u}{\delta t \delta x}$$

$$\frac{\delta^2 p}{\delta t^2} = -\gamma p_0 \frac{\delta^2 u}{\delta t \delta x}$$



$$\frac{\delta^2 p}{\delta t^2} = -\gamma p_0 \frac{\delta^2 u}{\delta t \delta x}$$

Now, take the partial derivative with respect to x of Newton's second law:

Newton's 2nd Law: $\frac{\delta p}{\delta x} = -\rho_0 \frac{\delta u}{\delta t}$

$$\frac{\delta^2 p}{\delta x^2} = -\rho_0 \frac{\delta^2 u}{\delta x \delta t}$$

Substitute $\frac{\delta^2 u}{\delta x \delta t}$ into the top-most equation:

$$\frac{\delta^2 p}{\delta x^2} = \frac{\rho_0}{\gamma p_0} \frac{\delta^2 p}{\delta t^2}$$

One- Dimensional Acoustic Wave Equation

$$\frac{\delta^2 \mathbf{p}}{\delta x^2} = \frac{\rho_0}{\gamma p_0}\frac{\delta^2 p}{\delta t^2} \text{ OR } \frac{\delta^2 \mathbf{p}}{\delta x^2} = \frac{1}{c^2}\frac{\delta^2 p}{\delta t^2}$$

Now, let's discuss solutions to the acoustic wave equation

03

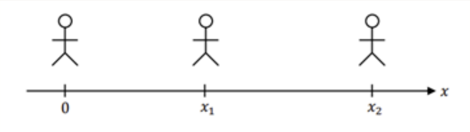
Wave Equation Solution

Now that we've derived the 1D Acoustic Wave Equation, let's find its solution

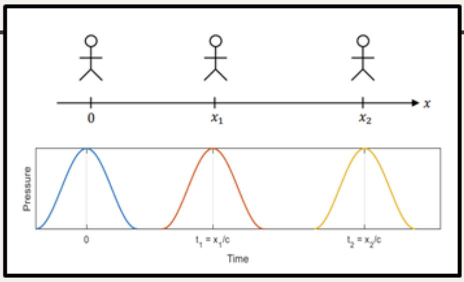
Solution Form

Before doing a formal derivation, let's intuitively construct its form

Consider the figure below showing three people located at $x = 0, x_1$, and x_2:



What will the difference in perceived sound be between person x_1 and person x_2?



The sound will arrive later to person x_2 than person x_1 shown in the above plot

Here we set time equal to the distance, x, over the speed of sound, c

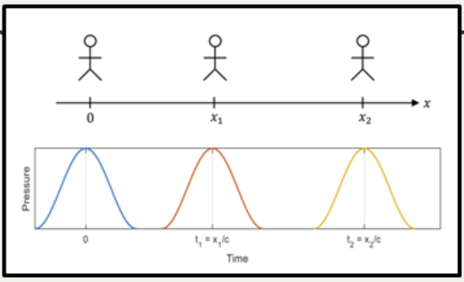
If the waveform at $x = 0$ is set equal to:

$$p(x = 0, t) = f(t)$$

Then the waveform heard by person x_1 and x_2 will be:

$$p(x = x_1, t) = f(t - t_1) = f\left(t - \frac{x_1}{c}\right)$$

$$p(x = x_2, t) = f(t - t_2) = f\left(t - \frac{x_2}{c}\right)$$



To generalize this to any continuous sound pressure waveform travelling in the positive x-direction:

$$p(x,t) = f_+\left(t - \frac{x}{c}\right)$$

And in the negative x-direction:

$$p(x,t) = f_-\left(t - \frac{x}{c}\right)$$

Solving The Wave Equation

- As previously derived the 1-D wave equation in pressure is defined as:

$$\frac{\delta^2 p(x,t)}{\delta x^2} = \frac{1}{c^2}\frac{\delta^2 p(x,t)}{\delta t^2}$$

- We'll solve this using the method of separation of variables whose solution takes the form:

$$p(x,t) = X(x)T(t)$$

- Plugging this into the wave equation yields:

$$\frac{\delta^2}{\delta x^2}(X(x)T(t)) = \frac{1}{c^2}\frac{\delta^2}{\delta t^2}(X(x)T(t))$$

Solving The Wave Equation

- As previously derived the 1-D wave equation in pressure is defined as:

$$\frac{\delta^2 p(x,t)}{\delta x^2} = \frac{1}{c^2}\frac{\delta^2 p(x,t)}{\delta t^2}$$

- We’ll solve this using the method of separation of variables whose solution takes the form:

$$p(x,t) = X(x)T(t)$$

- Plugging this into the wave equation yields:

$$\frac{\delta^2}{\delta x^2}(X(x)T(t)) = \frac{1}{c^2}\frac{\delta^2}{\delta t^2}(X(x)T(t))$$

$$\frac{\delta^2}{\delta x^2}(X(x)T(t)) = \frac{1}{c^2}\frac{\delta^2}{\delta t^2}(X(x)T(t))$$

$$T(t)\frac{\delta^2 X(x)}{\delta x^2} = \frac{1}{c^2}X(x)\frac{\delta^2 T(t)}{\delta t^2}$$

$$\frac{1}{X(x)}\frac{\delta^2 X(x)}{\delta x^2} = \frac{1}{c^2}\frac{1}{T(t)}\frac{\delta^2 T(t)}{\delta t^2}$$

- The variables have now been separated to either side of the equation
- Since each side of the equation holds for any x or t, each side must equal a constant

- To proceed, we'll make an educated guess at what the constant is:

$$\frac{1}{X(x)}\frac{\delta^2 X(x)}{\delta x^2} = \frac{1}{c^2}\frac{1}{T(t)}\frac{\delta^2 T(t)}{\delta t^2} = constant = -\frac{\omega^2}{c^2}$$

- Where ω is the real angular frequency and c is the wave speed.
- This guess is made for convenience
- We will now continue with each side of the equation beginning with the left:

$$\frac{1}{X(x)}\frac{\delta^2 X(x)}{\delta x^2} + \frac{\omega^2}{c^2} = 0$$

$$\frac{\delta^2 X(x)}{\delta x^2} + \frac{\omega^2}{c^2}X(x) = 0$$

$$\frac{\delta^2 X(x)}{\delta x^2} + \frac{\omega^2}{c^2}X(x) = 0$$

$X(x)$ will take the following complex form:

$$X(x) = X_0 e^{\alpha x}$$

Where X_0 and α are both complex. This yields:

$$\frac{\delta^2 X_0 e^{\alpha x}}{\delta x^2} + \frac{\omega^2}{c^2}X_0 e^{\alpha x} = 0$$

$$X_0 \alpha^2 e^{\alpha x} + \frac{\omega^2}{c^2}X_0 e^{\alpha x} = 0$$

$$X_0 e^{\alpha x}\left(\alpha^2 + \frac{\omega^2}{c^2}\right) = 0$$

$$X_0 e^{\alpha x}\left(\alpha^2 + \frac{\omega^2}{c^2}\right) = 0$$

- Analyzing this equation, we can see that setting $X_0 e^{\alpha x} = 0$ provides a trivial solution as it gives no information about the wave equation
- For this reason, we are only interested in solution in which $X_0 e^{\alpha x} \neq 0$

$$\alpha^2 + \frac{\omega^2}{c^2} = 0$$

- Given that ω and c are real, the two solutions are:

$$\alpha = \pm j\frac{\omega}{c} = \pm jk$$

$$\alpha = \pm j\frac{\omega}{c} = \pm jk$$

- Here, $k = \frac{\omega}{c}$ is known as the wave number
- The wave number (units of m^{-1}) can be thought of as the spatial counterpart to frequency (units of s^{-1}).
- This leaves:

$$X(x) = X_0 e^{\alpha x} = X_0 e^{\pm j\frac{\omega x}{c}} = X_0 e^{\pm jkx}$$

$$X(x) = X_0 e^{\pm jkx}$$

- Now let's work on the right side of the equation:

$$\frac{1}{c^2}\frac{1}{T(t)}\frac{\delta^2 T(t)}{\delta t^2}+\frac{\omega^2}{c^2}=0$$

$$\frac{\delta^2 T(t)}{\delta t^2}+\omega^2 T(t)=0$$

- Applying the same complex form to $T(t)$:

$$T(t)=T_0 e^{\beta t}$$

- Where T_0 and β are complex numbers. This yields:

$$\frac{\delta^2 T_0 e^{\beta t}}{\delta t^2}+\omega^2 T_0 e^{\beta t}=0$$

$$\frac{\delta^2 T_0 e^{\beta t}}{\delta t^2}+\omega^2 T_0 e^{\beta t}=0$$

$$T_0\beta^2 e^{\beta t}+\omega^2 T_0 e^{\beta t}=0$$

$$T_0 e^{\beta t}(\beta^2+\omega^2)=0$$

- Following the same logic as before, we are only interested in solutions where $T_0 e^{\beta t}\neq 0$

$$\beta^2+\omega^2=0$$

$$\beta^2=-\omega^2$$

$$\beta=\pm j\omega$$

- This leaves:

$$T(t)=T_0 e^{\beta t}=T_0 e^{\pm j\omega t}$$

$$p(x,t) = P_1 e^{j\omega\left(t+\frac{x}{c}\right)} + P_2 e^{j\omega\left(-t-\frac{x}{c}\right)} + P_3 e^{j\omega\left(t-\frac{x}{c}\right)} + P_4 e^{j\omega\left(-t+\frac{x}{c}\right)}$$

- Next, we combine complex conjugates, rewriting their coefficients:

$$p(x,t) = P_A e^{j\omega\left(t+\frac{x}{c}\right)} + {P_A}^* e^{j\omega\left(-t-\frac{x}{c}\right)} + P_B e^{j\omega\left(t-\frac{x}{c}\right)} + {P_B}^* e^{j\omega\left(-t+\frac{x}{c}\right)}$$

- The real components of complex pairs are equal:

$$p(x,t) = 2Re\left\{P_A e^{j\omega\left(t+\frac{x}{c}\right)}\right\} + 2Re\left\{P_B e^{j\omega\left(t-\frac{x}{c}\right)}\right\}$$

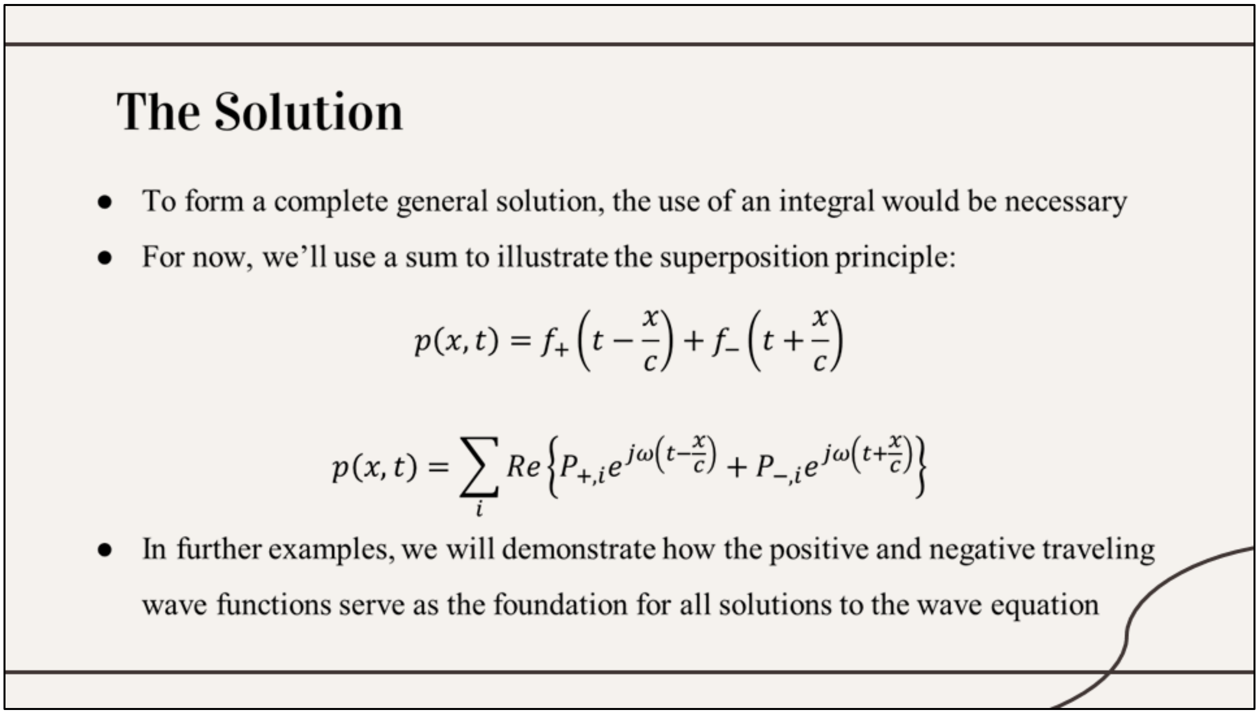
$$p(x,t) = 2Re\left\{P_A e^{j\omega\left(t+\frac{x}{c}\right)}\right\} + 2Re\left\{P_B e^{j\omega\left(t-\frac{x}{c}\right)}\right\}$$

- Rewriting this in a more informative way we set:

$$2P_A = P_- \text{ and } 2P_B = P_+$$

$$p(x,t) = Re\left\{P_+ e^{j\omega\left(t-\frac{x}{c}\right)} + P_- e^{j\omega\left(t+\frac{x}{c}\right)}\right\}$$

- Here, the first term is a wave travelling in the positive x-direction and the second is a wave travelling in the negative x-direction
- This gives the solution for a single frequency, but any signal can be created by a superposition of various frequencies



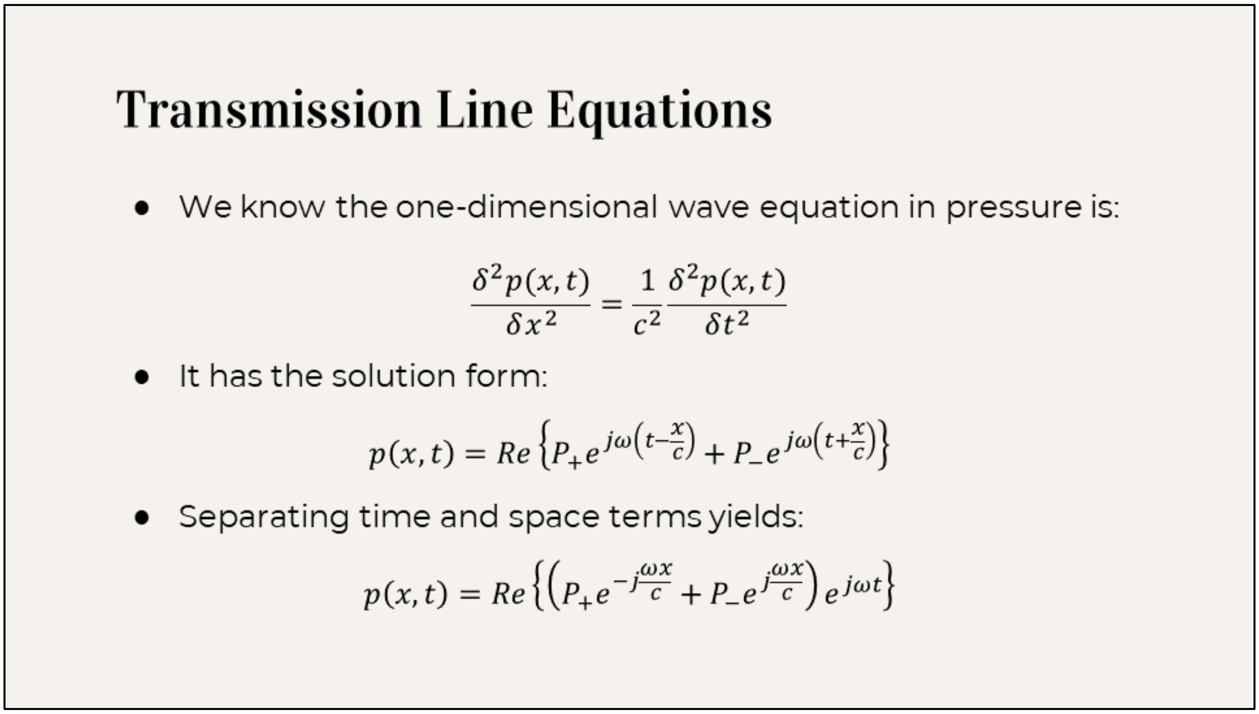
The Solution

- To form a complete general solution, the use of an integral would be necessary
- For now, we'll use a sum to illustrate the superposition principle:

$$p(x,t) = f_+\left(t - \frac{x}{c}\right) + f_-\left(t + \frac{x}{c}\right)$$

$$p(x,t) = \sum_i Re\left\{P_{+,i}e^{j\omega\left(t-\frac{x}{c}\right)} + P_{-,i}e^{j\omega\left(t+\frac{x}{c}\right)}\right\}$$

- In further examples, we will demonstrate how the positive and negative traveling wave functions serve as the foundation for all solutions to the wave equation



Transmission Line Equations

- We know the one-dimensional wave equation in pressure is:

$$\frac{\delta^2 p(x,t)}{\delta x^2} = \frac{1}{c^2}\frac{\delta^2 p(x,t)}{\delta t^2}$$

- It has the solution form:

$$p(x,t) = Re\left\{P_+e^{j\omega\left(t-\frac{x}{c}\right)} + P_-e^{j\omega\left(t+\frac{x}{c}\right)}\right\}$$

- Separating time and space terms yields:

$$p(x,t) = Re\left\{\left(P_+e^{-j\frac{\omega x}{c}} + P_-e^{j\frac{\omega x}{c}}\right)e^{j\omega t}\right\}$$

$$p(x,t) = Re\left\{\left(P_+ e^{-j\frac{\omega x}{c}} + P_- e^{j\frac{\omega x}{c}}\right) e^{j\omega t}\right\}$$

- Now the term that only depends on space is defined as:

$$p(x,\omega) = P_+ e^{-j\frac{\omega x}{c}} + P_- e^{j\frac{\omega x}{c}}$$

- This is defined as the complex amplitude of the waveform. For ease of notation, we will typically denote $j\omega$ by s, which represents the complex frequency:

$$s = \sigma + j\omega$$

- Rewriting the initial equation in terms of s:

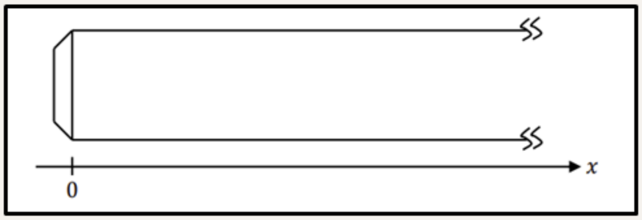
$$p(x,t) = Re\{P(x,s)e^{st}\}$$

- Where:

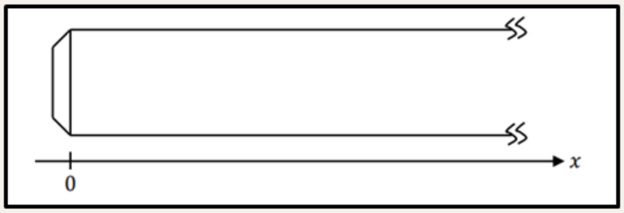
$$P(x,s) = P_+ e^{-\frac{sx}{c}} + P_- e^{\frac{sx}{c}}$$

Example 1: Steady-State Sinusoid

- Consider a semi-infinite tube shown below:



- It is common to use tubes to represent systems with place waves as plane waves in larger spaces are difficult to create, so using a tube gives a simpler representation to study wave behavior.



- In this example, there is one boundary condition: at x=0 a speaker moves to create a steady-state sinusoid with pressure:

$$p(x=0,t) = Acos(\omega t+\theta) = Re\{Ae^{j\theta}e^{j\omega t}\}$$

- Because the tube is infinite, there will be no reflections present:

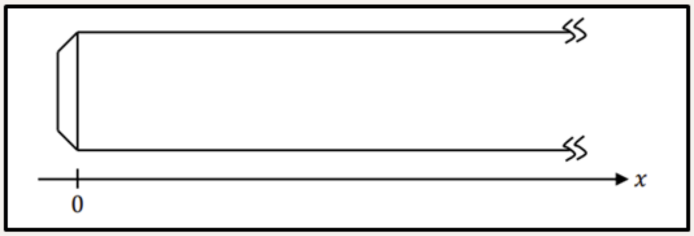
$$P_{-} = 0$$

- And

$$P_{+} = Ae^{j\theta}$$

- Therefore:

$$p(x,t) = Re\left\{Ae^{j\theta}e^{-j\frac{\omega x}{c}}e^{j\omega t}\right\}$$



$$p(x,t) = Re\left\{Ae^{j\theta}e^{-j\frac{\omega x}{c}}e^{j\omega t}\right\}$$

$$p(x,t) = Re\left\{Ae^{j\theta-j\frac{\omega x}{c}j\omega t}\right\}$$

$$p(x,t) = Re\left\{\mathrm{Acos}\left(j\theta - j\frac{\omega x}{c}j\omega t\right) + jAsin\left(j\theta - j\frac{\omega x}{c}j\omega t\right)\right\}$$

$$p(x,t) = Acos\left(\omega\left(t-\frac{x}{c}\right)+\theta\right)$$

- For all t and $x \geq 0$.

Example 2: Velocity waves

- Previously, we found that the wave equation for velocity in 1-D takes the same form as pressure:

$$\frac{\delta^2 u(x,t)}{\delta x^2} = \frac{1}{c^2}\frac{\delta^2 u(x,t)}{\delta t^2}$$

- The solution must then take the same form:

$$u(x,t) = Re\{U(x,s)e^{j\omega t}\}$$

$$U(x,s) = U_+ e^{-\frac{sx}{c}} + U_- e^{\frac{sx}{c}}$$

- Now we must find a relationship between velocity and pressure.

- To do this we'll take the result from Newton's second law:

$$\frac{\delta p(x,t)}{\delta x} = -\rho_0 \frac{\delta u(x,t)}{\delta t}$$

- Using the general expressions, we derived for pressure and velocity:

$$\frac{\delta}{\delta x}\left(Re\left\{\left(P_+ e^{-\frac{sx}{c}} + P_- e^{\frac{sx}{c}}\right)e^{st}\right\}\right) = -\rho_0 \frac{\delta}{\delta t} Re\left\{\left(U_+ e^{-\frac{sx}{c}} + U_- e^{\frac{sx}{c}}\right)e^{st}\right\}$$

$$Re\left\{\left(-\frac{s}{c}P_+ e^{-\frac{sx}{c}} + \frac{s}{c}P_- e^{\frac{sx}{c}}\right)e^{st}\right\} = -\rho_0 Re\left\{\left(U_+ e^{-\frac{sx}{c}} + U_- e^{\frac{sx}{c}}\right)se^{st}\right\}$$

$$Re\left\{\left(-\frac{s}{c}P_+ e^{-\frac{sx}{c}} + \frac{s}{c}P_- e^{\frac{sx}{c}}\right)e^{st}\right\} = Re\left\{\left(-\rho_0 sU_+ e^{-\frac{sx}{c}} - \rho_0 sU_- e^{\frac{sx}{c}}\right)e^{st}\right\}$$

- From the previous, the following equations must be true:

$$\begin{cases} -\frac{s}{c}P_+ = -\rho_0 s U_+ \\ \frac{s}{c}P_- = -\rho_0 s U_- \end{cases}$$

$$\begin{cases} \frac{P_+}{\rho_0 c} = U_+ \\ -\frac{P_-}{\rho_0 c} = U_- \end{cases}$$

- We can now rewrite the complex amplitude of velocity as:

$$U(x,s) = \frac{P_+}{\rho_0 c} e^{-\frac{sx}{c}} - \frac{P_-}{\rho_0 c} e^{\frac{sx}{c}}$$

$$U(x,s) = \frac{P_+}{\rho_0 c} e^{-\frac{sx}{c}} - \frac{P_-}{\rho_0 c} e^{\frac{sx}{c}}$$

- With the following definition:

$$Z_0 = \rho_0 c$$

- This is rewritten as:

$$U(x,s) = \frac{P_+}{Z_0} e^{-\frac{sx}{c}} - \frac{P_-}{Z_0} e^{\frac{sx}{c}}$$

- Where Z_0 is the characteristic impedance of the medium, which in this case is air. Now we have equations for both pressure and velocity with the same two unknowns!

- These are called the transmission line equations (Directly analogous to electric transmission lines).
- In summary, the transmission line equations are:

$$p(x,t) = Re\{P(x,s)e^{st}\}$$

$$u(x,t) = Re\{U(x,s)e^{st}\}$$

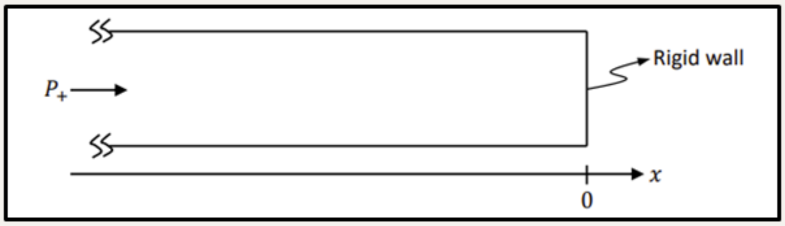
$$P(x,s) = P_+ e^{-\frac{sx}{c}} + P_- e^{\frac{sx}{c}}$$

$$U(x,s) = \frac{P_+}{Z_0} e^{-\frac{sx}{c}} - \frac{P_-}{Z_0} e^{\frac{sx}{c}}$$

- Again, where $Z_0 = \rho_0 c$ and $s = j\omega$.

Example 3: Transmission line equations Applied

- Consider the following 1D tube closed at the right end:



- With the rigid wall at $x = 0$, the boundary condition here is:

$$u(x = 0, t) = 0$$

- This means that in the frequency domain:

$$u(x = 0, s) = 0$$

- Because this is true for all of time, this means that the positive travelling wave reflects off the wall to become the negative travelling wave. This means:

$$\frac{P_+}{Z_0} - \frac{P_-}{Z_0} = 0$$

$$P_+ = P_-$$

- This means that the pressure transmission line equation becomes:

$$P(x,s) = P_+ e^{-\frac{sx}{c}} + P_+ e^{\frac{sx}{c}}$$

$$P(x,s) = P_+ e^{-j\frac{\omega x}{c}} + P_+ e^{j\frac{\omega x}{c}}$$

$$P(x,s) = P_+\left(cos\left(-\frac{\omega x}{c}\right) + jsin\left(-\frac{\omega x}{c}\right)\right) + P_+\left(cos\left(\frac{\omega x}{c}\right) + jsin\left(\frac{\omega x}{c}\right)\right)$$

$$P(x,s) = P_+\left(cos\left(-\frac{\omega x}{c}\right) + jsin\left(-\frac{\omega x}{c}\right)\right) + P_+\left(cos\left(\frac{\omega x}{c}\right) + jsin\left(\frac{\omega x}{c}\right)\right)$$

$$P(x,s) = P_+\left(cos\left(\frac{\omega x}{c}\right) - jsin\left(\frac{\omega x}{c}\right)\right) + P_+\left(cos\left(\frac{\omega x}{c}\right) + jsin\left(\frac{\omega x}{c}\right)\right)$$

$$P(x,s) = P_+\left(cos\left(\frac{\omega x}{c}\right) - jsin\left(\frac{\omega x}{c}\right)\right) + P_+\left(cos\left(\frac{\omega x}{c}\right) + jsin\left(\frac{\omega x}{c}\right)\right)$$

$$P(x,s) = 2P_+\cos\left(\frac{\omega x}{c}\right)$$

$$p(x,t) = Re\left\{2|P_+|e^{j\angle P_+}\cos\left(\frac{\omega x}{c}\right)e^{j\omega t}\right\} = 2|P_+|\cos\left(\frac{\omega x}{c}\right)Re\{e^{j\angle P_+}e^{j\omega t}\}$$

$$p(x,t) = 2|P_+|\cos\left(\frac{\omega x}{c}\right)cos(\omega t + \angle P_+)$$

- Similarly, for all $x \leq 0$, velocity is defined as:

$$U(x,s) = \frac{P_+}{Z_0}e^{-\frac{sx}{c}} - \frac{P_+}{Z_0}e^{\frac{sx}{c}}$$

$$U(x,s) = \frac{P_+}{Z_0}e^{-j\frac{\omega x}{c}} - \frac{P_+}{Z_0}e^{j\frac{\omega x}{c}}$$

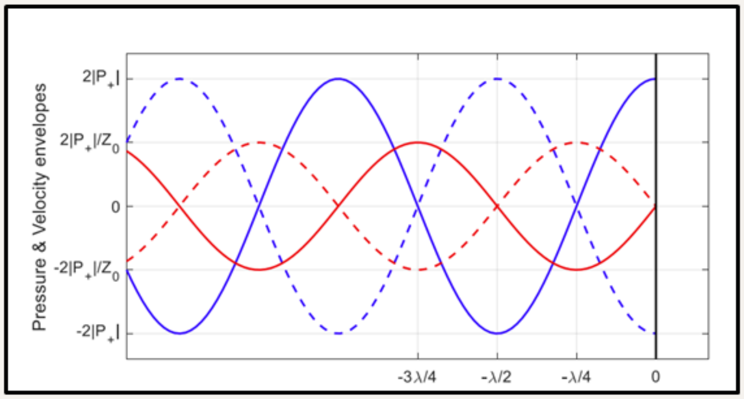
$$U(x,s) = -2j\frac{P_+}{Z_0}\sin\left(\frac{\omega x}{c}\right)$$

$$u(x,t) = Re\left\{-2j\frac{|P_+|e^{j\angle P_+}}{Z_0}\sin\left(\frac{\omega x}{c}\right)e^{j\omega t}\right\}$$

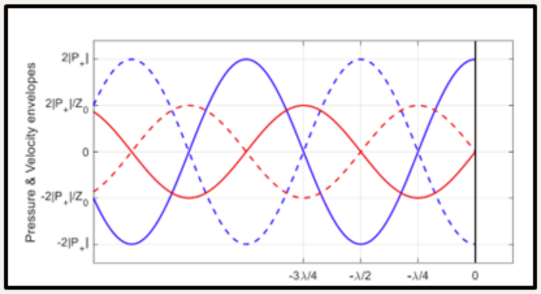
$$u(x,t) = 2\frac{|P_+|}{Z_0}sin\left(\frac{\omega x}{c}\right)Re\{-je^{j\angle P_+}e^{j\omega t}\}$$

$$u(x,t) = 2\frac{|P_+|}{Z_0}sin\left(\frac{\omega x}{c}\right)\sin(\omega t + \angle P_+)$$

- The figure below shows the pressure and velocity envelopes within the closed tube:



Pressure (blue) and velocity (red) envelopes in a closed tube.



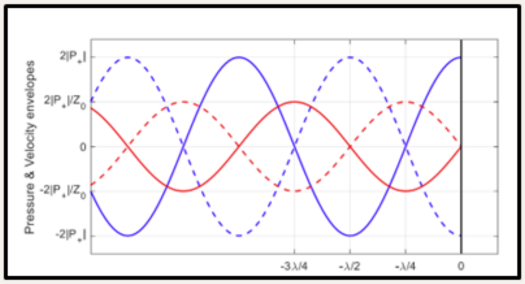
- The solid blue curve shows:

$$2|P_+|\cos\left(\frac{\omega x}{c}\right)$$

- Whereas the dashed blue curve is the negative of that. If a location is chosen within the tube, then the time varying term:

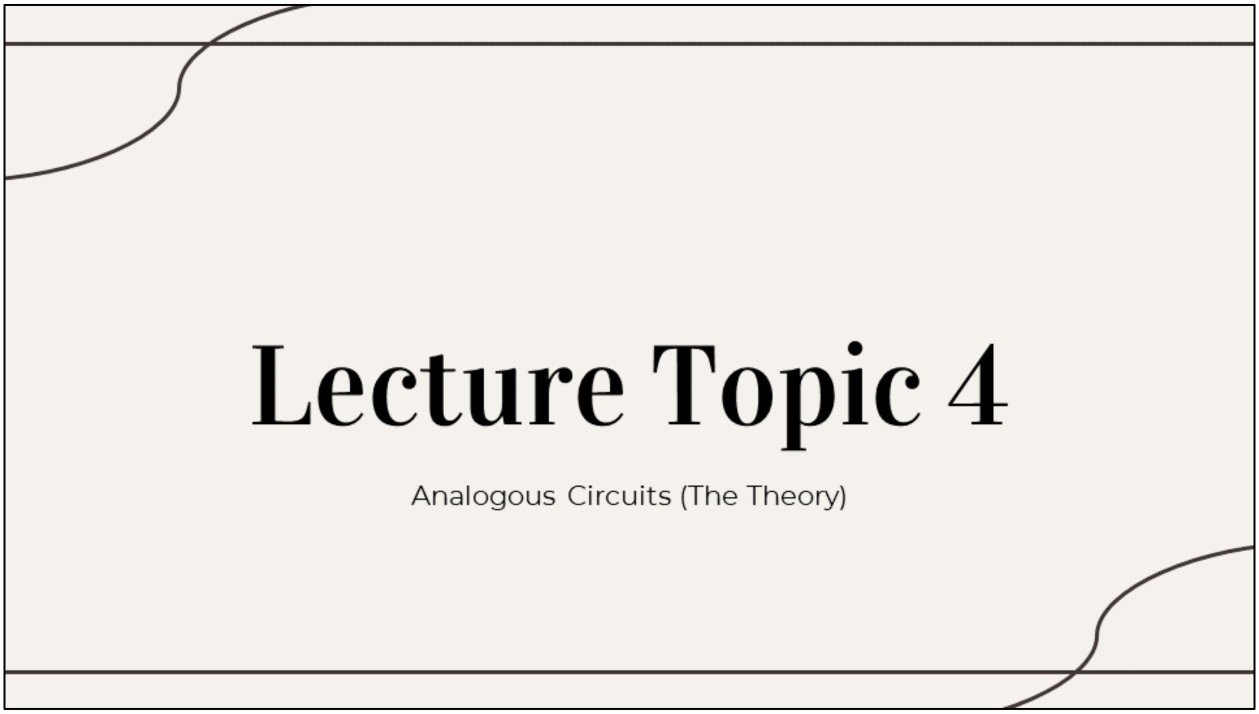
$$cos(\omega t + \angle P_+)$$

- will scale the pressure within the envelope



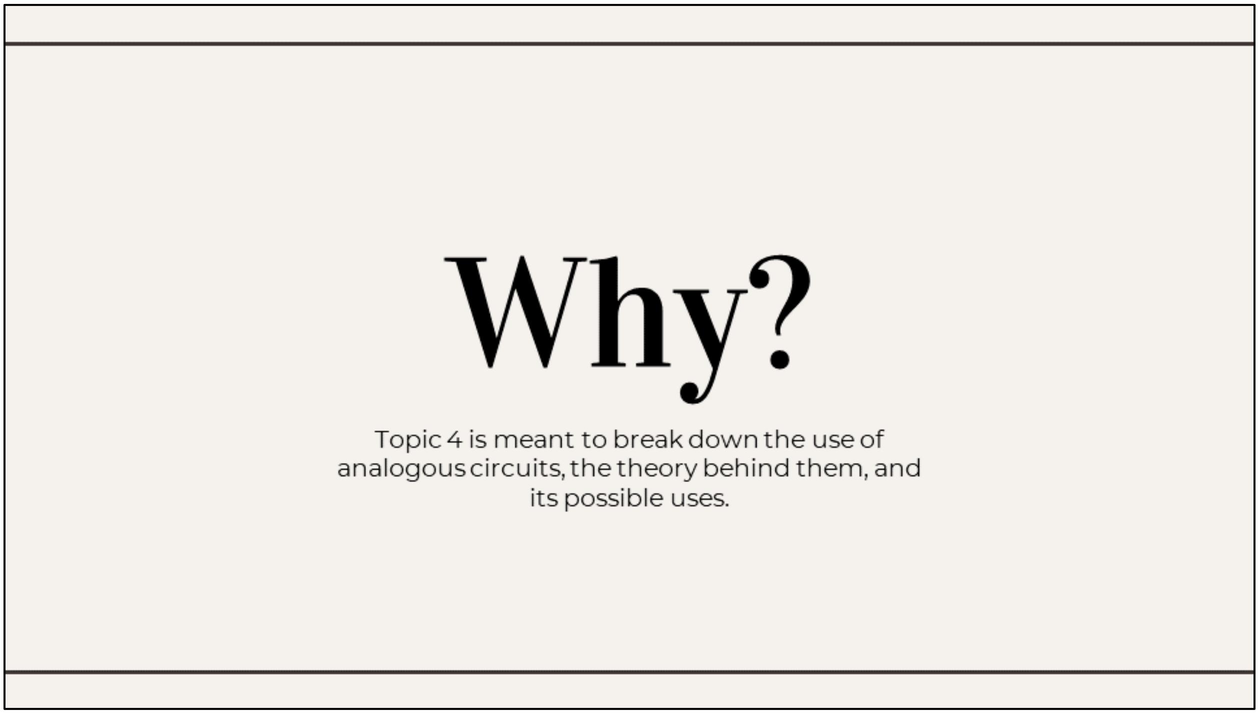
- You’ll notice an established pattern with the tube for both pressure and velocity where at specific points equally spaced there is a value of 0
- These points are called nodes, and the areas with maximum amplitudes are called antinodes
- This pattern of nodes and antinodes set up a seemingly stationary wave called a standing wave
- This wave, although perceived as stationary, is just the composition of 2 travelling waves as shown above.

Appendix D – Lecture Topic 4 Slides



Lecture Topic 4

Analogous Circuits (The Theory)

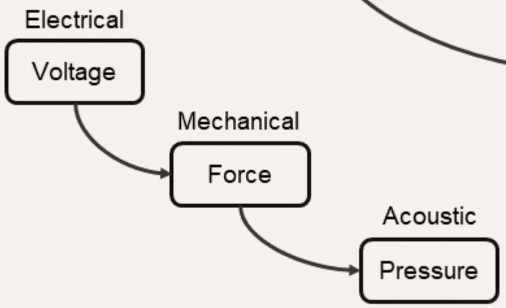


Why?

Topic 4 is meant to break down the use of analogous circuits, the theory behind them, and its possible uses.

Before we begin…

In order to understand the transition between electrical, mechanical, and acoustic domains you must understand the concept of **impedance**

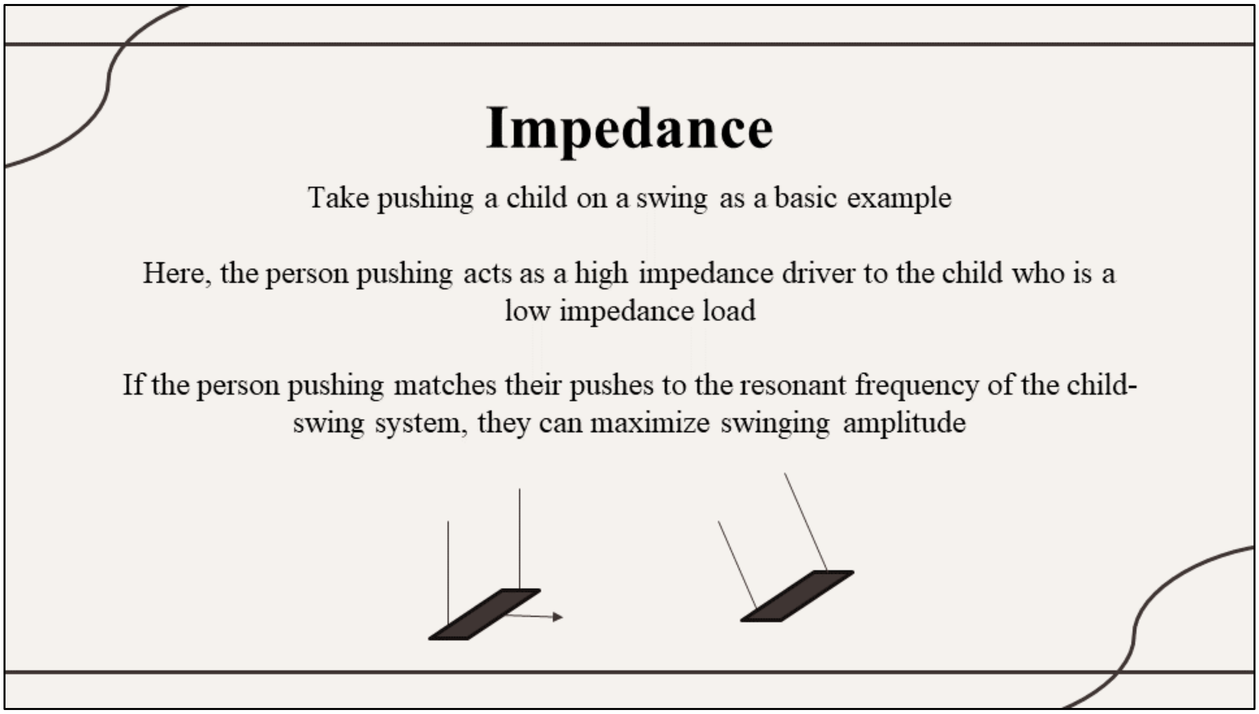


Impedance

Simply put impedance is a measure of how much resistance a system presents to some kind of flow

Electrical: Resistance to electric current
Mechanical: Resistance to mechanical force
Acoustical: Resistance to acoustic volume flow

This concept is incredibly helpful in understanding how systems will react to inputs

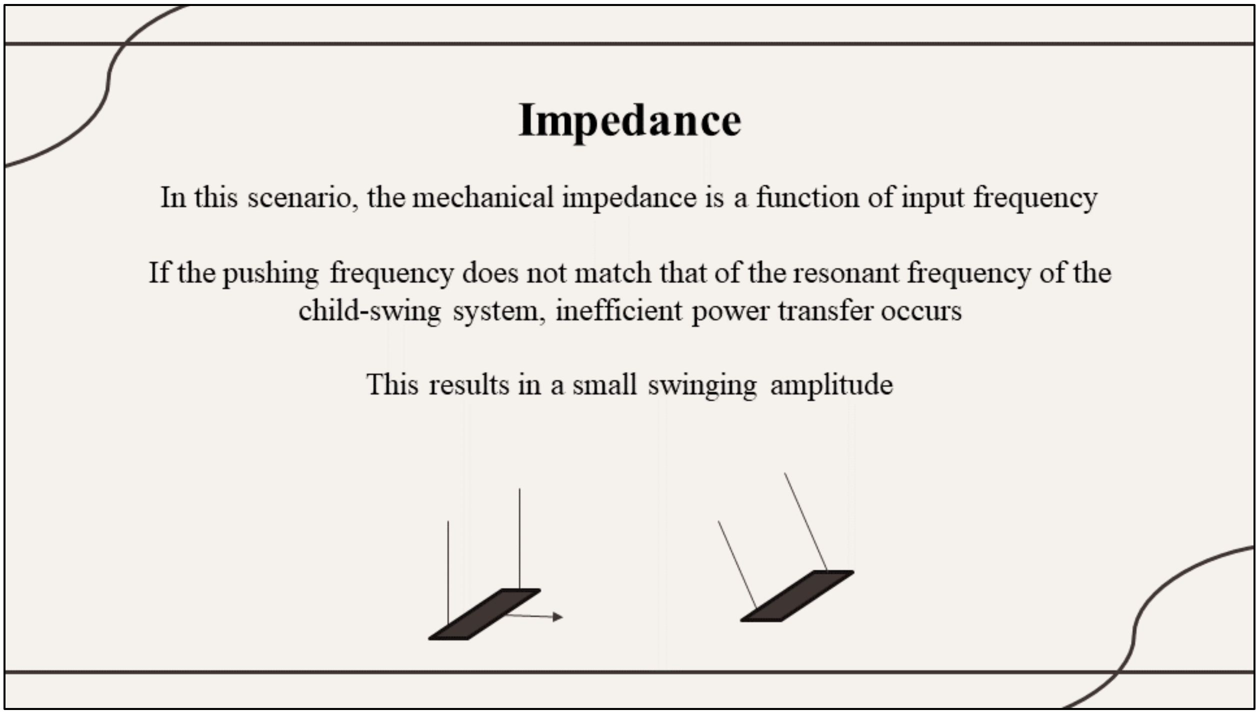


Impedance

Take pushing a child on a swing as a basic example

Here, the person pushing acts as a high impedance driver to the child who is a low impedance load

If the person pushing matches their pushes to the resonant frequency of the child-swing system, they can maximize swinging amplitude



Impedance

In this scenario, the mechanical impedance is a function of input frequency

If the pushing frequency does not match that of the resonant frequency of the child-swing system, inefficient power transfer occurs

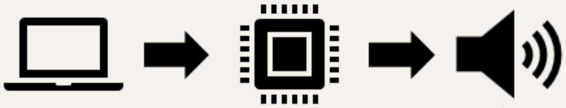
This results in a small swinging amplitude

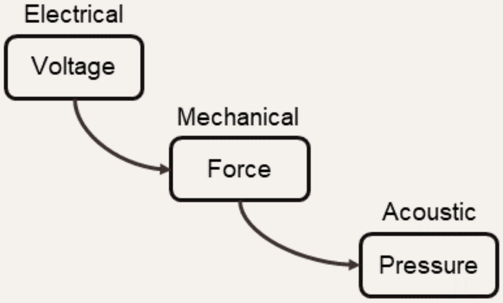
Impedance

In audio equipment, electrical impedance matching plays a crucial role in ensuring that proper power transfer occurs

In this system, the audio amplifier acts as a high impedance source and the loudspeaker as a low impedance load

To ensure efficient power transfer, achieving the best audio quality, the output impedance of the amplifier must match that of the loudspeaker





Relating Domains

By understanding the relationship between impedance in different domains, you can relate quantities like voltage, force, and pressure to aid in modeling systems

What are we learning?

01

Electric Domain

Cross and through variables in the electrical domain

02

Mechanical Domain

Cross and through variables in the mechanical domain

03

Acoustic Domain

Cross and through variables in the acoustic domain

04

Transformations

Transforming through various domains

01

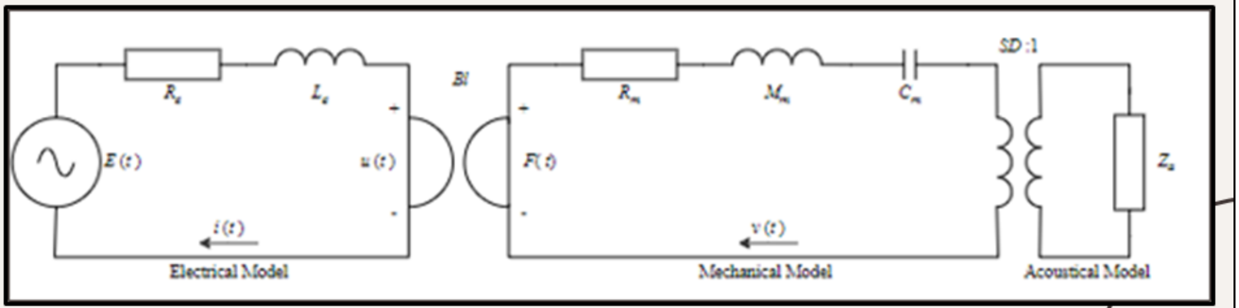
The Basics

Basic concepts and variables behind analogous circuits

Analogous Circuits – Acoustic Analogies

Electrical → Mechanical → Acoustical transition can be modeled easily!

Below is a model of a speaker you will make later in the course showing all three systems!



Analogous Circuits – Acoustic Analogies

Electrical → Mechanical → Acoustical transition can be modeled easily!

First let's look at their values…

Variable	**Electrical**	**Mechanical**	**Acoustic**
Cross	Voltage, e (Volts)	Velocity, v (m/sec)	Pressure, p (N/m^2, Pa)
Through	Current, I (Amperes)	force, f (Newtons)	Volume Velocity, U (m^3/sec)

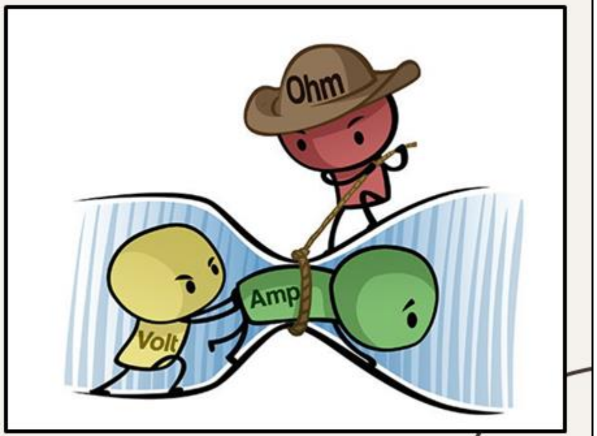
Two types of variables - Each system has 1 **cross** and 1 **through** variable that will be transformed through each section of the circuit

Analogous Circuits – Acoustic Analogies

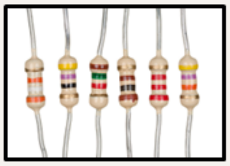
Two types of variables:

Effort Variables (**Cross**)
- Variables measured across a circuit
- Voltage (V)

Flow Variables (**Through**)
- Variables measured through a circuit
- Current (I)



Acoustic Analogies – Electrical Domain



Variable	**Electrical**	**Mechanical**	**Acoustic**
Cross	Voltage, e (Volts)	Velocity, v (m/sec)	Pressure, p (N/m^2, Pa)
Through	Current, I (Amperes)	force, f (Newtons)	Volume Velocity, U (m^3/sec)

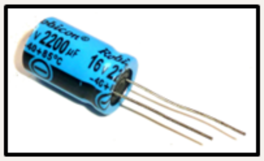
How do voltage and current correlate… Resistance!

$$R = \frac{V}{I}$$

$$V = I * R$$

R = Resistance in Ohms [Ω]
V = Voltage in Volts [V]
I = Current in Amps [A]

Acoustic Analogies – Electrical Domain



Variable	Electrical	Mechanical	Acoustic
Cross	Voltage, e (Volts)	Velocity, v (m/sec)	Pressure, p (N/m^2, Pa)
Through	Current, I (Amperes)	force, f (Newtons)	Volume Velocity, U (m^3/sec)

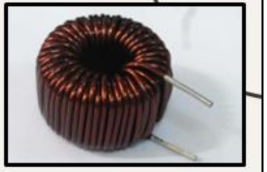
How else do voltage and current correlate…
Capacitance!

Capacitance – Ability of a circuit component to hold a charge for a duration of time

$$V = \frac{1}{c}\int_0^t Idt$$

t = Time in Seconds $[s]$
V = Voltage in Volts $[V]$
I = Current in Amps $[A]$
C = Capacitance in Farads $[F]$

Acoustic Analogies – Electrical Domain



Variable	Electrical	Mechanical	Acoustic
Cross	Voltage, e (Volts)	Velocity, v (m/sec)	Pressure, p (N/m^2, Pa)
Through	Current, I (Amperes)	force, f (Newtons)	Volume Velocity, U (m^3/sec)

How else do voltage and current correlate…
Inductance!

Inductance – Property that causes an electromotive force to be generated due to a rate of change in current

$$V = L\frac{1}{c}$$

L = Inductance in Henrys $[H]$
V = Voltage in Volts [V]
$\frac{dI}{dt}$ = Change in Current due to time $[\frac{Amps}{sec}]$

Acoustic Analogies – Mechanical Domain

Variable	**Electrical**	**Mechanical**	**Acoustic**
Cross	Voltage, e (Volts)	Velocity, v (m/sec)	Pressure, p (N/m^2, Pa)
Through	Current, I (Amperes)	force, f (Newtons)	Volume Velocity, U (m^3/sec)

How do velocity and force correlate…

Mechanical Responsiveness!

$$\frac{v(t)}{r_{RMS}} = f(t)$$

$f(t)$ = Force [N]
r_{RMS} = Mechanical Responsiveness [m/Ns]
$v(t)$ = Velocity [m/s]

$$v(t) = v(t) * r_{RMS}$$

Acoustic Analogies – Mechanical Domain

Variable	**Electrical**	**Mechanical**	**Acoustic**
Cross	Voltage, e (Volts)	Velocity, v (m/sec)	Pressure, p (N/m^2, Pa)
Through	Current, I (Amperes)	force, f (Newtons)	Volume Velocity, U (m^3/sec)

How else do velocity and force correlate… Mechanical Mass!

$$v(t) = \frac{1}{m}\int_{-\infty}^{t} f(\tau)d\tau$$

Mechanical Mass – Represents the inertia of the moving parts of the mechanical system

$f(\tau)$= Force [N]m
m = mass [kg]
$v(t)$= Velocity [m/s]

Wait… where did τ come from???

Relation to Newton's 2nd Law

$f(\tau)$ = Force [N]m
m = mass [kg]
v(t) = Velocity [m/s]

$$v(t) = \frac{1}{m}\int_{-\infty}^{t} f(\tau)d\tau$$

$$m * v(t) = \int_{-\infty}^{t} f(\tau)d\tau$$

$$\frac{d}{dt}(m * v(t)) = \frac{d}{dt}\int_{-\infty}^{t} f(\tau)d\tau$$

$$m * a(t) = f(t)$$

$$F = ma$$

Acoustic Analogies – Mechanical Domain

Variable	**Electrical**	**Mechanical**	**Acoustic**
Cross	Voltage, e (Volts)	Velocity, v (m/sec)	Pressure, p (N/m^2, Pa)
Through	Current, I (Amperes)	force, f (Newtons)	Volume Velocity, U (m^3/sec)

How else do velocity and force correlate… Mechanical compliance!

Mechanical Compliance – The property that indicates the flexibility or elasticity of mechanical parts

$$v(t) = Comp\frac{df(t)}{dt}$$

Comp = Mechanical Compliance [m/N]
Comp = $\frac{1}{Stiffness}$
v(t) = Velocity [m/s]
f(t) = Force [N]

Acoustic Analogies – Acoustic Domain

Variable	**Electrical**	**Mechanical**	**Acoustic**
Cross	Voltage, e (Volts)	Velocity, v (m/sec)	Pressure, p (N/m^2, Pa)
Through	Current, I (Amperes)	force, f (Newtons)	Volume Velocity, U (m^3/sec)

How do pressure and volume correlate… Acoustic loss!

$$\frac{P(t)}{Loss} = U(t)$$

P(t) = Acoustic Pressure [μb] (Micro Bar)
U(t) = Volume Velocity [m^3/s]
Loss = Acoustic Loss [dB] (decibels)

$$P(t) = U(t) * Loss$$

Acoustic Analogies – Acoustic Domain

Variable	**Electrical**	**Mechanical**	**Acoustic**
Cross	Voltage, e (Volts)	Velocity, v (m/sec)	Pressure, p (N/m^2, Pa)
Through	Current, I (Amperes)	force, f (Newtons)	Volume Velocity, U (m^3/sec)

How else do pressure and volume correlate… Acoustic Volume!

Acoustic Volume – Elements of a system that act as compliances, used in finding acoustic compliance

$$p(t) = \frac{\rho c^2}{Volume}\int_{-\infty}^{t} U(t)dt$$

p(t) = Acoustic Pressure [μb] (Micro Bar)
U(t) = Volume Velocity [m^3/s]
Volume = Acoustic Volume [m^3]
ρ = Density [kg/ m^3]
c = speed of sound [m/s]

Acoustic Analogies – Acoustic Domain

Variable	Electrical	Mechanical	Acoustic
Cross	Voltage, e (Volts)	Velocity, v (m/sec)	Pressure, p (N/m^2, Pa)
Through	Current, I (Amperes)	force, f (Newtons)	Volume Velocity, U (m^3/sec)

How else do pressure and volume correlate… Acoustic Port!

Acoustic Port – The lumped acoustic mass that models loudspeaker ports

$$p(t) = \frac{\rho c^2}{Volume} \int_{-\infty}^{t} U(t)dt$$

p(t) = Acoustic Pressure [μb] (Micro Bar)
U(t) = Volume Velocity [m^3/s]
Volume = Acoustic Volume [m^3]
ρ = Density [kg/ m^3]
c = speed of sound [m/s]

Analogous Circuits – Acoustic Analogies

Variable	Electrical	Mechanical	Acoustic
Cross	Voltage, e (Volts)	Velocity, v (m/sec)	Pressure, p (N/m^2, Pa)
Through	Current, I (Amperes)	force, f (Newtons)	Volume Velocity, U (m^3/sec)

$$R = \frac{V}{I}$$

(Electrical)

$$\frac{v(t)}{r_{RMS}} = f(t)$$

(Mechanical)

$$\frac{P(t)}{Loss} = U(t)$$

(Acoustic)

Did you notice all equations look similar?

Wait there's more!

Analogous Circuits – Acoustic Analogies

Electrical	**Mechanical**	**Acoustic**
Voltage, e (Volts)	Velocity, v (m/sec)	Pressure, p (N/m^2, Pa)
Current, I (Amperes)	force, f (Newtons)	Volume Velocity, U (m^3/sec)

$$V = \frac{1}{c}\int_0^t I dt \qquad v(t) = \frac{1}{m}\int_{-\infty}^{t} f(\tau)d\tau \qquad p(t) = \frac{\rho c^2}{Volume}\int_{-\infty}^{t} U(t)dt$$

$$V = L\frac{1}{c} \qquad v(t) = Comp\frac{df(t)}{dt} \qquad p(t) = \frac{\rho Length}{Area} * \frac{dU(t)}{dt}$$

(Electrical) (Mechanical) (Acoustic)

Analogous Circuits – Acoustic Analogies

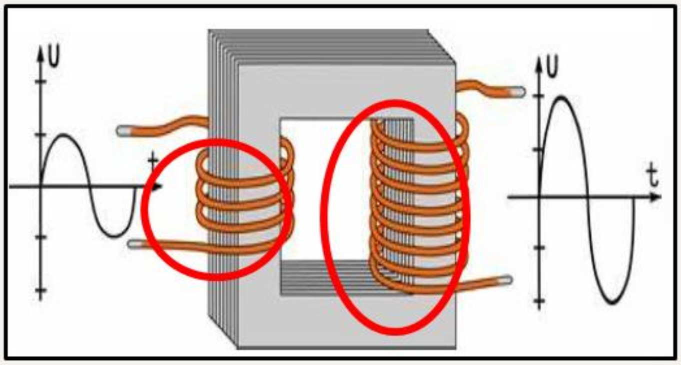
Variable	**Electrical**	**Mechanical**	**Acoustic**
Cross	Voltage, e (Volts)	Velocity, v (m/sec)	Pressure, p (N/m^2, Pa)
Through	Current, I (Amperes)	force, f (Newtons)	Volume Velocity, U (m^3/sec)

How do you convert voltage/current (Electrical) to velocity/force (Mechanical)
and
Velocity/force to pressure/volume (Acoustic)

All are dependent on the sensitivity of a transformer!

Basic Element - Transformer

The different number of coils can adjust the voltage!



Transformation of Variables

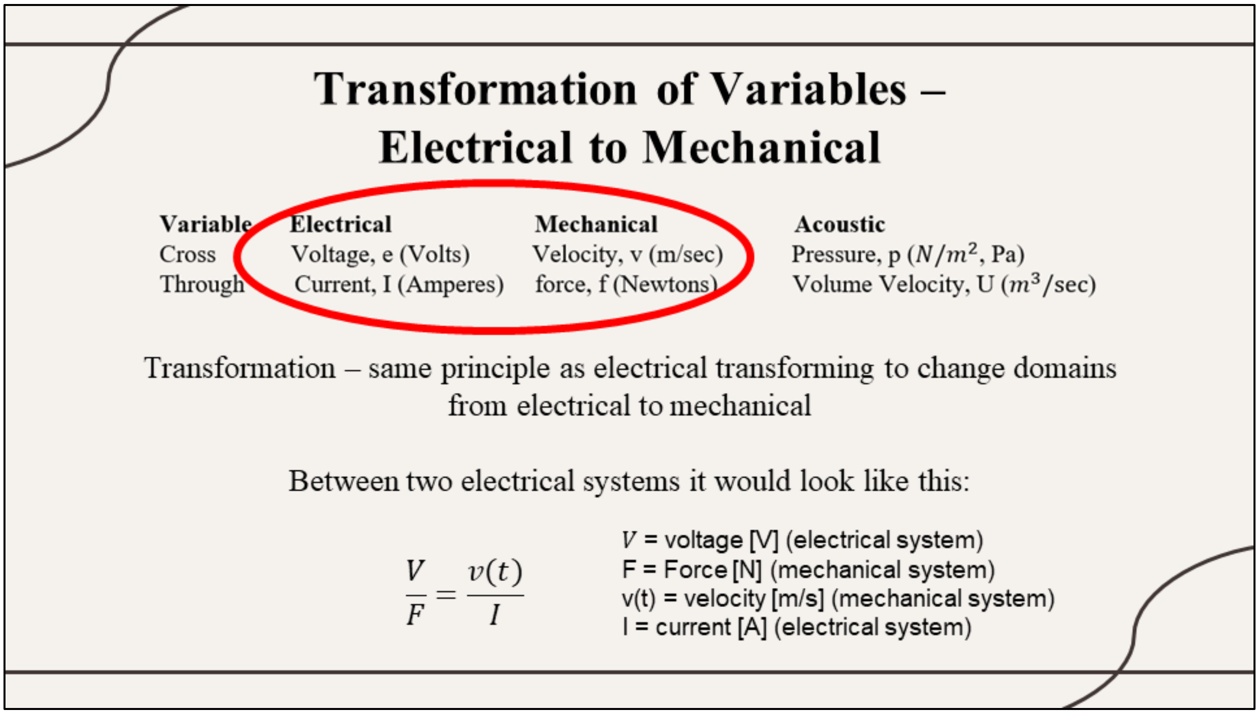
Variable	Electrical	Mechanical	Acoustic
Cross	Voltage, e (Volts)	Velocity, v (m/sec)	Pressure, p (N/m^2, Pa)
Through	Current, I (Amperes)	force, f (Newtons)	Volume Velocity, U (m^3/sec)

Transformation – changing of an Across variable either to a higher or lower magnitude between two systems.

Between two electrical systems it would look like this:

$$\frac{V_1}{V_2} = \frac{I_2}{I_1}$$

V_1 = incoming voltage [V]
V_2= outgoing voltage [V]
I_1 = incoming current [A]
I_2= outgoing current [A]



Transformation of Variables – Electrical to Mechanical

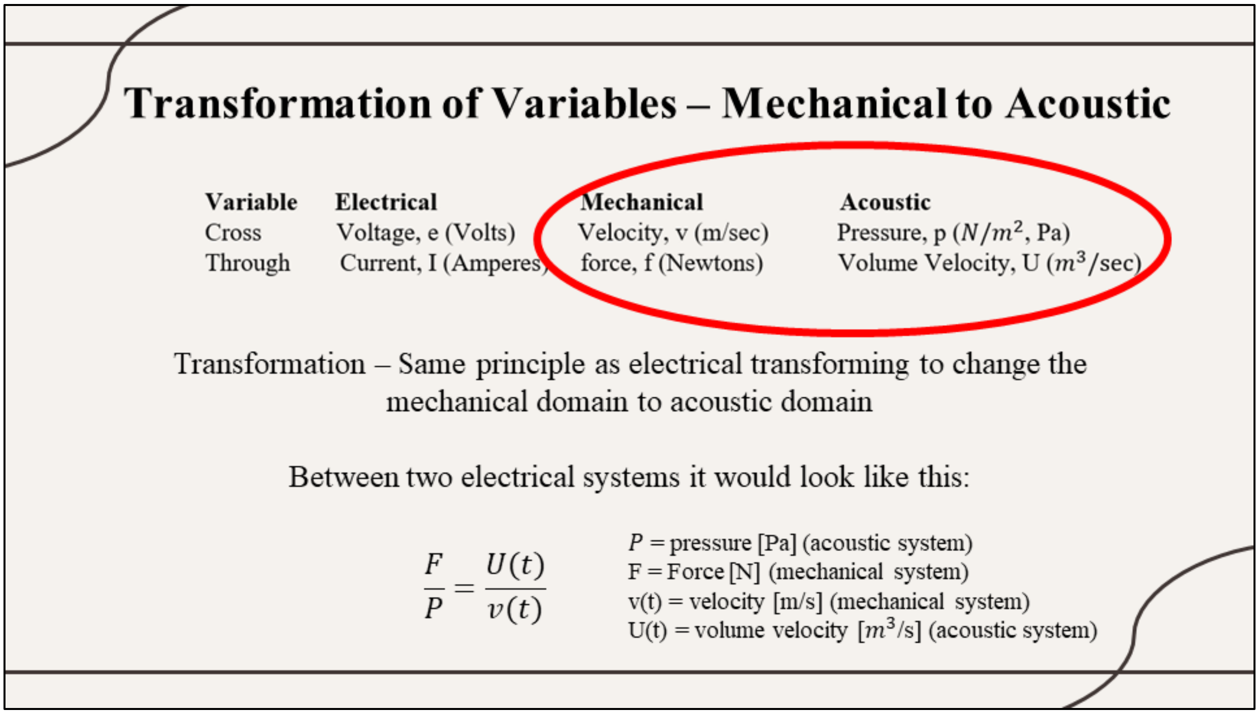
Variable	Electrical	Mechanical	Acoustic
Cross	Voltage, e (Volts)	Velocity, v (m/sec)	Pressure, p (N/m^2, Pa)
Through	Current, I (Amperes)	force, f (Newtons)	Volume Velocity, U (m^3/sec)

Transformation – same principle as electrical transforming to change domains from electrical to mechanical

Between two electrical systems it would look like this:

$$\frac{V}{F} = \frac{v(t)}{I}$$

V = voltage [V] (electrical system)
F = Force [N] (mechanical system)
v(t) = velocity [m/s] (mechanical system)
I = current [A] (electrical system)



Transformation of Variables – Mechanical to Acoustic

Variable	Electrical	Mechanical	Acoustic
Cross	Voltage, e (Volts)	Velocity, v (m/sec)	Pressure, p (N/m^2, Pa)
Through	Current, I (Amperes)	force, f (Newtons)	Volume Velocity, U (m^3/sec)

Transformation – Same principle as electrical transforming to change the mechanical domain to acoustic domain

Between two electrical systems it would look like this:

$$\frac{F}{P} = \frac{U(t)}{v(t)}$$

P = pressure [Pa] (acoustic system)
F = Force [N] (mechanical system)
v(t) = velocity [m/s] (mechanical system)
U(t) = volume velocity [m^3/s] (acoustic system)

Analogous Circuits – Acoustic Analogies

- Analogies and circuit theory are useful in studying complex systems

Ex. Loud Speakers

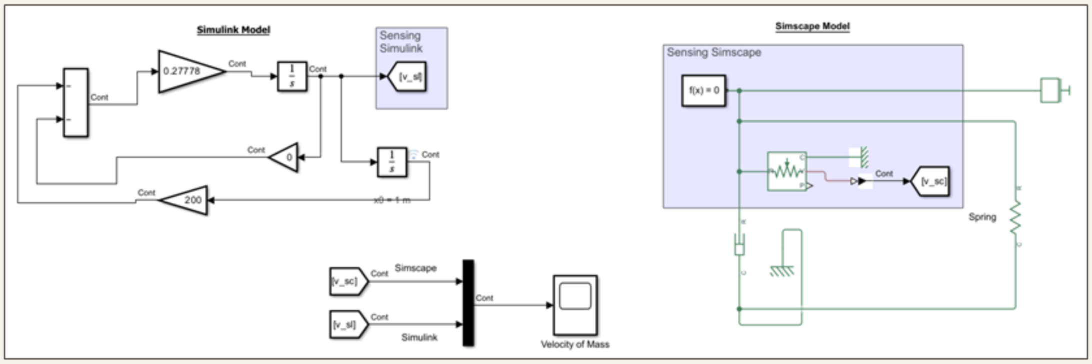
- Transform electrical signals → mechanical forces → acoustical pressure
- This can be modeled through equations and different simulators (MATLAB!)

Appendix E – Mass on a Spring Modeling Slides

Mass on a Spring w/ Simulink and Simscape

Introduction

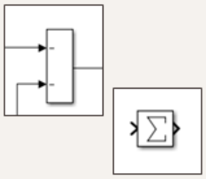
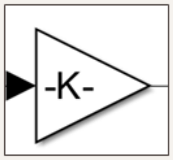
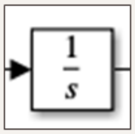
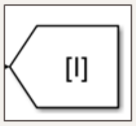
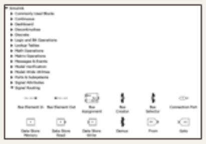
The complete model setup should look like the diagram below

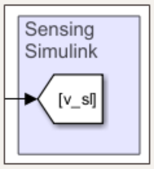


Construct this model following this slide deck

We will begin with undamped simple oscillations

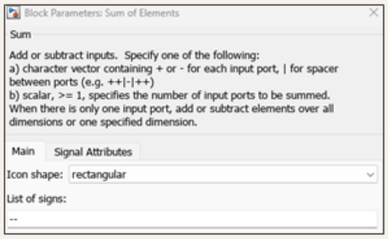
Simulink Model

Symbol				
Part Name	**Sum of Elements**	**Gain**	**Integrator**	**Goto**
Location	**Search Part Name**			

Symbol		
Part Name	**Insert Area (Purple Box) – add annotation for the text**	**Logging Signal**
Location		**Right Click the Connector Line and Select “Log Selected Signals”**

Simulink Model Inputs

Sum of Elements Input

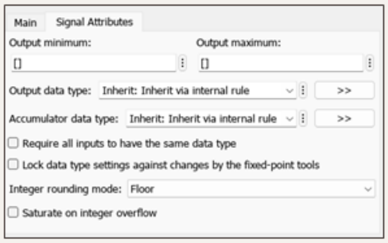


Sum

Add or subtract inputs. Specify one of the following:
a) character vector containing + or - for each input port, | for spacer between ports (e.g. ++|-|++)
b) scalar, >= 1, specifies the number of input ports to be summed.
When there is only one input port, add or subtract elements over all dimensions or one specified dimension.

Icon shape: rectangular

List of signs: --



Output minimum: []

Output maximum: []

Output data type: Inherit: Inherit via internal rule

Accumulator data type: Inherit: Inherit via internal rule

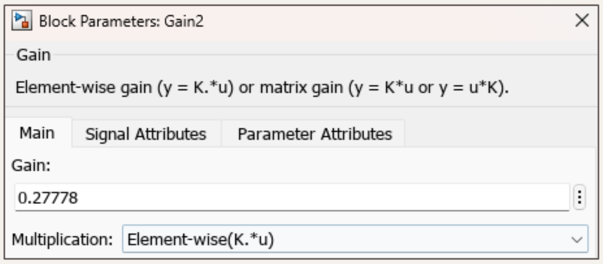
☐ Require all inputs to have the same data type

☐ Lock data type settings against changes by the fixed-point tools

Integer rounding mode: Floor

☐ Saturate on integer overflow

Top Gain Input



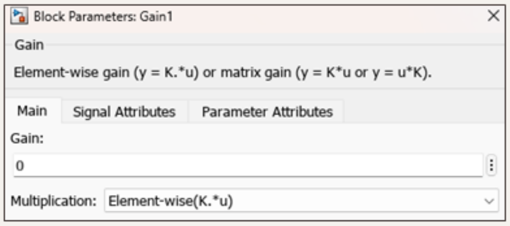
Gain

Element-wise gain (y = K.*u) or matrix gain (y = K*u or y = u*K).

Gain: 0.27778

Multiplication: Element-wise(K.*u)

Middle Gain Input


Block Parameters: Gain1

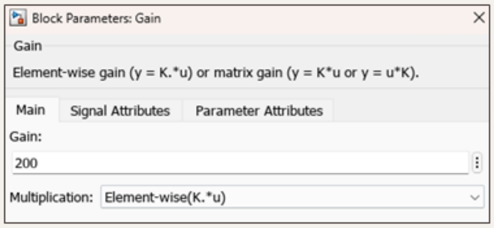
Gain

Element-wise gain (y = K.*u) or matrix gain (y = K*u or y = u*K).

Gain: 0

Multiplication: Element-wise(K.*u)

Bottom Gain Input


Block Parameters: Gain

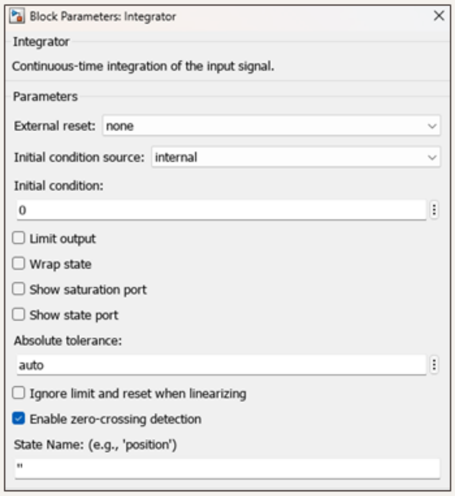
Gain

Element-wise gain (y = K.*u) or matrix gain (y = K*u or y = u*K).

Gain: 200

Multiplication: Element-wise(K.*u)

Top Integrator Input


Block Parameters: Integrator

Integrator

Continuous-time integration of the input signal.

Parameters

External reset: none

Initial condition source: internal

Initial condition: 0

☐ Limit output

☐ Wrap state

☐ Show saturation port

☐ Show state port

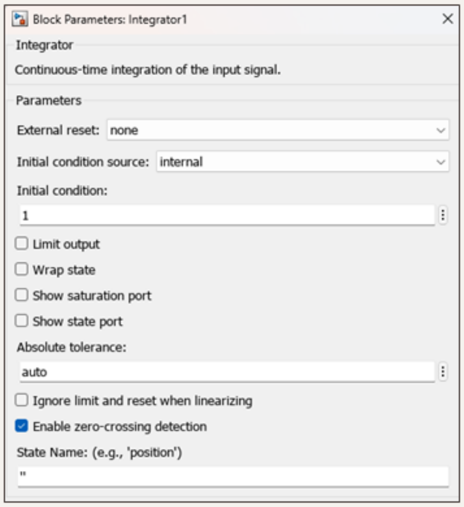
Absolute tolerance: auto

☐ Ignore limit and reset when linearizing

☑ Enable zero-crossing detection

State Name: (e.g., 'position') ''

Bottom Integrator Input


Block Parameters: Integrator1

Integrator

Continuous-time integration of the input signal.

Parameters

External reset: none

Initial condition source: internal

Initial condition: 1

☐ Limit output

☐ Wrap state

☐ Show saturation port

☐ Show state port

Absolute tolerance: auto

☐ Ignore limit and reset when linearizing

☑ Enable zero-crossing detection

State Name: (e.g., 'position') ''

Goto Input

Goto

Send signals to From blocks that have the specified tag. If tag visibility is 'scoped', then a Goto Tag Visibility block must be used to define the visibility of the tag. The block icon displays the selected tag name (local tags are enclosed in brackets, [], and scoped tag names are enclosed in braces, {}).

Parameters

Goto tag: v_sl

Tag visibility: local

From Input

From

Receive signals from the Goto block with the specified tag. If the tag is defined as 'scoped' in the Goto block, then a Goto Tag Visibility block must be used to define the visibility of the tag. After 'Update Diagram', the block icon displays the selected tag name (local tags are enclosed in brackets, [], and scoped tag names are enclosed in braces, {}).

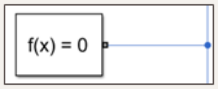
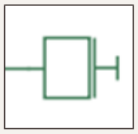
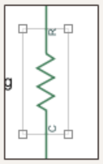
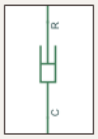
Parameters

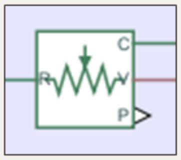
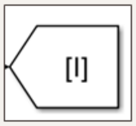
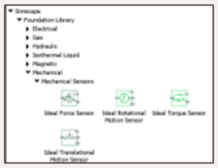
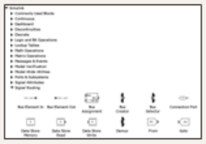
Goto tag: v_sl

Goto source: MQP_Models_MassonSpring/Goto

Icon display: Tag

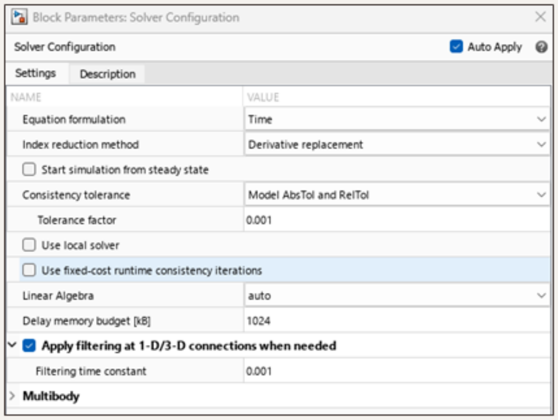
Simscape Model

Symbol	 		 	
Part Name	**Solver Configuration**	**Mass**	**Translational Spring**	**Translational Damper**
Location	**Search Part Name**			

Symbol		 		
Part Name	**Mechanical Translational Reference**	**Ideal Translational Motion Sensor**	**PS-Simulink Converter**	**Goto**
Location		 	**Search Part Name**	

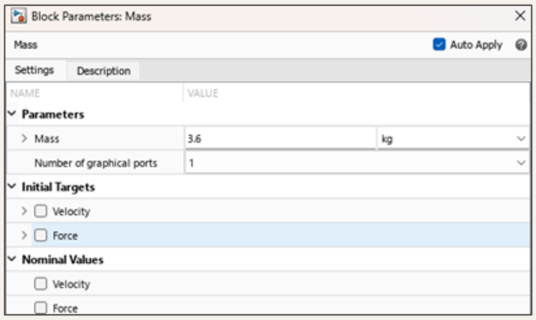
Simscape Model Inputs

Solver Configuration Input



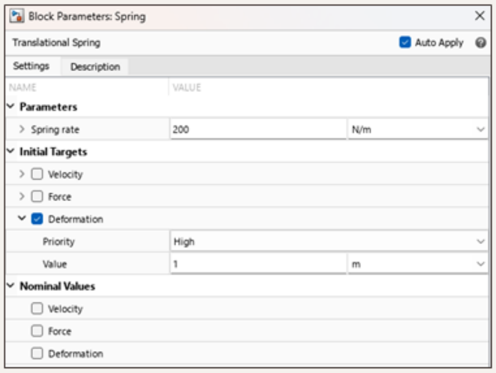
NAME	VALUE
Equation formulation	Time
Index reduction method	Derivative replacement
☐ Start simulation from steady state	
Consistency tolerance	Model AbsTol and RelTol
Tolerance factor	0.001
☐ Use local solver	
☐ Use fixed-cost runtime consistency iterations	
Linear Algebra	auto
Delay memory budget [kB]	1024
☑ **Apply filtering at 1-D/3-D connections when needed**	
Filtering time constant	0.001
Multibody	

Mass Input



NAME	VALUE	
Parameters		
Mass	3.6	kg
Number of graphical ports	1	
Initial Targets		
☐ Velocity		
☐ Force		
Nominal Values		
☐ Velocity		
☐ Force		

Translational Spring Input



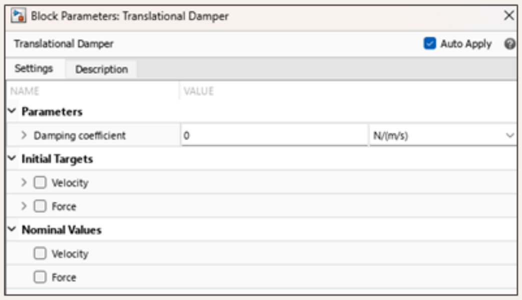
Block Parameters: Spring

Translational Spring ☑ Auto Apply

Settings | Description

NAME	VALUE	
Parameters		
Spring rate	200	N/m
Initial Targets		
☐ Velocity		
☐ Force		
☑ Deformation		
Priority	High	
Value	1	m
Nominal Values		
☐ Velocity		
☐ Force		
☐ Deformation		

Translational Damper Input



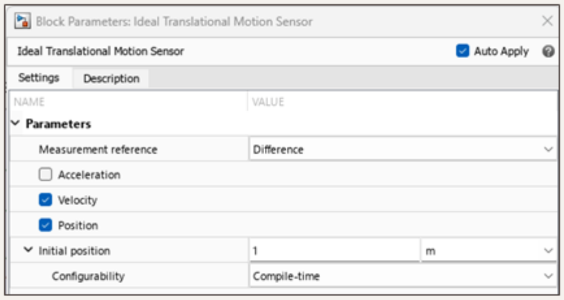
Block Parameters: Translational Damper

Translational Damper ☑ Auto Apply

Settings | Description

NAME	VALUE	
Parameters		
Damping coefficient	0	N/(m/s)
Initial Targets		
☐ Velocity		
☐ Force		
Nominal Values		
☐ Velocity		
☐ Force		

Translational Motion Sensor Input



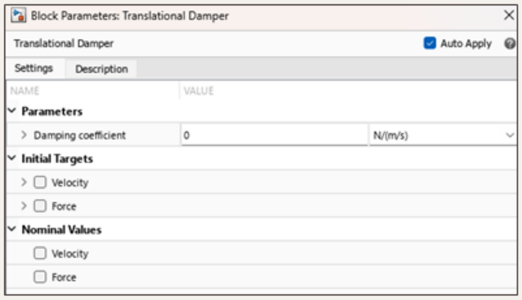
Block Parameters: Ideal Translational Motion Sensor

Ideal Translational Motion Sensor ☑ Auto Apply

Settings | Description

NAME	VALUE	
Parameters		
Measurement reference	Difference	
☐ Acceleration		
☑ Velocity		
☑ Position		
Initial position	1	m
Configurability	Compile-time	

Translational Damper Input



Block Parameters: Translational Damper

Translational Damper ☑ Auto Apply

Settings | Description

NAME	VALUE	
Parameters		
Damping coefficient	0	N/(m/s)
Initial Targets		
☐ Velocity		
☐ Force		
Nominal Values		
☐ Velocity		
☐ Force		

Goto Input

Goto

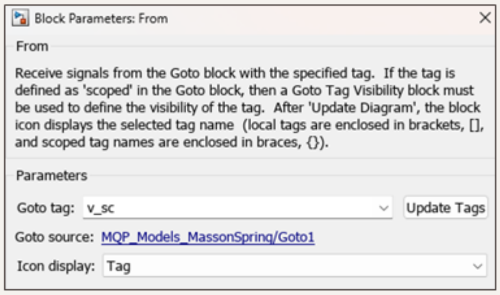
Send signals to From blocks that have the specified tag. If tag visibility is 'scoped', then a Goto Tag Visibility block must be used to define the visibility of the tag. The block icon displays the selected tag name (local tags are enclosed in brackets, [], and scoped tag names are enclosed in braces, {}).

Parameters

Goto tag: v_sc

Tag visibility: local

From Input



From

Receive signals from the Goto block with the specified tag. If the tag is defined as 'scoped' in the Goto block, then a Goto Tag Visibility block must be used to define the visibility of the tag. After 'Update Diagram', the block icon displays the selected tag name (local tags are enclosed in brackets, [], and scoped tag names are enclosed in braces, {}).

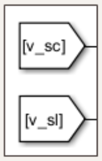
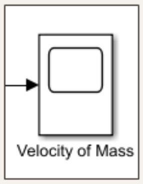
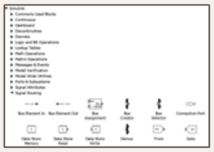
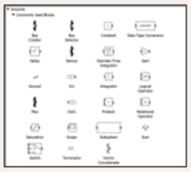
Parameters

Goto tag: v_sc

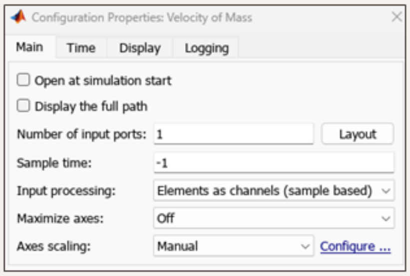
Goto source: MQP_Models_MassonSpring/Goto1

Icon display: Tag

Data Measurement

Symbol			
Part Name	**From x2**	**Mux**	**Scope**
Location			

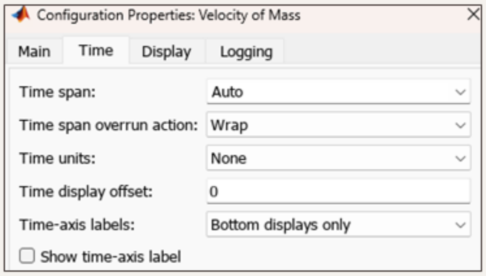
Scope Inputs



Configuration Properties: Velocity of Mass

Main | Time | Display | Logging

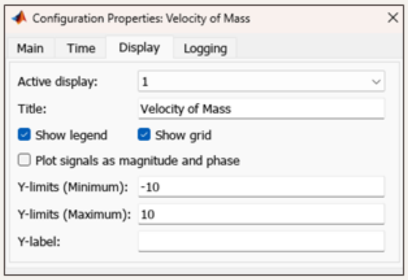
☐ Open at simulation start
☐ Display the full path
Number of input ports: 1
Sample time: -1
Input processing: Elements as channels (sample based)
Maximize axes: Off
Axes scaling: Manual



Configuration Properties: Velocity of Mass

Main | Time | Display | Logging

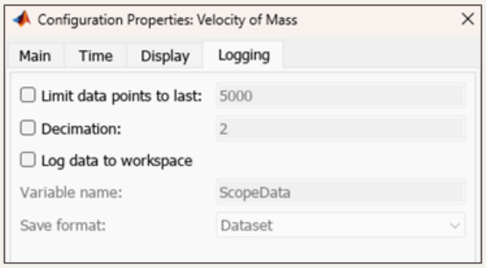
Time span: Auto
Time span overrun action: Wrap
Time units: None
Time display offset: 0
Time-axis labels: Bottom displays only
☐ Show time-axis label



Configuration Properties: Velocity of Mass

Main | Time | Display | Logging

Active display: 1
Title: Velocity of Mass
☑ Show legend ☑ Show grid
☐ Plot signals as magnitude and phase
Y-limits (Minimum): -10
Y-limits (Maximum): 10
Y-label:



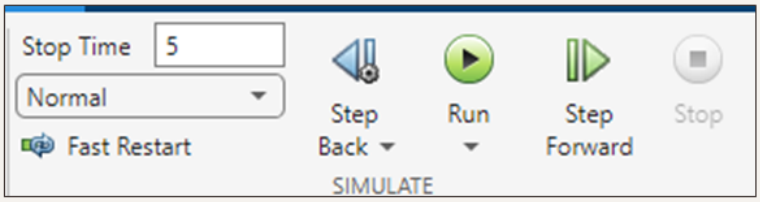
Configuration Properties: Velocity of Mass

Main | Time | Display | Logging

☐ Limit data points to last: 5000
☐ Decimation: 2
☐ Log data to workspace
Variable name: ScopeData
Save format: Dataset

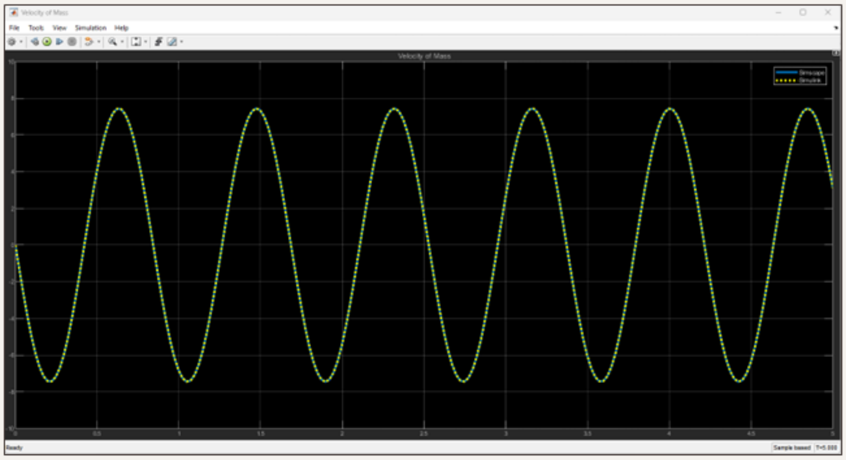
Run the Simulation

- Make sure Stop time = 5



Stop Time: 5
Normal

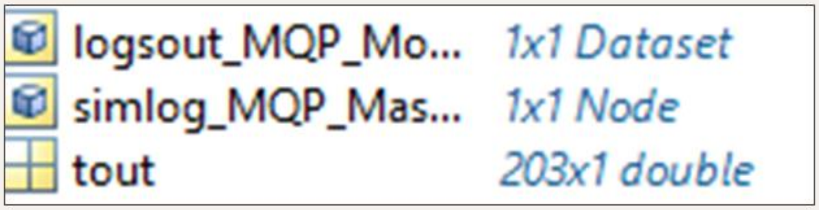
If Everything is Correct, Run the Simulation



Does Your Output Look Like This?

If your simulation output matches, proceed to integrate the code seen on the following slides in order to plot the simulation results.

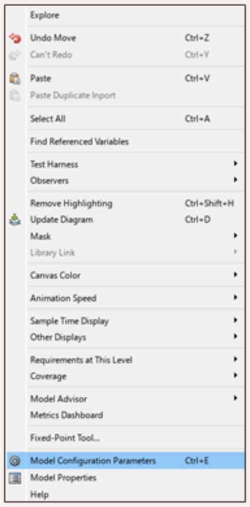
Variables Needed for MATLAB Code



logsout_MQP_Mo...	1x1 Dataset
simlog_MQP_Mas...	1x1 Node
tout	203x1 double

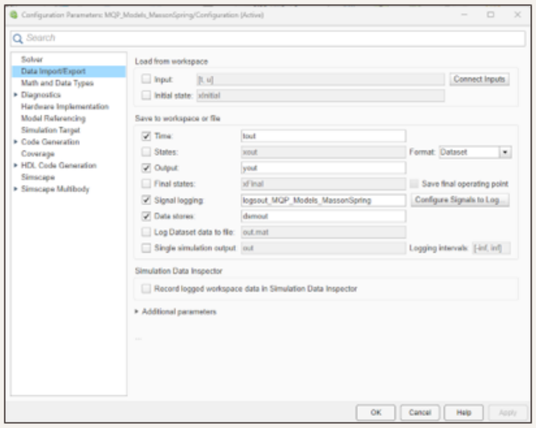
How to Get Variables

Back on Simulation Screen, right click and select "Model Configuration Parameters"



How to Get Variables

How to get signal logging variable



Load from workspace

☐ Input: [t, u]

☐ Initial state: xInitial

Save to workspace or file

☑ Time: tout

☐ States: xout   Format: Dataset

☑ Output: yout

☐ Final states: xFinal   ☐ Save final operating point

☑ Signal logging: logsout_MQP_Models_MassonSpring

☑ Data stores: dsmout

☐ Log Dataset data to file: out.mat

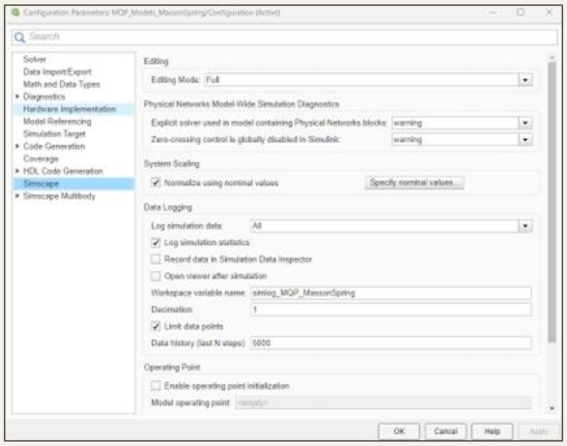
☐ Single simulation output: out   Logging intervals: [-inf, inf]

Simulation Data Inspector

☐ Record logged workspace data in Simulation Data Inspector

How to Get Variables

How to get logged simulation data



Editing

Editing Mode: Full

Physical Networks Model-Wide Simulation Diagnostics

Explicit solver used in model containing Physical Networks blocks: warning

Zero-crossing control is globally disabled in Simulink: warning

System Scaling

☑ Normalize using nominal values

Data Logging

Log simulation data: All

☑ Log simulation statistics

☐ Record data in Simulation Data Inspector

☐ Open viewer after simulation

Workspace variable name: simlog_MQP_MassonSpring

Decimation: 1

☑ Limit data points

Data history (last N steps): 5000

Operating Point

☐ Enable operating point initialization

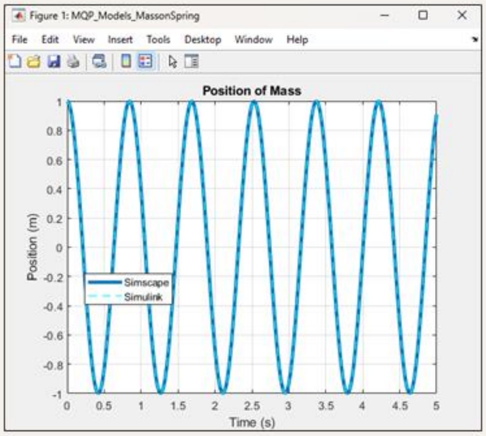
Model operating point: <empty>

Insert this MATLAB Code for Simulation Result Plot

```
% Code to plot simulation results from MQP_Models_MassonSpring


% Reuse figure if it exists, else create new figure
try
figure(h1_MassSpring)
catch
h1_MassSpring=figure('Name', 'Mass on Spring');
end


% Generate simulation results if they don't exist
if(~exist('simlog_MQP_Models_MassonSpring','var')) %This is the name given to the data logging variable
sim('MQP_Models_MassonSpring') %'MQP_Models_MassonSpring' is the file name used. Your code must have your file name inserted.
end


% Get simulation results
temp_x1_sl=get(logsout_MQP_Models_MassonSpring,'Integrator_R_Simulink'); % Simulink
%This is the name given to the signal logging variable.
temp_x1_sc=simlog_MQP_MassonSpring.Spring.x.series; % Simscape
%This is the name given to the data logging variable


% Plot results
plot(temp_x1_sc.time,temp_x1_sc.values,'LineWidth',3)
hold on
plot(temp_x1_sl.Values.Time,temp_x1_sl.Values.Data,'c--','LineWidth',1)
hold off
grid on
xlabel('Time (s)');
ylabel('Position (m)');
title('Position of Mass');
legend({'Simscape','Simulink'},'Location','Best');


% Remove temporary variables
clear temp_x1_sl temp_x1_sc
```

Minor Changes may be Needed. See Code Comments.

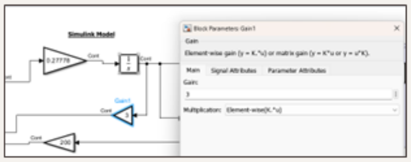
Run the Simulation again and the Run the Code and the plot should look like the following



What changes when a Damper is introduced to the system?

Within the Simulink Model, adjust the following:

Middle Gain Input



Block Parameters: Gain1

Gain

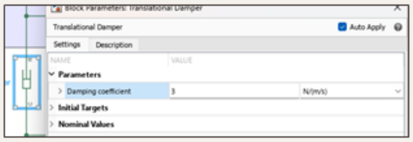
Element-wise gain (y = K.*u) or matrix gain (y = K*u or y = u*K).

Gain: 3

Multiplication: Element-wise(K.*u)

Within the Simscape Model, adjust the following:

Translational Damper

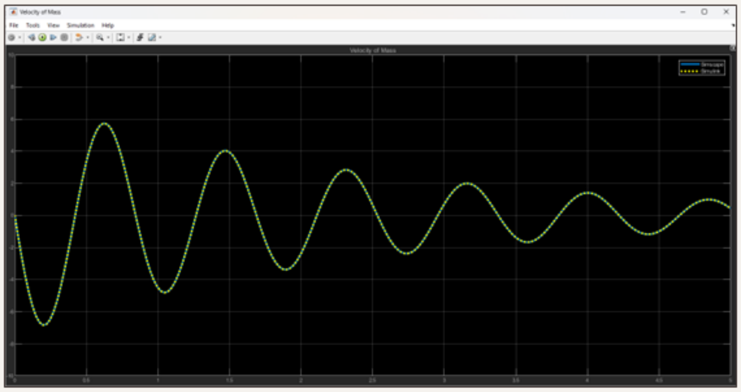

Block Parameters: Translational Damper

Translational Damper

NAME	VALUE	
Parameters		
Damping coefficient	3	N/(m/s)
Initial Targets		
Nominal Values		

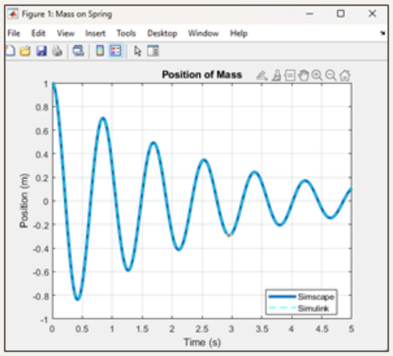
If Everything is Correct, Run the Simulation

Does Your Output Look Like This?



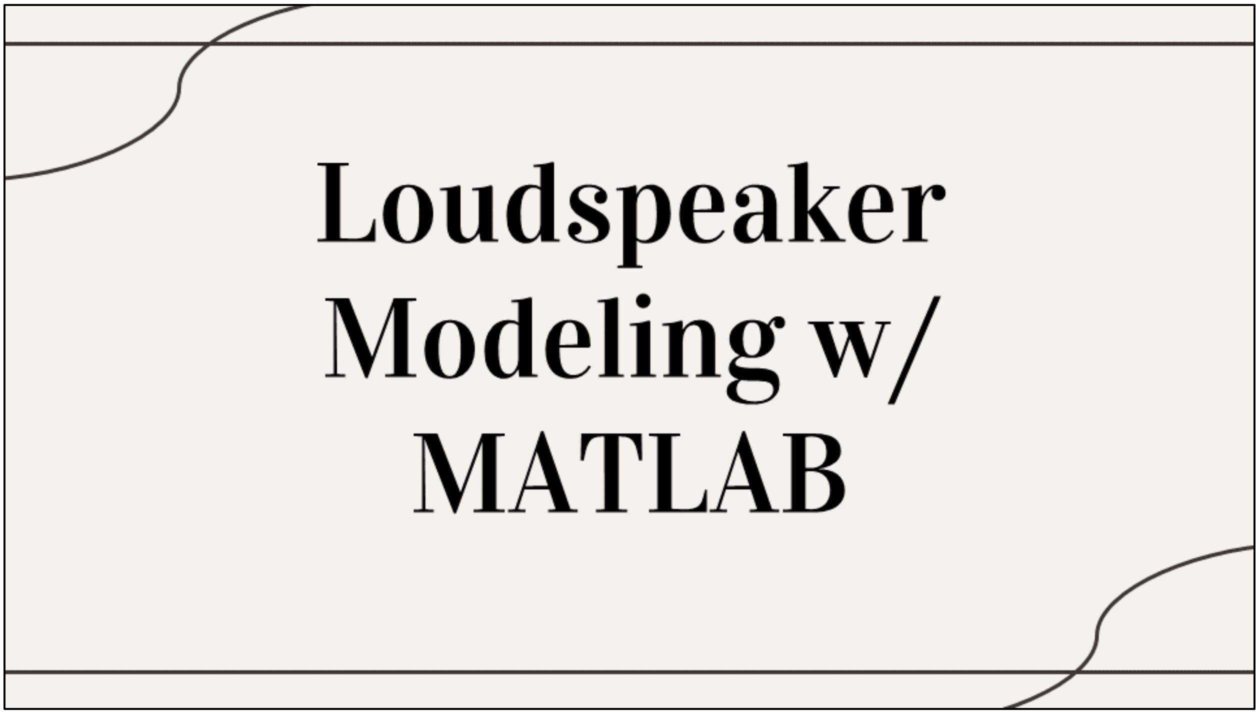
Following Correct Simulation, Run the Code

Does Your Output Look Like This?

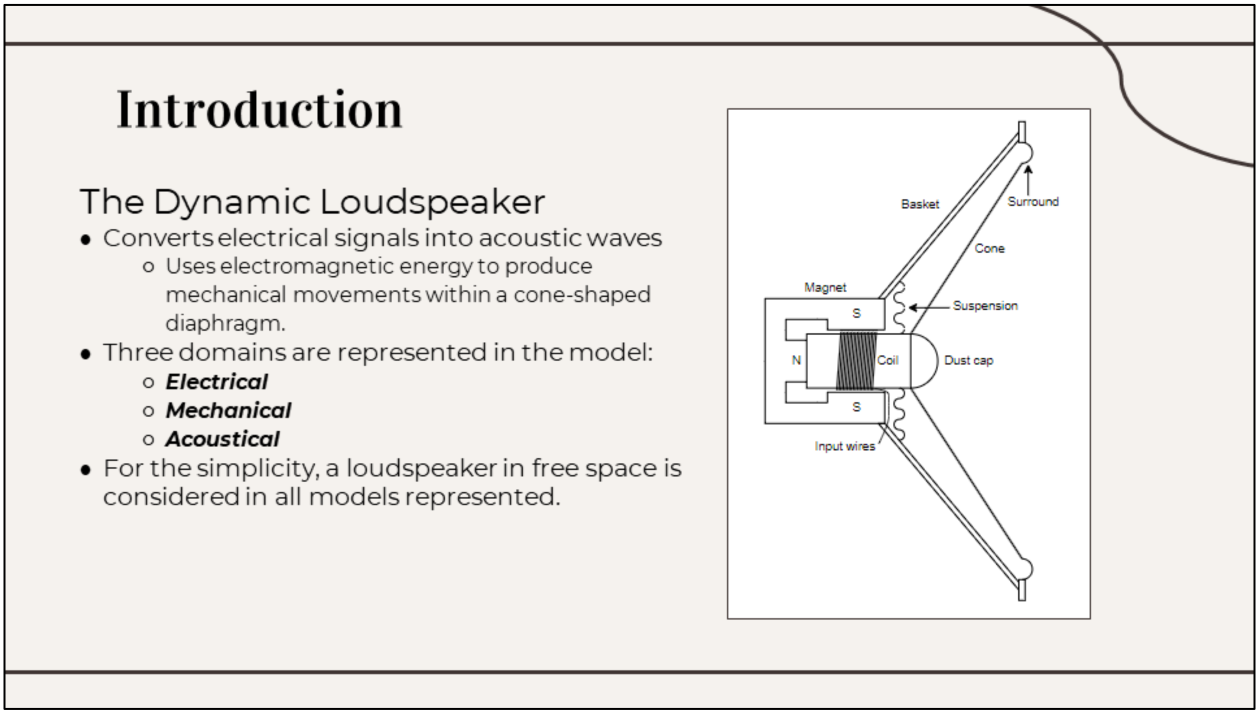


How is this output different to the prior simulation with no damper?

Appendix F – Loudspeaker Modeling Slides



Loudspeaker Modeling w/ MATLAB



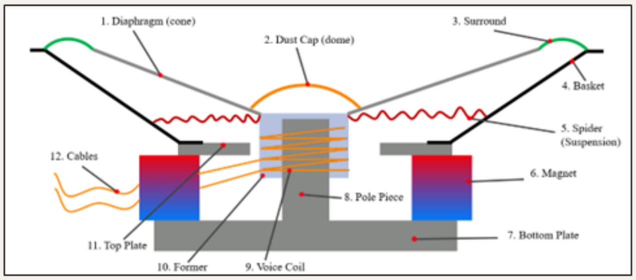
Introduction

The Dynamic Loudspeaker

- Converts electrical signals into acoustic waves
  - Uses electromagnetic energy to produce mechanical movements within a cone-shaped diaphragm.
- Three domains are represented in the model:
  - ***Electrical***
  - ***Mechanical***
  - ***Acoustical***
- For the simplicity, a loudspeaker in free space is considered in all models represented.

Introduction - Loudspeaker Make-Up

1. **Diaphragm (Cone)**: Moves in and out to push air and make sound.
2. **Dust cap (dome)**: Protects the voice coil from dust and dirt.
3. **Surround**: A piece of elastic rubber, foam, or textile that flexibly fastens the diaphragm to the basket (outer frame).
4. **Basket**: The sturdy metal framework around which the speaker is built.
5. **Spider (suspension)**: A flexible, corrugated support that holds the voice coil in place, while allowing it to move freely.
6. **Magnet**: Typically made from ferrite or powerful neodymium.
7. **Bottom Plate**: Made of soft iron.
8. **Pole Piece**: Concentrates the magnetic field produced by the voice coil.
9. **Voice Coil**: The coil that moves the diaphragm back and forth.
10. **Former**: A cylinder of cardboard or other material onto which the coil is wound.
11. **Top Plate**: Also made of soft iron.
12. **Cables**: Connect stereo amplifier unit to voice coil.

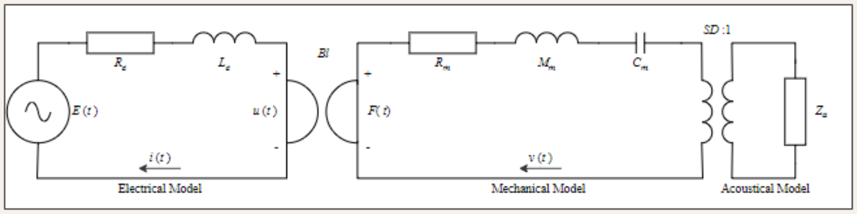


Introduction

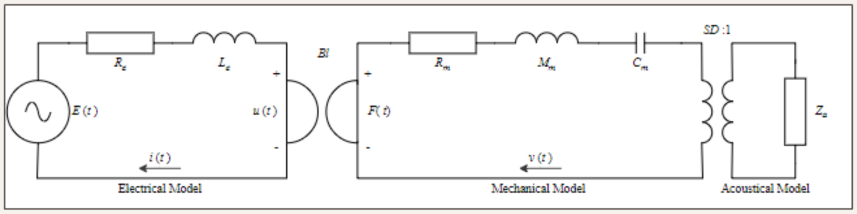
The examples illustrated are for the dynamic loudspeaker in which linear and nonlinear lumped element models are utilizied.

- Linear Elements: Show a linear relationship between voltage and current
  - Examples: Resistors, Inductors, Capacitors, etc.
- Nonlinear Elements: Do not show a linear relation b/w voltage and current
  - Examples: Voltage and Current Sources

For a loudspeaker, a common model is to represent the system as an electrical circuit, a lumped element model



Electrical Circuit Explained

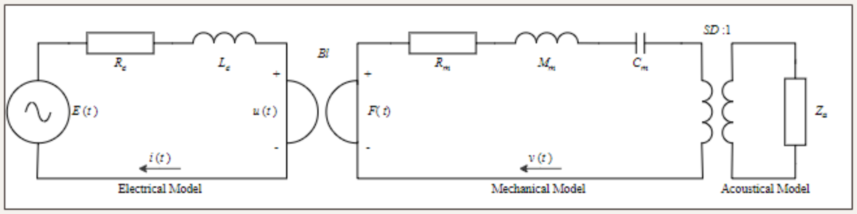


Inside the electrical model:

- Illustrated here is the voice coil and magnet, together creating the motor for the loudspeaker
  - ***E(t) is the voltage that drives the voice coil***
  - ***R_e and L_e are the resistance and inductance of the voice coil***
    - L_e is also impacted by the magnet because of its ferrous core

---

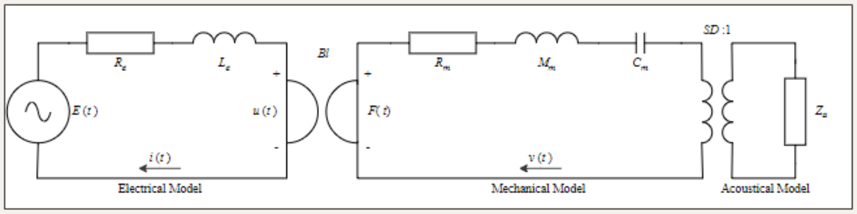
Electrical Circuit Explained



Inside the electrical model:

- The magnet has a magnetic field with a flux density B.
  - When multiplied by the wire length (l) the force factor is created, ***Bl***
  - The force factor is the conversion factor b/w electrical and mechanical domains.
    - This is the force applied to the voice coil, ***F(t) = Bl*i(t)*** where ***i(t)*** = electrical current applied at the input.
- Inversely, there is a voltage, ***u(t) = Bl*v(t)*** where ***v(t)*** is analogous to the cone velocity.
- As seen in the circuit diagram, the conversion b/w electrical and mechanical domains is through a ***gyrator***.
  - ***Velocity, v(t), corresponds to electrical current, i(t)***
  - ***Force, F(t), corresponds to voltage, u(t)***

Electrical Circuit Explained

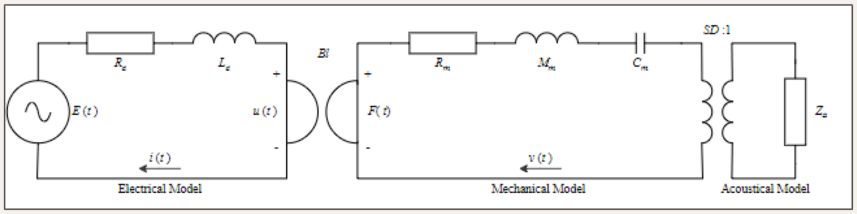


Inside the Mechanical model:

The electrical components are analogous to mechanical properities like mass and compliance.

- ***M_m = the inertia of the total moving mass***
  - Includes the coil, cone, and dust cap.
- ***C_m = the stiffness of the suspension and spider***
- ***R_m = mechanical loss in the suspension system***

Electrical Circuit Explained



Inside the Acoustical model:

The connection b/w the mechanical and acoustical model is analogous to a ***transformer***.

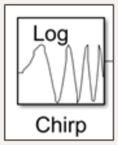
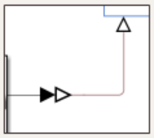
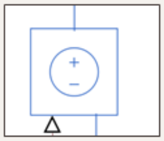
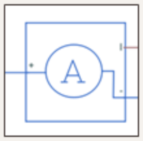
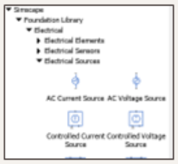
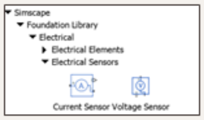
- Connection is where the driver cone surface interfaces with air.
  - For a given mass, more mechanical energy is transformed to acoustic energy through a larger cone.
- ***Z_a = an impedance (acoustic impedance)***, analogous to radiation resistance for the front and back of the cone.
  - Formed by R_a = acoustic resistance, C_a = acoustic compliance, and M_a = mass of air moving in and out.
  - Assume Z_a = 0 since modeling loudspeaker in free space.

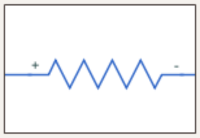
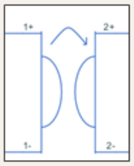
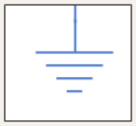
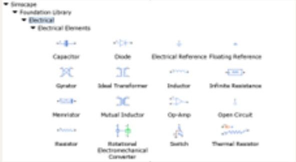
How to Find all of the Parts While Modeling

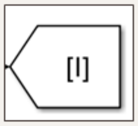
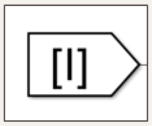
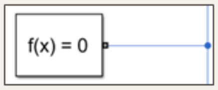
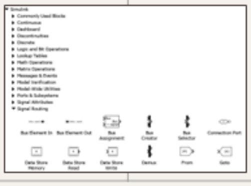
All parts can be searched by their name in the library browser.

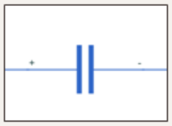
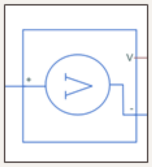
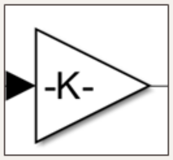
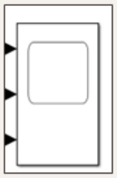
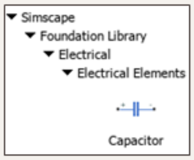
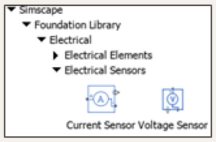
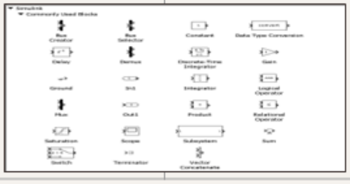
In addition, the following slides have a detailed break down of every part name, symbol, and location in the library browser.

- Simulink
- Aerospace Blockset
- Audio Toolbox
- Automated Driving Toolbox
- AUTOSAR Blockset
- C2000 Microcontroller Blockset
- Communications Toolbox
- Communications Toolbox HDL Support
- Computer Vision Toolbox
- Control System Toolbox
- Data Acquisition Toolbox
- DDS Blockset
- Deep Learning Toolbox
- DSP HDL Toolbox
- DSP System Toolbox
- DSP System Toolbox HDL Support
- Embedded Coder
- Fixed-Point Designer
- Fixed-Point Designer HDL Support
- Fuzzy Logic Toolbox
- HDL Coder
- HDL Verifier
- Image Acquisition Toolbox
- Industrial Communication Toolbox
- Instrument Control Toolbox
- Lidar Toolbox

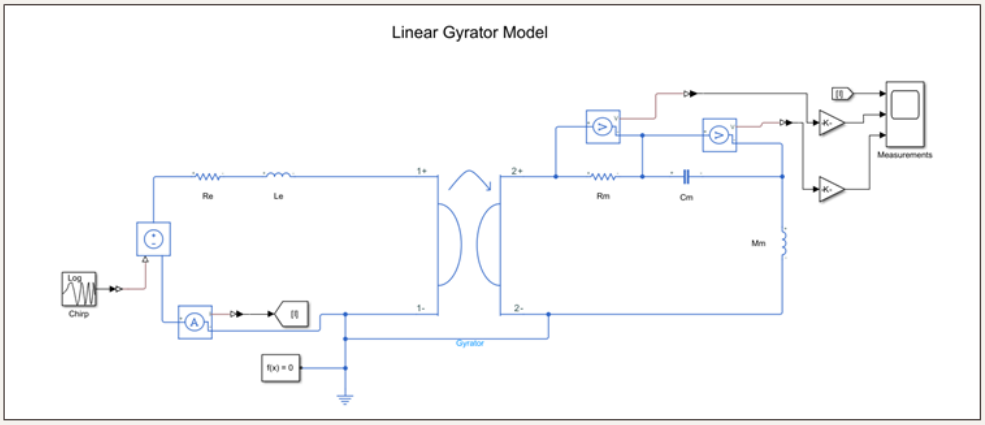
Symbol				
Part Name	**Chirp**	**Simulink-PS Converter**	**Controlled Voltage Source**	**Current Sensor**
Location	DSP System Toolbox Dataflow Estimation Feature Extractors Filtering Math Functions Quantizers Signal Management Signal Operations Sinks Sources Binary File Reader Chirp	**Search Part Name**	Simscape Foundation Library Electrical Electrical Elements Electrical Sensors Electrical Sources AC Current Source AC Voltage Source Controlled Current Source Controlled Voltage Source	Simscape Foundation Library Electrical Electrical Elements Electrical Sensors Current Sensor Voltage Sensor

Symbol				
Part Name	Resistor	Inductor	Gyrator	Electrical Reference (Ground)
Location				

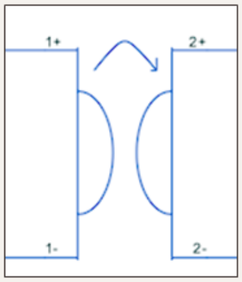
Symbol				
Part Name	PS-Simulink Converter	Goto	From	Solver Configuration
Location	Search Part Name			Search Part Name

Symbol				
Part Name	**Capacitor**	**Voltage Sensor**	**Gain**	**Scope**
Location				

Construct the Linear Gyrator Circuit as Shown Below



Note: Do not worry about specific input values, simply connect the parts as shown here



This is the Gyrator.

Specifically in this model circuit design, the gyrator is implemented to convert the mechanical domain of the loudspeaker circuit into the electrical domain.

Linear Gyrator Inputs

Log Chirp Inputs

Chirp (mask) (link)

Linear, Logarithmic, and Quadratic modes generate a swept-frequency cosine with instantaneous frequency values specified by the frequency and time parameters. The Swept cosine mode generates a swept-frequency cosine with a linear instantaneous output frequency that may differ from the one specified by the frequency and time parameters.

Parameters

Frequency sweep: Logarithmic

Sweep mode: Unidirectional

Initial frequency (Hz): 20

Target frequency (Hz): 24000

Target time (s): 1

Sweep time (s): 1

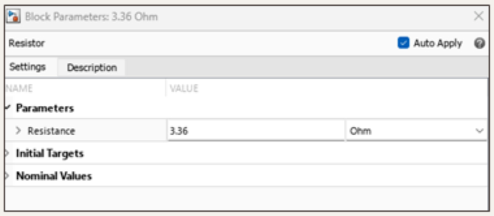
Initial phase (rad): 0

Sample time: 2.0833e-05

Samples per frame: 1

Output data type: Double

Resistor (Re) Input



Resistor

☑ Auto Apply

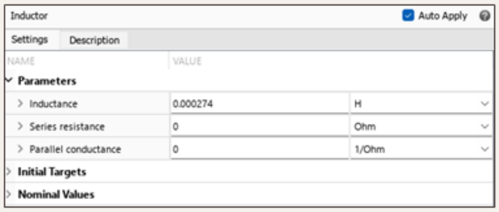
Parameters

Resistance	3.36	Ohm

Initial Targets

Nominal Values

Inductor (Le) Input



Inductor

☑ Auto Apply

Parameters

Inductance	0.000274	H
Series resistance	0	Ohm
Parallel conductance	0	1/Ohm

Initial Targets

Nominal Values

Goto Block Input

Goto

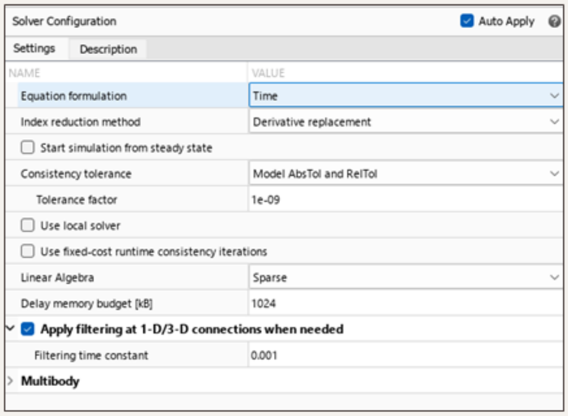
Send signals to From blocks that have the specified tag. If tag visibility is 'scoped', then a Goto Tag Visibility block must be used to define the visibility of the tag. The block icon displays the selected tag name (local tags are enclosed in brackets, [], and scoped tag names are enclosed in braces, {}).

Parameters

Goto tag: I

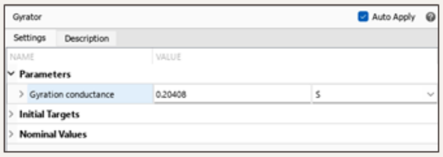
Tag visibility: local

Solver Configuration Input



NAME	VALUE
Equation formulation	Time
Index reduction method	Derivative replacement
☐ Start simulation from steady state	
Consistency tolerance	Model AbsTol and RelTol
Tolerance factor	1e-09
☐ Use local solver	
☐ Use fixed-cost runtime consistency iterations	
Linear Algebra	Sparse
Delay memory budget [kB]	1024
☑ **Apply filtering at 1-D/3-D connections when needed**	
Filtering time constant	0.001
Multibody	

Gyrator Input



NAME	VALUE	
Parameters		
Gyration conductance	0.20408	S
Initial Targets		
Nominal Values		

Right Loop Inputs For:

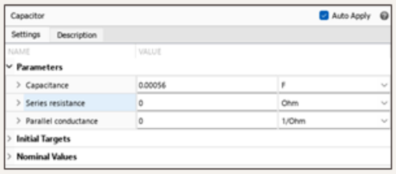
Resistor (Rm):

NAME	VALUE	
Parameters		
Resistance	0.784	Ohm

Inductor (Mm):

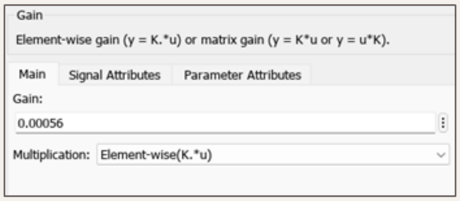
NAME	VALUE	
Parameters		
Inductance	0.0147	H
Series resistance	0	Ohm
Parallel conductance	0	1/Ohm
Initial Targets		
Nominal Values		

Capacitor (Cm):



NAME	VALUE	
Parameters		
Capacitance	0.00056	F
Series resistance	0	Ohm
Parallel conductance	0	1/Ohm
Initial Targets		
Nominal Values		

Gain Input (Both)



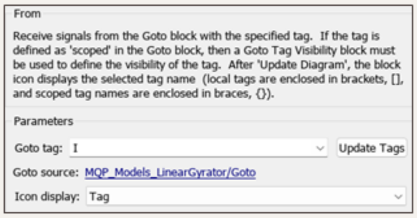
Gain

Element-wise gain (y = K.*u) or matrix gain (y = K*u or y = u*K).

Gain: 0.00056

Multiplication: Element-wise(K.*u)

From Block Input



From

Receive signals from the Goto block with the specified tag. If the tag is defined as 'scoped' in the Goto block, then a Goto Tag Visibility block must be used to define the visibility of the tag. After 'Update Diagram', the block icon displays the selected tag name (local tags are enclosed in brackets, [], and scoped tag names are enclosed in braces, {}).

Parameters

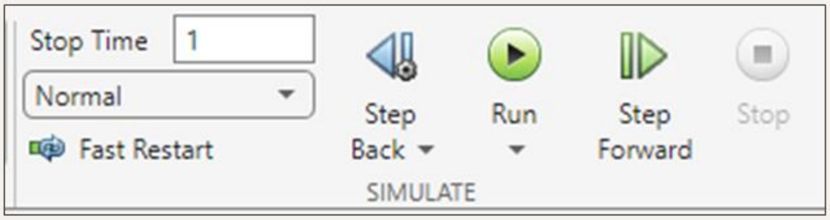
Goto tag: I

Goto source: MQP_Models_LinearGyrator/Goto

Icon display: Tag

Run the Simulation

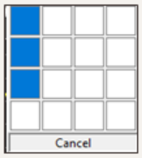
- Make sure Stop time = 1



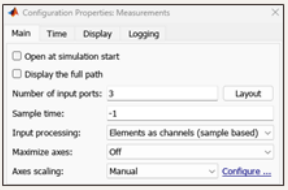
Stop Time: 1

Final Graph Data

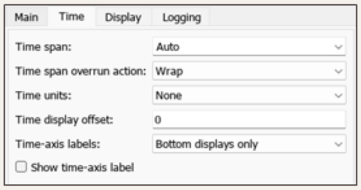
For the correct graph output, make sure you have 3 layouts stacked vertical on top of each other.
- If you do not, go to view tab, click layout, and drag till three blue boxes line up on each other.



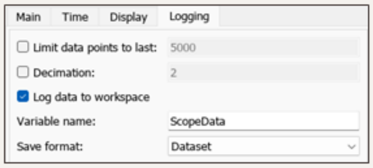
The Following are the Scope Settings:



☐ Open at simulation start
☐ Display the full path
Number of input ports: 3
Sample time: -1
Input processing: Elements as channels (sample based)
Maximize axes: Off
Axes scaling: Manual



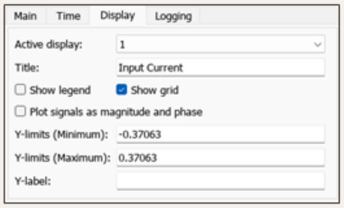
Time span: Auto
Time span overrun action: Wrap
Time units: None
Time display offset: 0
Time-axis labels: Bottom displays only
☐ Show time-axis label



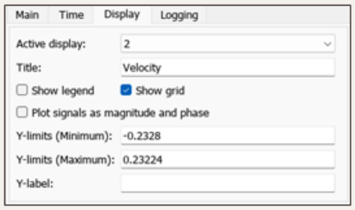
☐ Limit data points to last: 5000
☐ Decimation: 2
☑ Log data to workspace
Variable name: ScopeData
Save format: Dataset

Final Graph Data

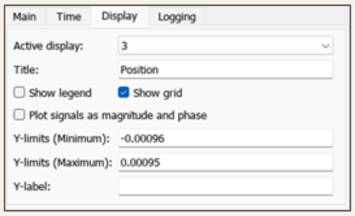
The Scope Display Settings are as follows:



Active display: 1
Title: Input Current
☐ Show legend ☑ Show grid
☐ Plot signals as magnitude and phase
Y-limits (Minimum): -0.37063
Y-limits (Maximum): 0.37063
Y-label:

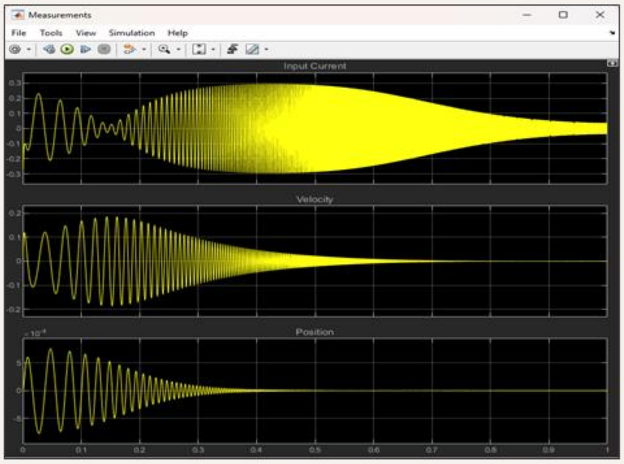


Active display: 2
Title: Velocity
☐ Show legend ☑ Show grid
☐ Plot signals as magnitude and phase
Y-limits (Minimum): -0.2328
Y-limits (Maximum): 0.23224
Y-label:



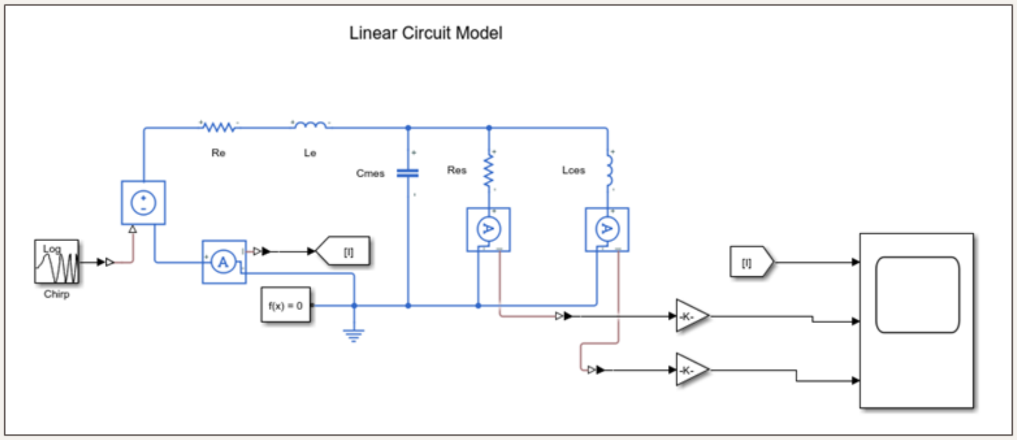
Active display: 3
Title: Position
☐ Show legend ☑ Show grid
☐ Plot signals as magnitude and phase
Y-limits (Minimum): -0.00096
Y-limits (Maximum): 0.00095
Y-label:

If Everything is Correct, Re-Run the Simulation



Do your Scope Outputs Look Like This?

Now Construct the Linear Circuit Model



Linear Circuit Model Inputs

Log Chirp Inputs


Block Parameters: Chirp

Chirp (mask) (link)

Linear, Logarithmic, and Quadratic modes generate a swept-frequency cosine with instantaneous frequency values specified by the frequency and time parameters. The Swept cosine mode generates a swept-frequency cosine with a linear instantaneous output frequency that may differ from the one specified by the frequency and time parameters.

Parameters

Frequency sweep: Logarithmic

Sweep mode: Unidirectional

Initial frequency (Hz): 20

Target frequency (Hz): 24000

Target time (s): 1

Sweep time (s): 1

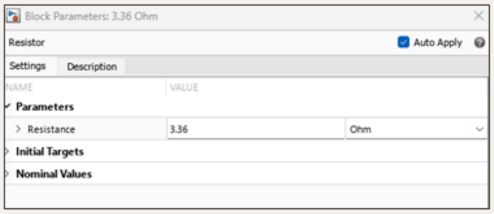
Initial phase (rad): 0

Sample time: 2.0833e-05

Samples per frame: 1

Output data type: Double

Resistor (Re) Input

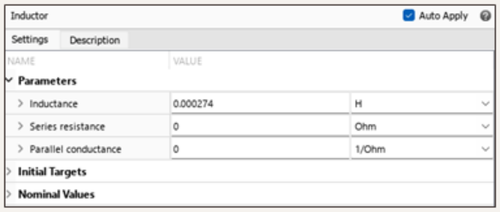

Block Parameters: 3.36 Ohm

Resistor

☑ Auto Apply

NAME	VALUE	
Parameters		
Resistance	3.36	Ohm
Initial Targets		
Nominal Values		

Inductor (Le) Input



NAME	VALUE	
Parameters		
Inductance	0.000274	H
Series resistance	0	Ohm
Parallel conductance	0	1/Ohm
Initial Targets		
Nominal Values		

Goto Block Input

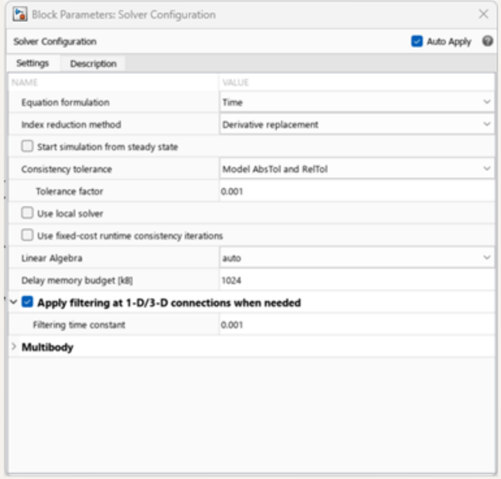
Send signals to From blocks that have the specified tag. If tag visibility is 'scoped', then a Goto Tag Visibility block must be used to define the visibility of the tag. The block icon displays the selected tag name (local tags are enclosed in brackets, [], and scoped tag names are enclosed in braces, {}).

Parameters

Goto tag: I

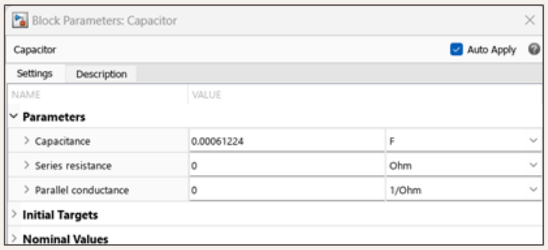
Tag visibility: local

Solver Configuration Input



NAME	VALUE
Equation formulation	Time
Index reduction method	Derivative replacement
☐ Start simulation from steady state	
Consistency tolerance	Model AbsTol and RelTol
Tolerance factor	0.001
☐ Use local solver	
☐ Use fixed-cost runtime consistency iterations	
Linear Algebra	auto
Delay memory budget [kB]	1024
☑ **Apply filtering at 1-D/3-D connections when needed**	
Filtering time constant	0.001
Multibody	

Capacitance Input

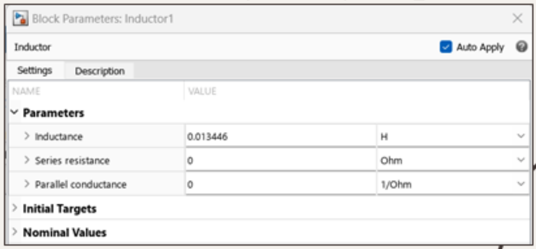


NAME	VALUE	
Parameters		
Capacitance	0.00061224	F
Series resistance	0	Ohm
Parallel conductance	0	1/Ohm
Initial Targets		
Nominal Values		

Resistor (Res) Input

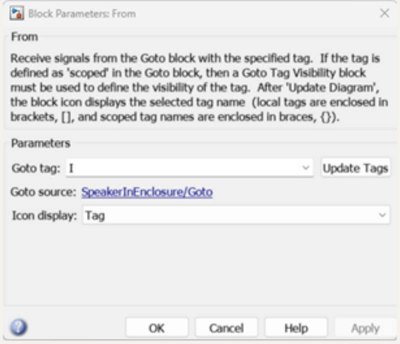
NAME	VALUE	
Parameters		
Resistance	30.625	Ohm
Initial Targets		
Nominal Values		

Inductor (Lces) Input



NAME	VALUE	
Parameters		
Inductance	0.013446	H
Series resistance	0	Ohm
Parallel conductance	0	1/Ohm
Initial Targets		
Nominal Values		

From Block Input



From

Receive signals from the Goto block with the specified tag. If the tag is defined as 'scoped' in the Goto block, then a Goto Tag Visibility block must be used to define the visibility of the tag. After 'Update Diagram', the block icon displays the selected tag name (local tags are enclosed in brackets, [], and scoped tag names are enclosed in braces, {}).

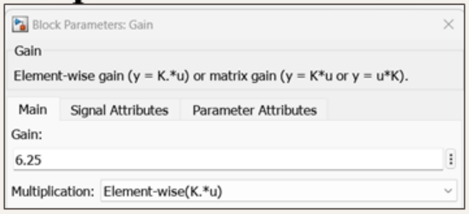
Parameters

Goto tag: I

Goto source: SpeakerInEnclosure/Goto

Icon display: Tag

Top Gain



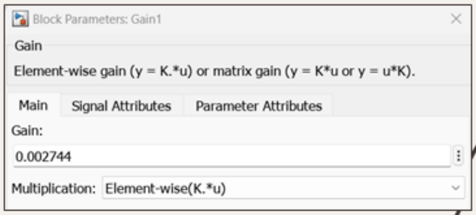
Gain

Element-wise gain (y = K.*u) or matrix gain (y = K*u or y = u*K).

Gain: 6.25

Multiplication: Element-wise(K.*u)

Bottom Gain



Gain

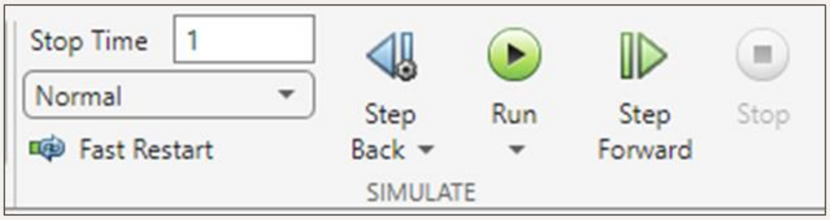
Element-wise gain (y = K.*u) or matrix gain (y = K*u or y = u*K).

Gain: 0.002744

Multiplication: Element-wise(K.*u)

Run the Simulation

- Make sure Stop time = 1

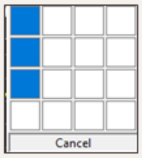


Stop Time 1

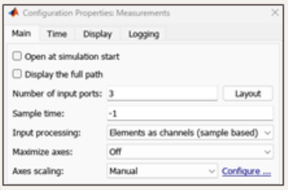
Normal

Final Graph Data

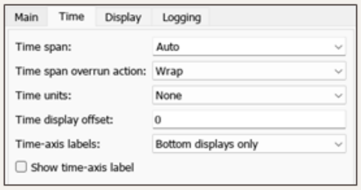
For the correct graph output, make sure you have 3 layouts stacked vertical on top of each other.
- If you do not, go to view tab, click layout, and drag till three blue boxes line up on each other.



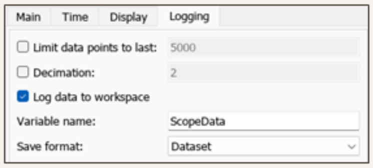
The Following are the Scope Settings:



☐ Open at simulation start
☐ Display the full path
Number of input ports: 3
Sample time: -1
Input processing: Elements as channels (sample based)
Maximize axes: Off
Axes scaling: Manual



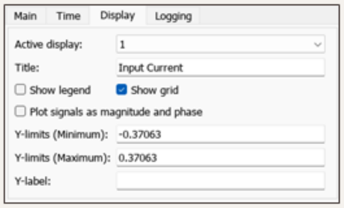
Time span: Auto
Time span overrun action: Wrap
Time units: None
Time display offset: 0
Time-axis labels: Bottom displays only
☐ Show time-axis label



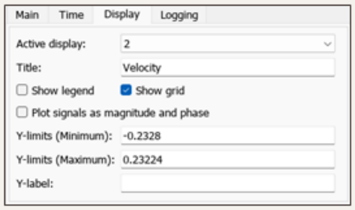
☐ Limit data points to last: 5000
☐ Decimation: 2
☑ Log data to workspace
Variable name: ScopeData
Save format: Dataset

Final Graph Data

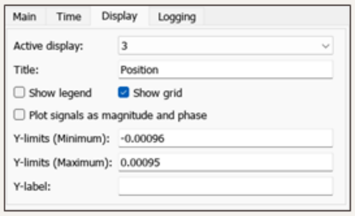
The Scope Display Settings are as follows:



Active display: 1
Title: Input Current
☐ Show legend ☑ Show grid
☐ Plot signals as magnitude and phase
Y-limits (Minimum): -0.37063
Y-limits (Maximum): 0.37063
Y-label:

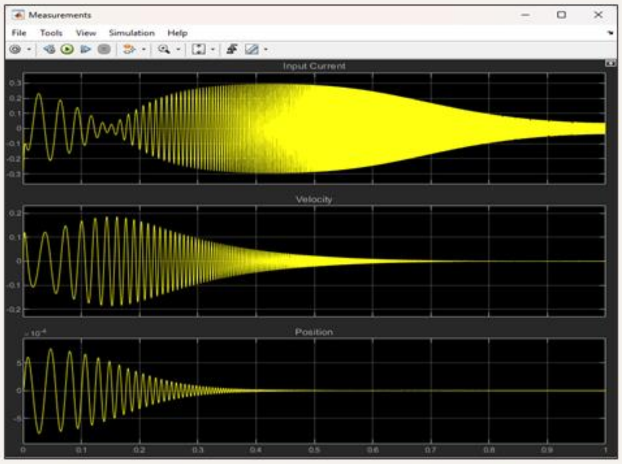


Active display: 2
Title: Velocity
☐ Show legend ☑ Show grid
☐ Plot signals as magnitude and phase
Y-limits (Minimum): -0.2328
Y-limits (Maximum): 0.23224
Y-label:



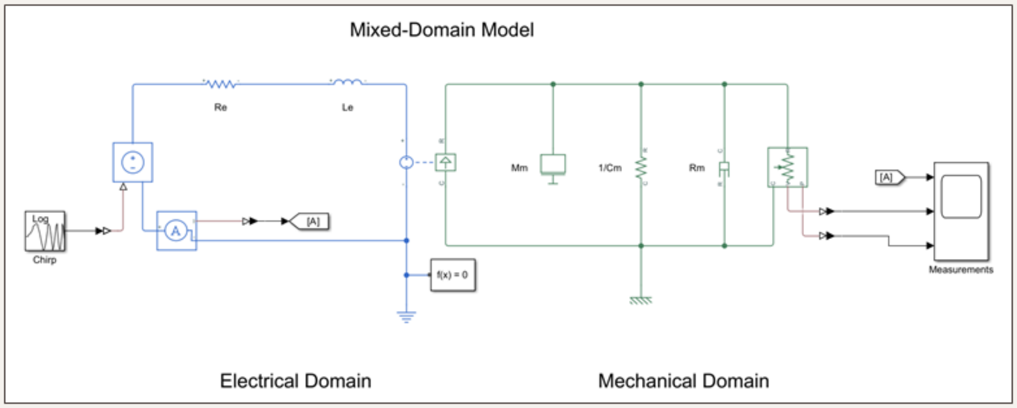
Active display: 3
Title: Position
☐ Show legend ☑ Show grid
☐ Plot signals as magnitude and phase
Y-limits (Minimum): -0.00096
Y-limits (Maximum): 0.00095
Y-label:

If Everything is Correct, Re-Run the Simulation



Do your Scope Outputs Look Like This?

Now Construct the Mixed Domain Model Below



Mixed-Domain Model Inputs

Log Chirp Inputs

Block Parameters: Chirp

Chirp (mask) (link)

Linear, Logarithmic, and Quadratic modes generate a swept-frequency cosine with instantaneous frequency values specified by the frequency and time parameters. The Swept cosine mode generates a swept-frequency cosine with a linear instantaneous output frequency that may differ from the one specified by the frequency and time parameters.

Parameters

Frequency sweep: Logarithmic

Sweep mode: Unidirectional

Initial frequency (Hz): 20

Target frequency (Hz): 24000

Target time (s): 1

Sweep time (s): 1

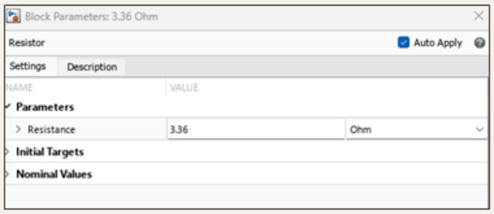
Initial phase (rad): 0

Sample time: 2.0833e-05

Samples per frame: 1

Output data type: Double

Resistor (Re) Input

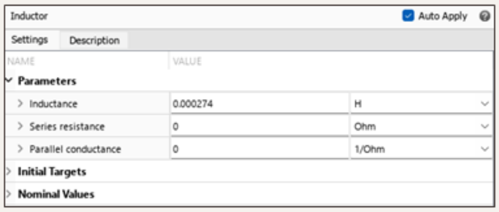


Block Parameters: 3.36 Ohm

Resistor ☑ Auto Apply

NAME	VALUE	
Parameters		
Resistance	3.36	Ohm
Initial Targets		
Nominal Values		

Inductor (Le) Input



NAME	VALUE	
Parameters		
Inductance	0.000274	H
Series resistance	0	Ohm
Parallel conductance	0	1/Ohm
Initial Targets		
Nominal Values		

Goto Block Input

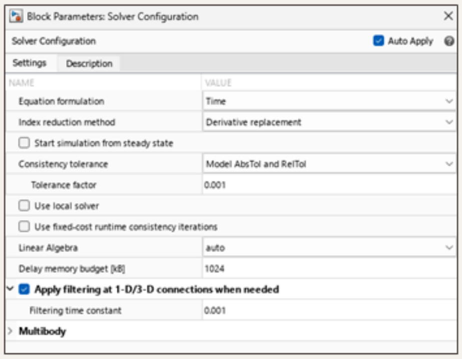
Send signals to From blocks that have the specified tag. If tag visibility is 'scoped', then a Goto Tag Visibility block must be used to define the visibility of the tag. The block icon displays the selected tag name (local tags are enclosed in brackets, [], and scoped tag names are enclosed in braces, {}).

Parameters

Goto tag: A

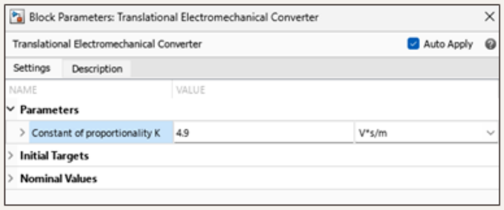
Tag visibility: local

Solver Configuration Input



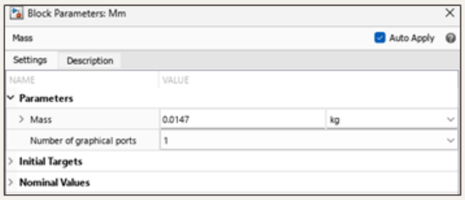
NAME	VALUE
Equation formulation	Time
Index reduction method	Derivative replacement
☐ Start simulation from steady state	
Consistency tolerance	Model AbsTol and RelTol
Tolerance factor	0.001
☐ Use local solver	
☐ Use fixed-cost runtime consistency iterations	
Linear Algebra	auto
Delay memory budget [kB]	1024
☑ **Apply filtering at 1-D/3-D connections when needed**	
Filtering time constant	0.001
Multibody	

Translational Electromechanical Converter Input



NAME	VALUE	
Parameters		
Constant of proportionality K	4.9	V*s/m
Initial Targets		
Nominal Values		

Mass (Mm) Input


Block Parameters: Mm

Mass

Parameters

Mass: 0.0147 kg

Number of graphical ports: 1

Initial Targets

Nominal Values

Translational Spring (1/Cm) Input


Block Parameters: 1/Cm

Translational Spring

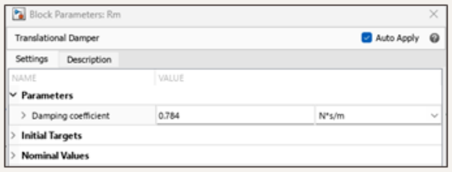
Parameters

Spring rate: 1785.7 N/m

Initial Targets

Nominal Values

Translational Damper (Rm) Input


Block Parameters: Rm

Translational Damper

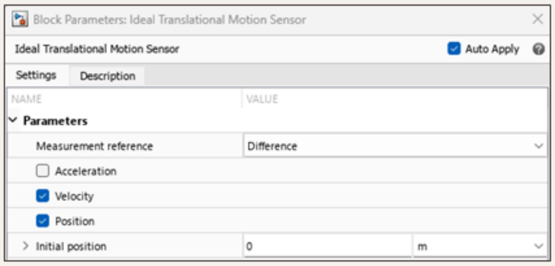
Parameters

Damping coefficient: 0.784 N*s/m

Initial Targets

Nominal Values

Ideal Translational Motion Sensor Input


Block Parameters: Ideal Translational Motion Sensor

Ideal Translational Motion Sensor

Parameters

Measurement reference: Difference

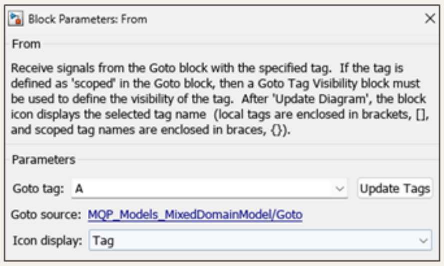
☐ Acceleration

☑ Velocity

☑ Position

Initial position: 0 m

From Block Input


Block Parameters: From

From

Receive signals from the Goto block with the specified tag. If the tag is defined as 'scoped' in the Goto block, then a Goto Tag Visibility block must be used to define the visibility of the tag. After 'Update Diagram', the block icon displays the selected tag name (local tags are enclosed in brackets, [], and scoped tag names are enclosed in braces, {}).

Parameters

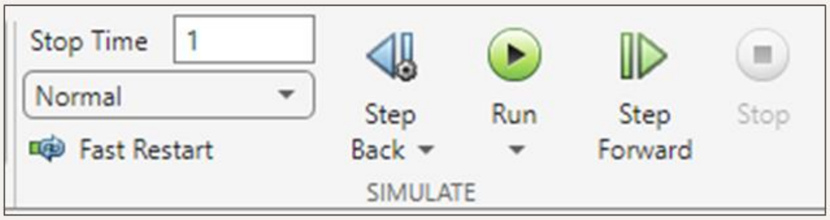
Goto tag: A

Goto source: MQP_Models_MixedDomainModel/Goto

Icon display: Tag

Run the Simulation

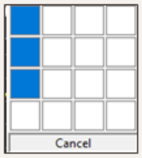
- Make sure Stop time = 1


Stop Time: 1

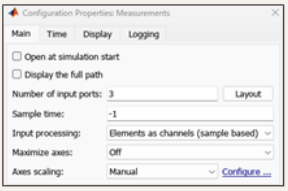
Normal

Final Graph Data

For the correct graph output, make sure you have 3 layouts stacked vertical on top of each other.
- If you do not, go to view tab, click layout, and drag till three blue boxes line up on each other.



The Following are the Scope Settings:


Configuration Properties: Measurements

☐ Open at simulation start

☐ Display the full path

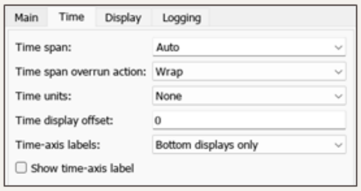
Number of input ports: 3

Sample time: -1

Input processing: Elements as channels (sample based)

Maximize axes: Off

Axes scaling: Manual



Time span: Auto

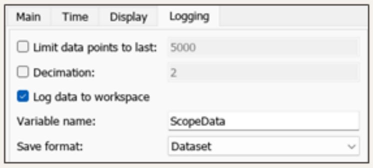
Time span overrun action: Wrap

Time units: None

Time display offset: 0

Time-axis labels: Bottom displays only

☐ Show time-axis label



☐ Limit data points to last: 5000

☐ Decimation: 2

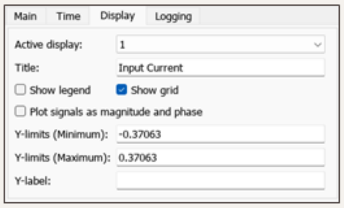
☑ Log data to workspace

Variable name: ScopeData

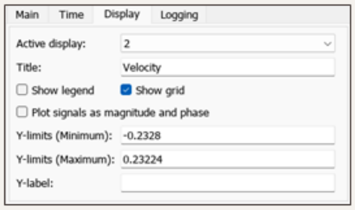
Save format: Dataset

Final Graph Data

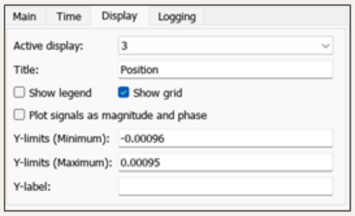
The Scope Display Settings are as follows:



Active display: 1
Title: Input Current
☐ Show legend ☑ Show grid
☐ Plot signals as magnitude and phase
Y-limits (Minimum): -0.37063
Y-limits (Maximum): 0.37063
Y-label:

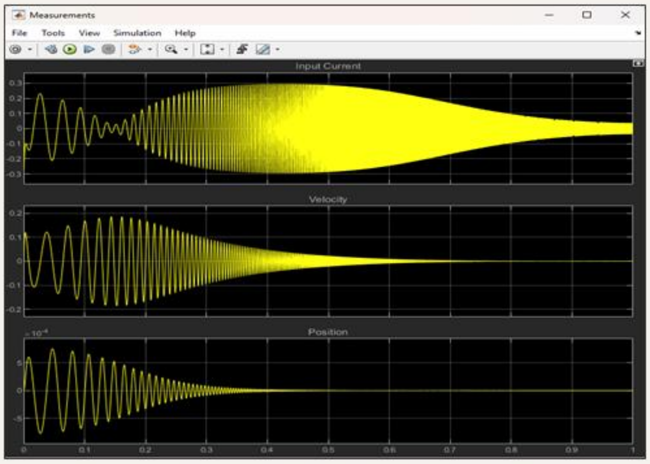


Active display: 2
Title: Velocity
☐ Show legend ☑ Show grid
☐ Plot signals as magnitude and phase
Y-limits (Minimum): -0.2328
Y-limits (Maximum): 0.23224
Y-label:



Active display: 3
Title: Position
☐ Show legend ☑ Show grid
☐ Plot signals as magnitude and phase
Y-limits (Minimum): -0.00096
Y-limits (Maximum): 0.00095
Y-label:

If Everything is Correct, Re-Run the Simulation

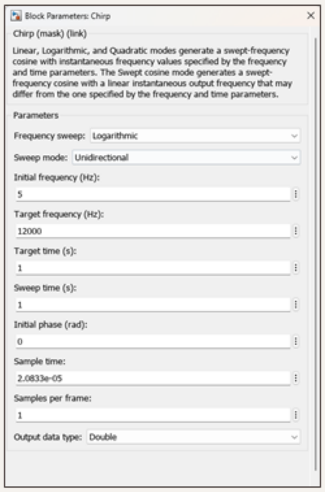


Do your Scope Outputs Look Like This?

Look back, what similarities are there between the graphs? Why?

Let's Change the Frequencies and Observe What Happens.

In all 3 Models, Change the Following



Chirp (mask) (link)

Linear, Logarithmic, and Quadratic modes generate a swept-frequency cosine with instantaneous frequency values specified by the frequency and time parameters. The Swept cosine mode generates a swept-frequency cosine with a linear instantaneous output frequency that may differ from the one specified by the frequency and time parameters.

Parameters

Frequency sweep: Logarithmic

Sweep mode: Unidirectional

Initial frequency (Hz): 5

Target frequency (Hz): 12000

Target time (s): 1

Sweep time (s): 1

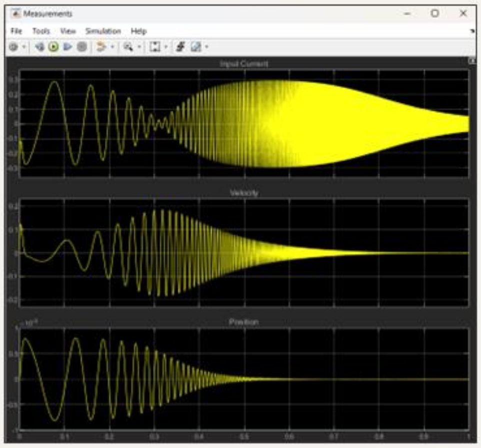
Initial phase (rad): 0

Sample time: 2.0833e-05

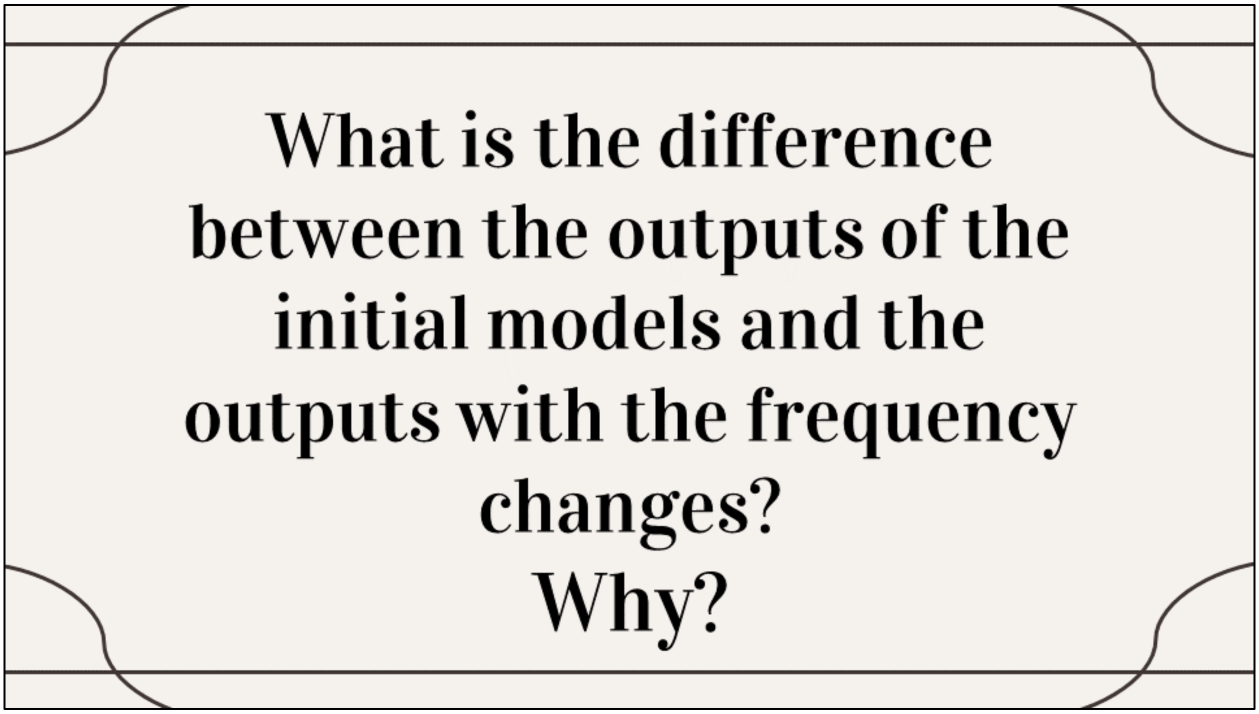
Samples per frame: 1

Output data type: Double

If Everything is Correct, Re-Run the Simulation



Do your Scope Outputs Look Like This?

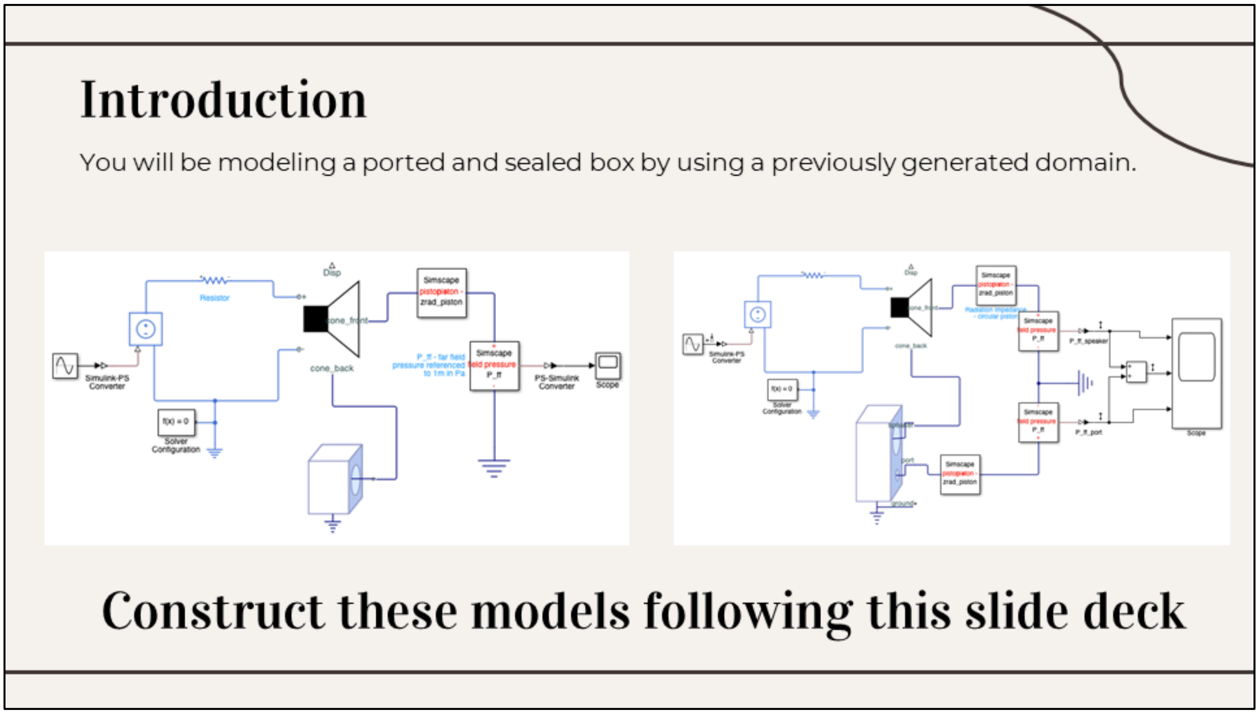


What is the difference between the outputs of the initial models and the outputs with the frequency changes?

Why?

Appendix G – Speakers in Enclosure Modeling Slides

Speakers in Enclosures Simscape



Introduction

You will be modeling a ported and sealed box by using a previously generated domain.

Construct these models following this slide deck

Installing the Domain

Because certain acoustic parts are not properly modeled in basic Matlab, we will need to install a domain where they are modeled.

To start, click the link below

https://www.mathworks.com/matlabcentral/fileexchange/109029-acoustical-domain-for-simscape?s_tid=srchtitle_support_results_3_loudspeaker%2520enclosure

Download the domain

Acoustical Domain for Simscape

Version 1.2.0 (2.64 MB) by Stephen Thompson

This library is an Acoustical Domain for use with Simscape. It follows the conventions used for decades in analog circuit acoustical models.

This library provides an Acoustical Domain for use with Simscape. This domain follows the conventions that have been used for decades with analog circuit models of acoustical systems. An early example of these methods is the book by Olsen (first edition published in 1943). A relevant current reference is Beranek and Mellow. These methods were initially used to model lumped parameter systems, systems that are analogous to simple RLC electrical circuits. Those lumped parameter components are included in the acoustical domain library. An example of the use of these simple components is a simple loudspeaker model. The current version of the acoustical library also includes components beyond the simple lumped elements. These include several kinds of acoustical transmission lines, piezoelectric pieces, and assemblies of simpler components such as approximate models radiation impedance and several models of moving coil speakers.

With version 1.2 of the library, several moving coil loudspeaker components are included, and several enclosures for to mount moving coil drivers have been added.

Requires

MATLAB

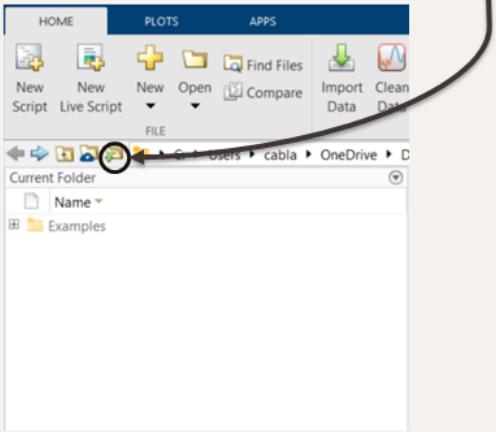
Simulink

Simscape

Frequency domain analysis requires the Control System Toolbox.

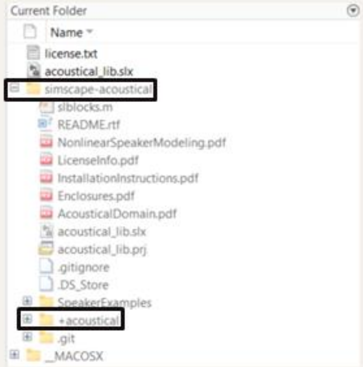
Unzip the folder under downloaded files on your computer.

Open Matlab and select this icon to change the directory



Select the unzipped file to set it as the new directory.

Restart Matlab and then open the "simscape-acoustical" folder and ensure that the "+acoustical" folder is visible.

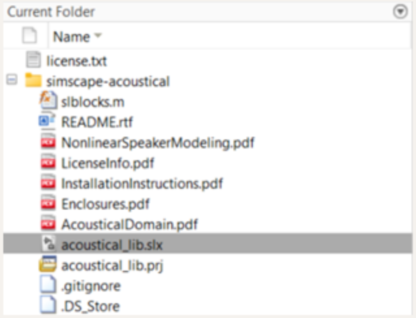


- license.txt
- acoustical_lib.slx
- simscape-acoustical
  - slblocks.m
  - README.rtf
  - NonlinearSpeakerModeling.pdf
  - LicenseInfo.pdf
  - InstallationInstructions.pdf
  - Enclosures.pdf
  - AcousticalDomain.pdf
  - acoustical_lib.slx
  - acoustical_lib.prj
  - .gitignore
  - .DS_Store
  - SpeakerExamples
  - +acoustical
  - .git
- __MACOSX

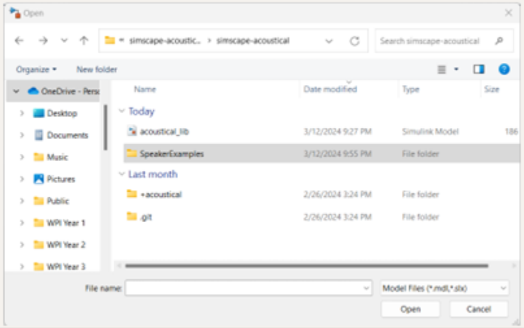
In the Matlab command window enter the following function to build the library for the acoustical elements:

```
>> ssc_build acoustical
Generating Simulink library 'acoustical_lib' in the current directory 'C:\Users\cabla\Downloads\simscape-acoustical_20240110 (2)\simscape-acoustic
```

Double-click the "acoustical_lib.slx" file in the simscape-acoustical folder on Matlab, and then open the sealed box and ported box examples.

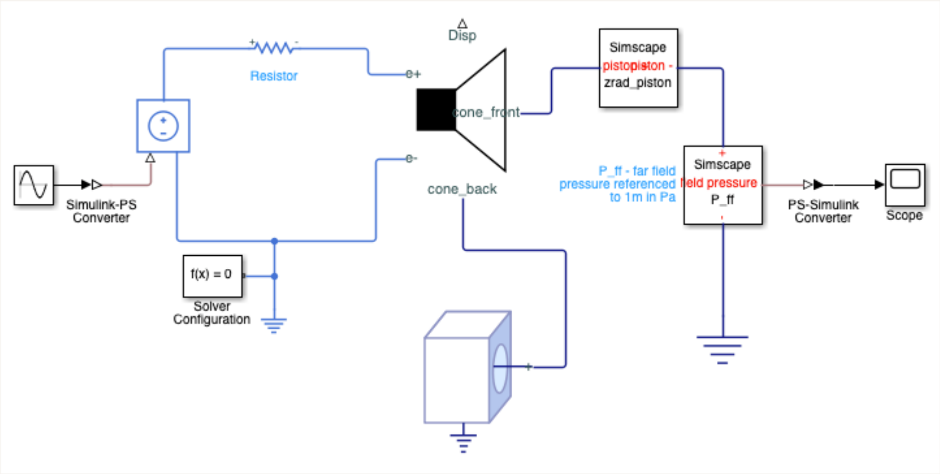


- license.txt
- simscape-acoustical
  - slblocks.m
  - README.rtf
  - NonlinearSpeakerModeling.pdf
  - LicenseInfo.pdf
  - InstallationInstructions.pdf
  - Enclosures.pdf
  - AcousticalDomain.pdf
  - acoustical_lib.slx
  - acoustical_lib.prj
  - .gitignore
  - .DS_Store



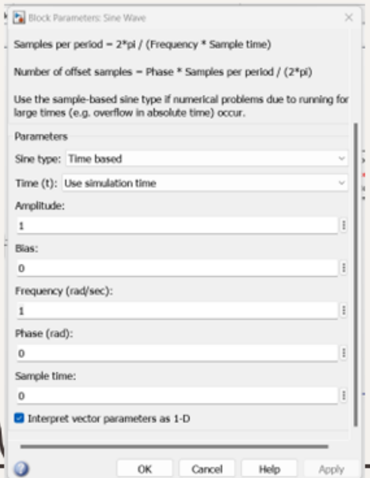
Name	Date modified	Type	Size
Today			
acoustical_lib	3/12/2024 9:27 PM	Simulink Model	186
SpeakerExamples	3/12/2024 9:55 PM	File folder	
Last month			
+acoustical	2/26/2024 3:24 PM	File folder	
.git	2/26/2024 3:24 PM	File folder	

Sealed Box Example Simulink Model:



Values for the parts should be as follows:

Sine Wave



Samples per period = 2*pi / (Frequency * Sample time)

Number of offset samples = Phase * Samples per period / (2*pi)

Use the sample-based sine type if numerical problems due to running for large times (e.g. overflow in absolute time) occur.

Parameters

Sine type: Time based

Time (t): Use simulation time

Amplitude: 1

Bias: 0

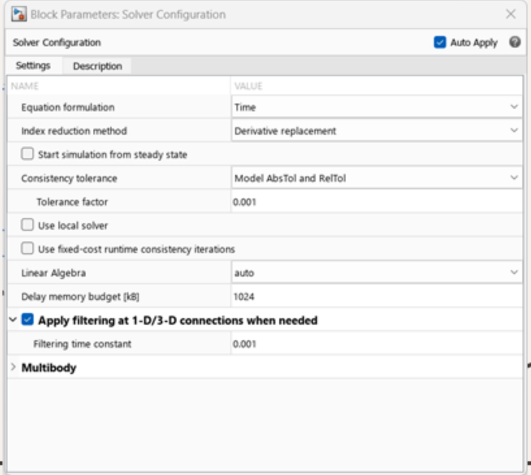
Frequency (rad/sec): 1

Phase (rad): 0

Sample time: 0

☑ Interpret vector parameters as 1-D

Solver Configuration

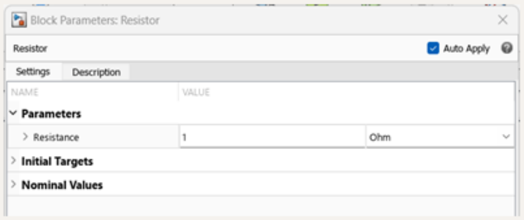


☑ Auto Apply

NAME	VALUE
Equation formulation	Time
Index reduction method	Derivative replacement
☐ Start simulation from steady state	
Consistency tolerance	Model AbsTol and RelTol
Tolerance factor	0.001
☐ Use local solver	
☐ Use fixed-cost runtime consistency iterations	
Linear Algebra	auto
Delay memory budget [kB]	1024
☑ **Apply filtering at 1-D/3-D connections when needed**	
Filtering time constant	0.001
Multibody	

Values for the parts should be as follows:

Resistor

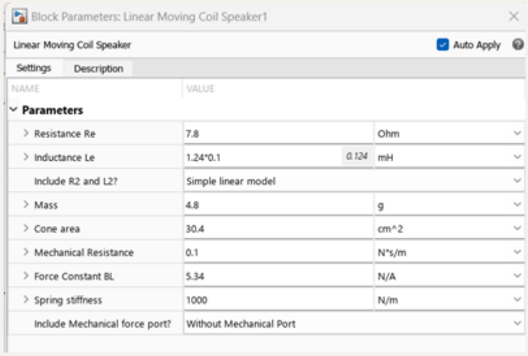


Block Parameters: Resistor

Resistor

NAME	VALUE	
Parameters		
Resistance	1	Ohm
Initial Targets		
Nominal Values		

Linear moving coil speaker 1



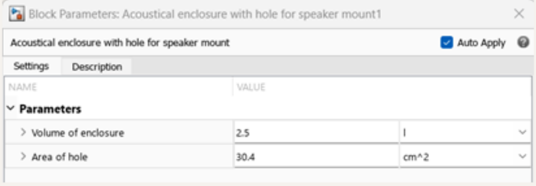
Block Parameters: Linear Moving Coil Speaker1

Linear Moving Coil Speaker

NAME	VALUE	
Parameters		
Resistance Re	7.8	Ohm
Inductance Le	1.24*0.1 (0.124)	mH
Include R2 and L2?	Simple linear model	
Mass	4.8	g
Cone area	30.4	cm^2
Mechanical Resistance	0.1	N*s/m
Force Constant BL	5.34	N/A
Spring stiffness	1000	N/m
Include Mechanical force port?	Without Mechanical Port	

Values for the parts should be as follows:

Enclosure with hole

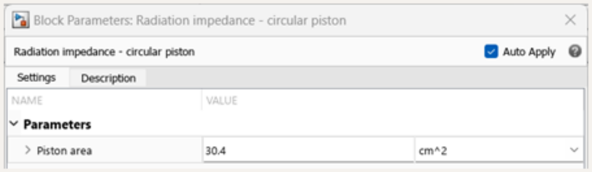


Block Parameters: Acoustical enclosure with hole for speaker mount1

Acoustical enclosure with hole for speaker mount

NAME	VALUE	
Parameters		
Volume of enclosure	2.5	l
Area of hole	30.4	cm^2

Radiation Impedance



Block Parameters: Radiation impedance - circular piston

Radiation impedance - circular piston

NAME	VALUE	
Parameters		
Piston area	30.4	cm^2

Far field pressure reference

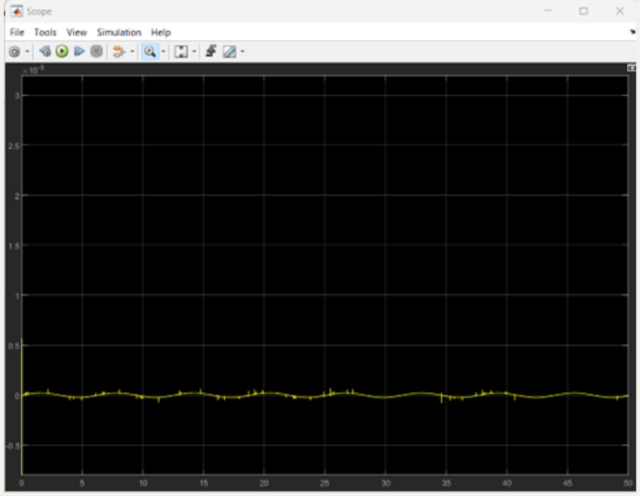
Block Parameters: P_ff - far field pressure referenced to 1m in Pa

P_ff - far field pressure referenced to 1m in Pa

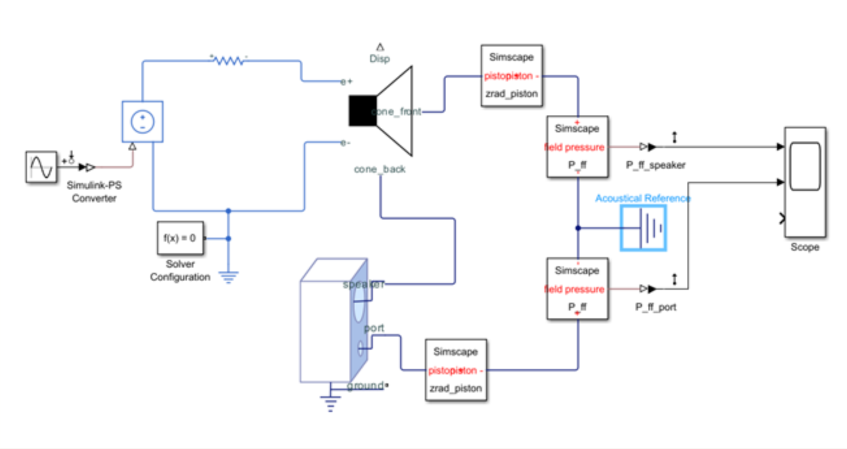
NAME	VALUE	
Parameters		
dist	1	m
Initial Targets		
Nominal Values		

Using the values provided through the domain, run the circuit after setting the stop time to 10 seconds.

The produced graph from selecting the scope icon should be identical to the one shown:

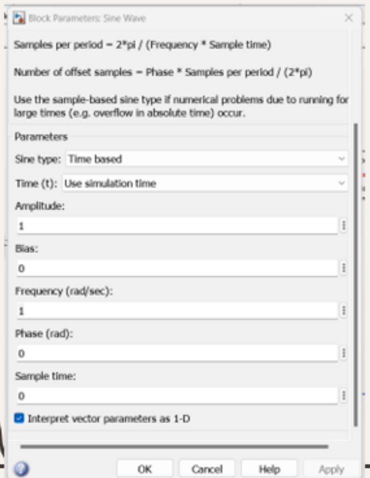


Ported Box Example Simulink Model:



Values for the parts should be as follows:

Sine Wave



Block Parameters: Sine Wave

Samples per period = 2*pi / (Frequency * Sample time)

Number of offset samples = Phase * Samples per period / (2*pi)

Use the sample-based sine type if numerical problems due to running for large times (e.g. overflow in absolute time) occur.

Parameters

Sine type: Time based

Time (t): Use simulation time

Amplitude: 1

Bias: 0

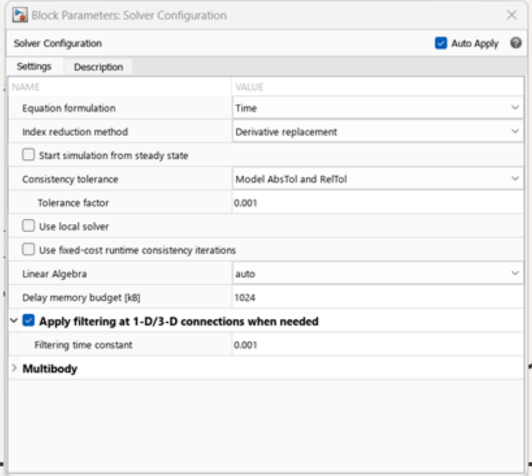
Frequency (rad/sec): 1

Phase (rad): 0

Sample time: 0

☑ Interpret vector parameters as 1-D

Solver Configuration



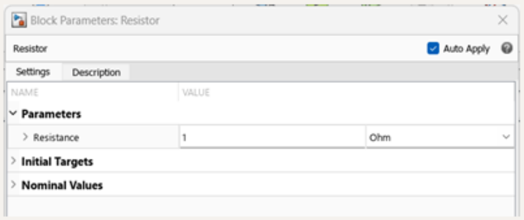
Block Parameters: Solver Configuration

Solver Configuration ☑ Auto Apply

NAME	VALUE
Equation formulation	Time
Index reduction method	Derivative replacement
☐ Start simulation from steady state	
Consistency tolerance	Model AbsTol and RelTol
Tolerance factor	0.001
☐ Use local solver	
☐ Use fixed-cost runtime consistency iterations	
Linear Algebra	auto
Delay memory budget [kB]	1024
☑ **Apply filtering at 1-D/3-D connections when needed**	
Filtering time constant	0.001
Multibody	

Values for the parts should be as follows:

Resistor

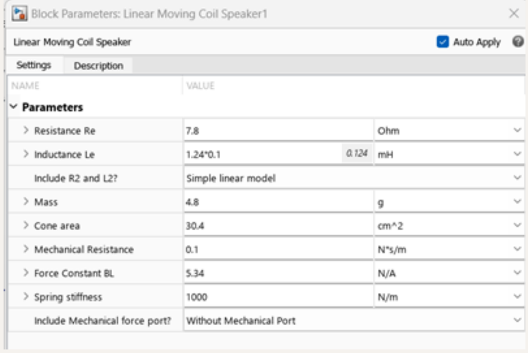


Block Parameters: Resistor

Resistor ☑ Auto Apply

NAME	VALUE	
Parameters		
Resistance	1	Ohm
Initial Targets		
Nominal Values		

Linear moving coil speaker 1



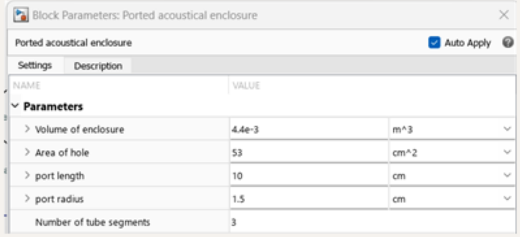
Block Parameters: Linear Moving Coil Speaker1

Linear Moving Coil Speaker ☑ Auto Apply

NAME	VALUE	
Parameters		
Resistance Re	7.8	Ohm
Inductance Le	1.24*0.1 0.124	mH
Include R2 and L2?	Simple linear model	
Mass	4.8	g
Cone area	30.4	cm^2
Mechanical Resistance	0.1	N*s/m
Force Constant BL	5.34	N/A
Spring stiffness	1000	N/m
Include Mechanical force port?	Without Mechanical Port	

Values for the parts should be as follows:

Ported acoustical enclosure



Block Parameters: Ported acoustical enclosure

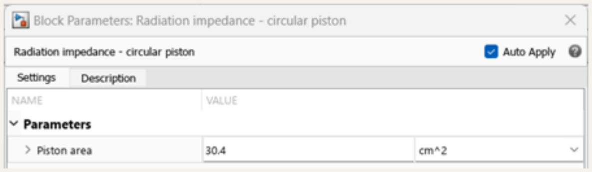
Ported acoustical enclosure

☑ Auto Apply

Settings | Description

NAME	VALUE	
Parameters		
Volume of enclosure	4.4e-3	m^3
Area of hole	53	cm^2
port length	10	cm
port radius	1.5	cm
Number of tube segments	3	

Both Radiation Impedances



Block Parameters: Radiation impedance - circular piston

Radiation impedance - circular piston

☑ Auto Apply

Settings | Description

NAME	VALUE	
Parameters		
Piston area	30.4	cm^2

Both Far field pressure references

Block Parameters: P_ff - far field pressure referenced to 1m in Pa

P_ff - far field pressure referenced to 1m in Pa

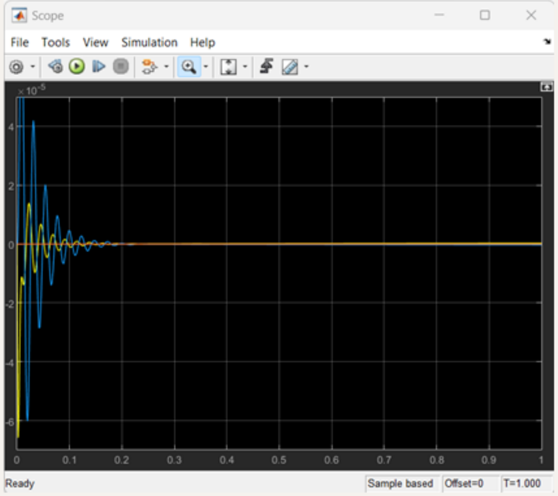
☑ Auto Apply

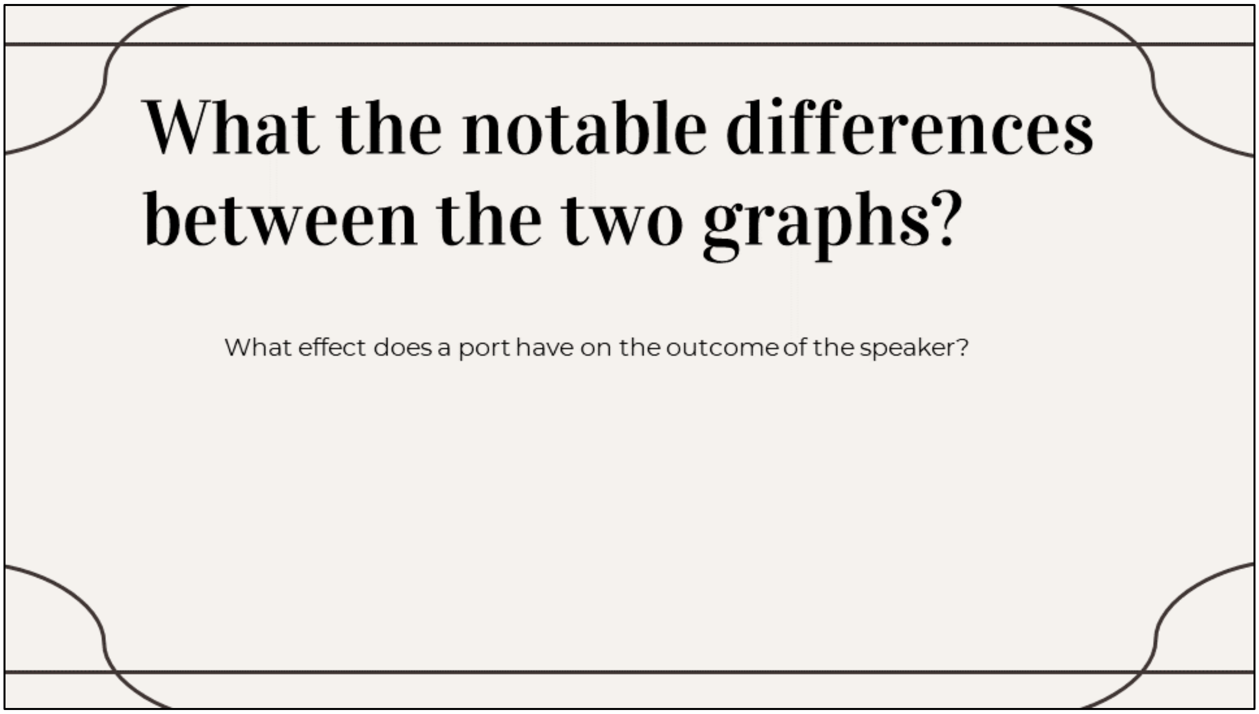
Settings | Description

NAME	VALUE	
Parameters		
dist	1	m
Initial Targets		
Nominal Values		

Use the values provided through the domain and then run the circuit after setting the stop time to 1 second.

The produced graph from selecting the scope icon should be identical to the one shown:





What the notable differences between the two graphs?

What effect does a port have on the outcome of the speaker?